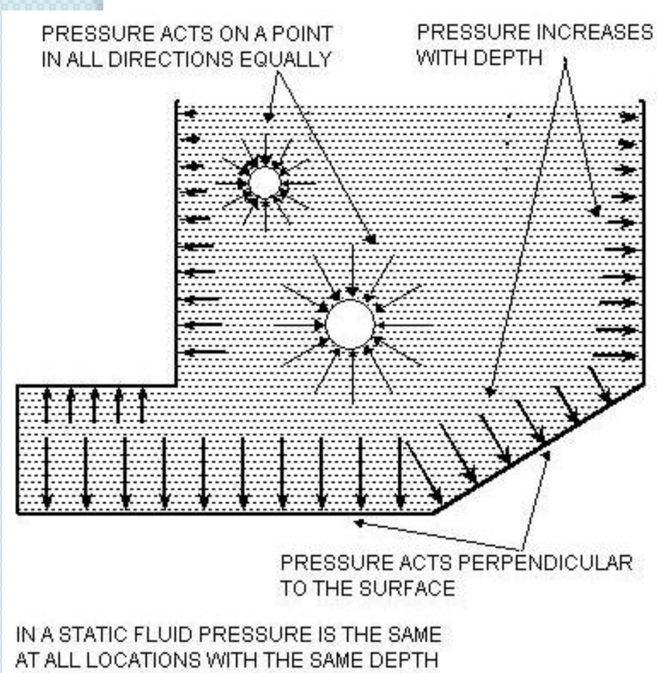
EQUILIBRIUM OF RIGID BODIES IN TWO DIMENSIONS

If the resultant of all external forces acting on a rigid body is zero, then the body is said to be in **equilibrium**. Therefore, in order for the rigid body to be in equilibrium, both the resultant force and the resultant couple must be zero.

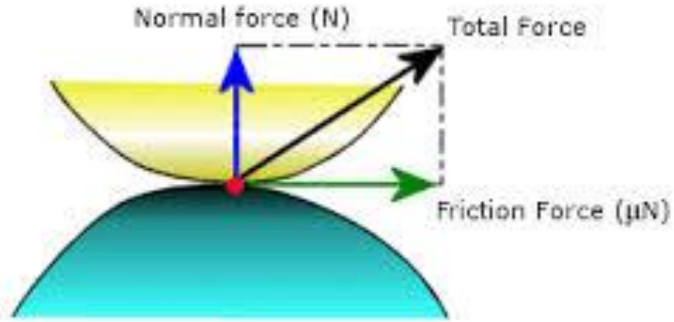
$$\vec{R} = \Sigma\vec{F} = R_x\vec{i} + R_y\vec{j} + R_z\vec{k} = 0$$

$$\Sigma\vec{M} = \Sigma M_x\vec{i} + \Sigma M_y\vec{j} + \Sigma M_z\vec{k} = 0$$

Fluid pressure acting to the wall of a water tank or a force exerted by the tire of a truck to the road is all external forces. The weight of a body is also an external force.

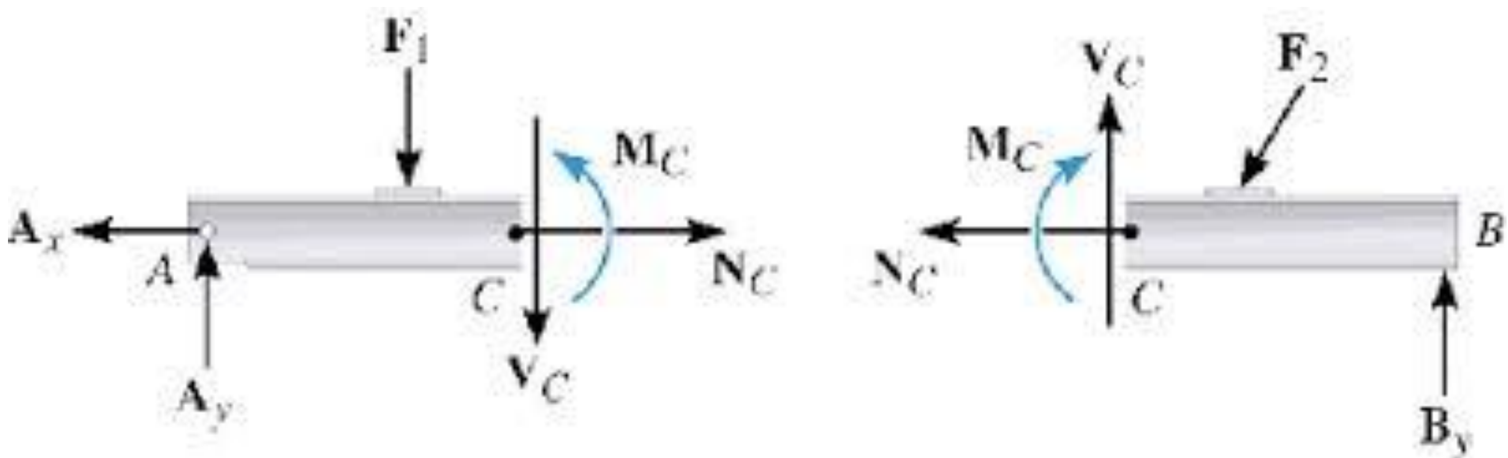


Forces and moments acting on a rigid body could be external forces/moments or internal forces/moments.



Forces acting from one body to another by direct physical contact or from the Earth are examples of external forces.

Internal forces, on the other hand, keep the particles which constitute the body intact. Since internal forces occur in pairs that are equal in magnitude opposite in direction, they are not considered in the equilibrium of rigid bodies.



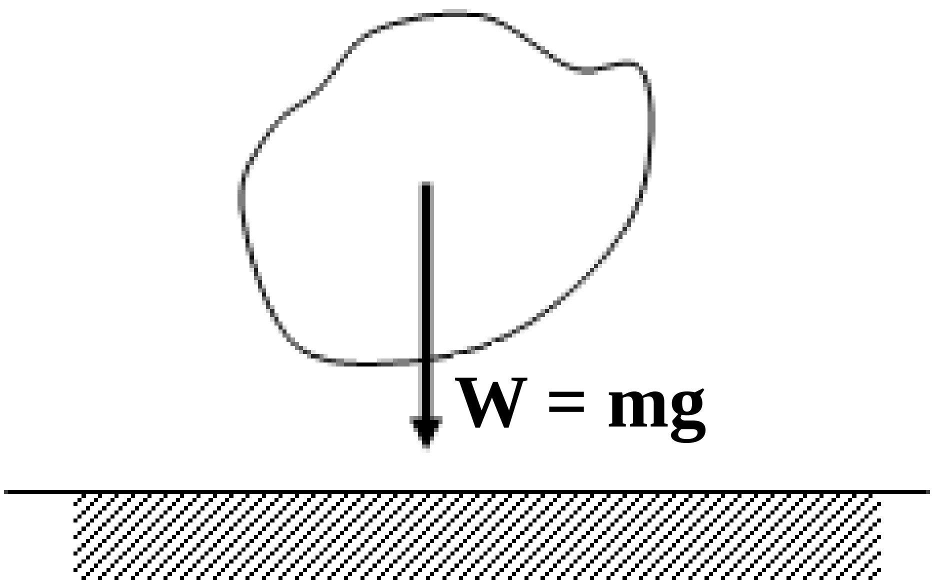
When examining the equilibrium of rigid bodies, it is extremely important to consider **all** the forces acting **on** the body and keep out all the other forces that are not directly exerted on the body. Therefore, the first step in the analysis of the equilibrium of rigid bodies must be to draw the "**free body diagram**" of the body in question. Since the rigid body is considered by its real shape and dimensions, the correct placement of the forces on the diagram is of great importance.

Common Support / Connection Element Types in Two Dimensional Analysis

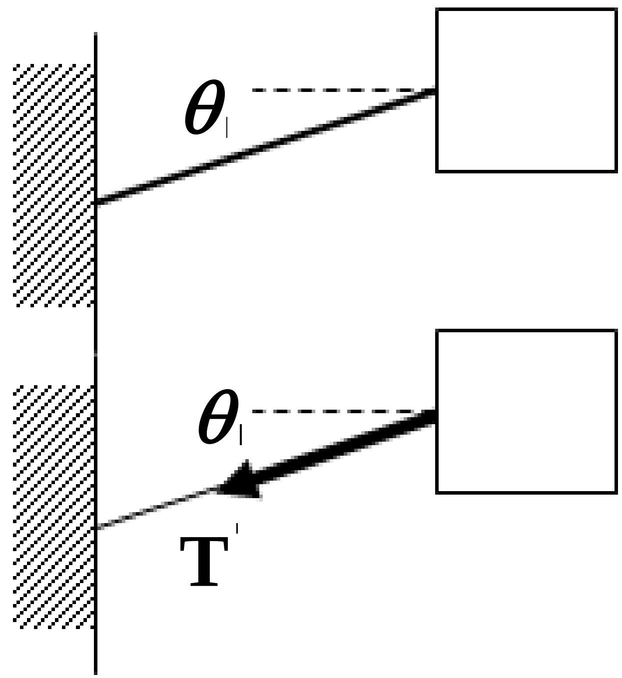
In rigid bodies subjected to two dimensional force systems, the forces exerted from supports and connection elements are shown in the free body diagram as follows:

It should be kept in mind that reaction will occur along the direction in which the motion of the body is restricted.

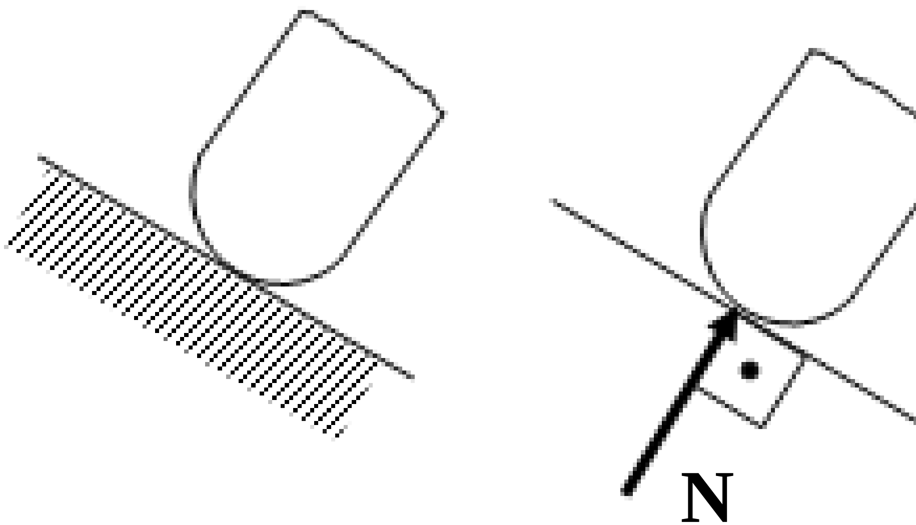
Gravity Force



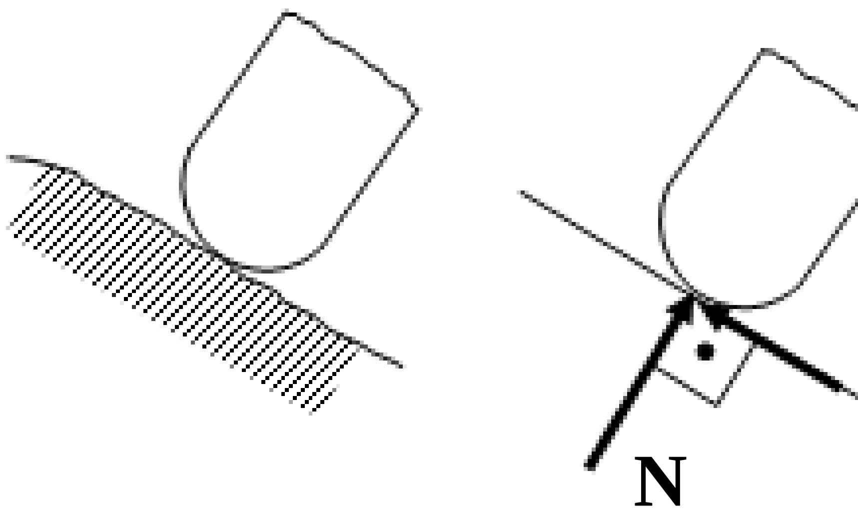
Forces in Cables, Strings, Chains, Ropes, Belts and Cords



Contact with a Smooth Surface

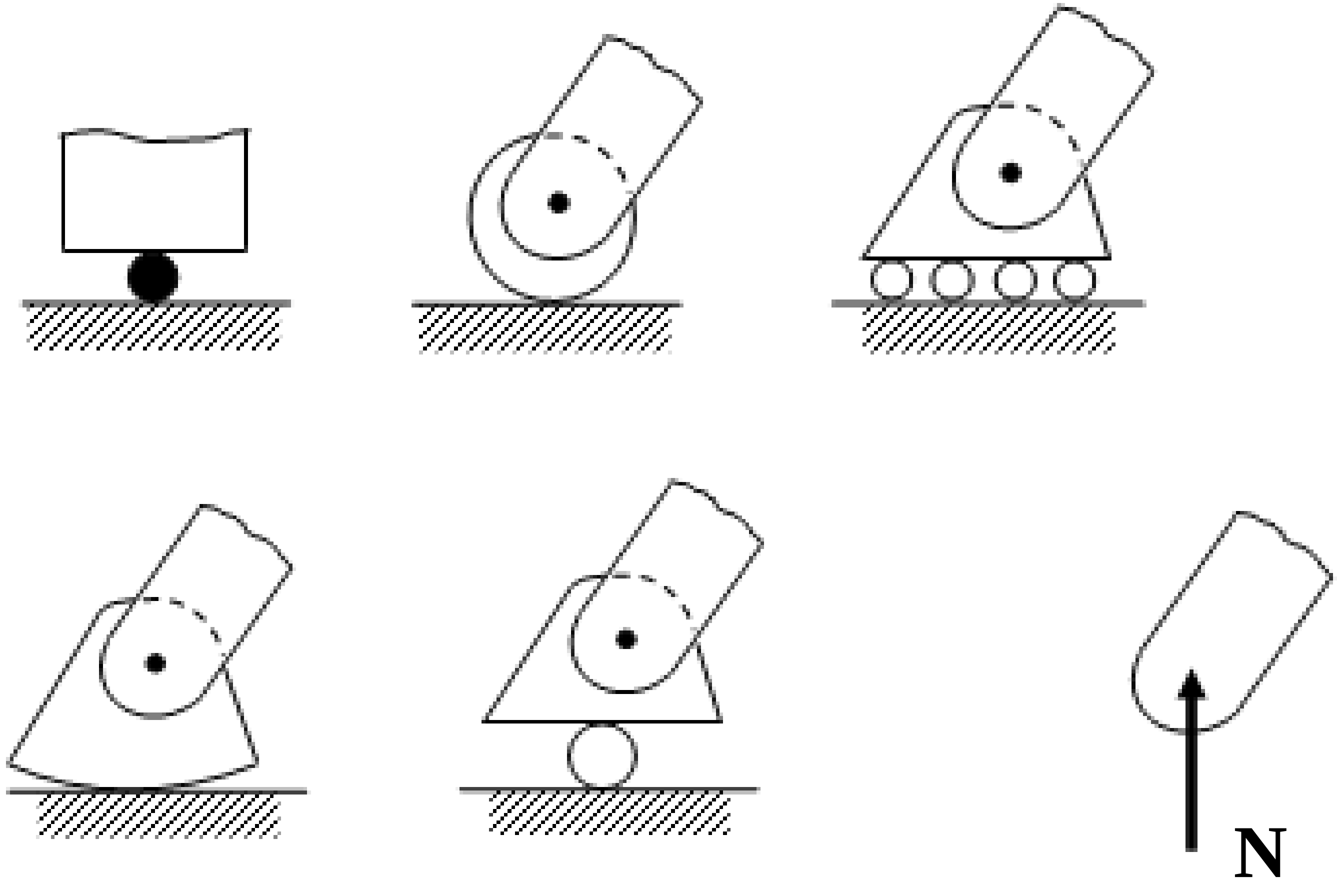


Contact with a Rough Surface

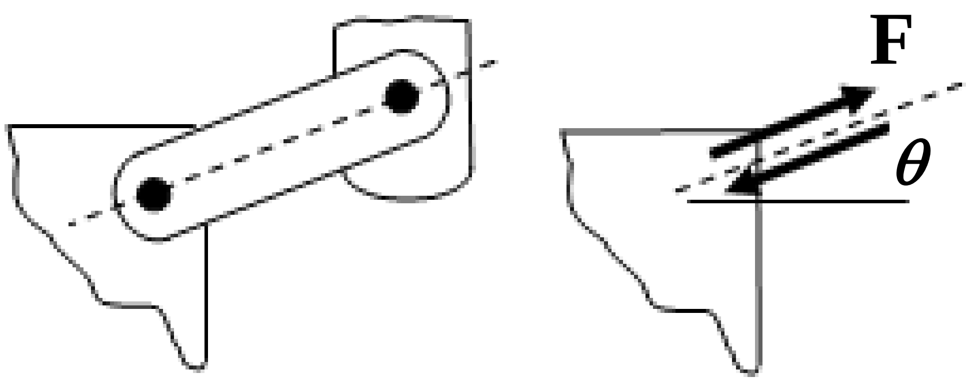


$\mathbf{F_f}$ (occurs in the direction opposite to the pending motion)

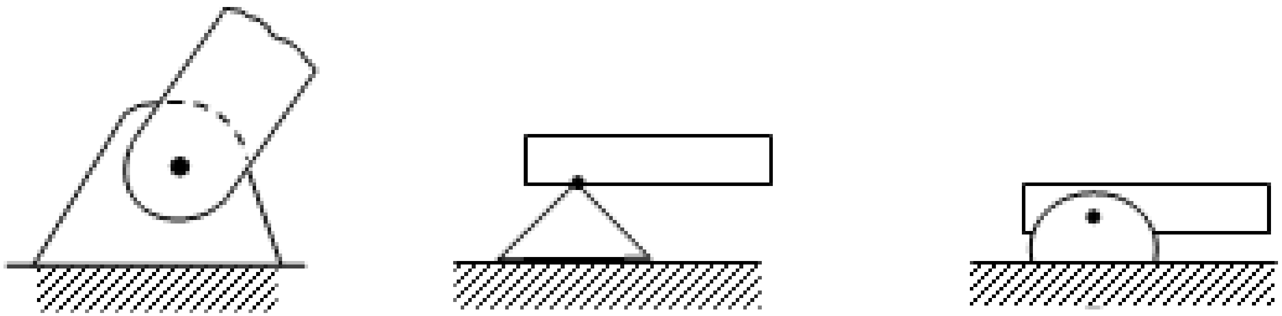
Roller Support or Sliding Support

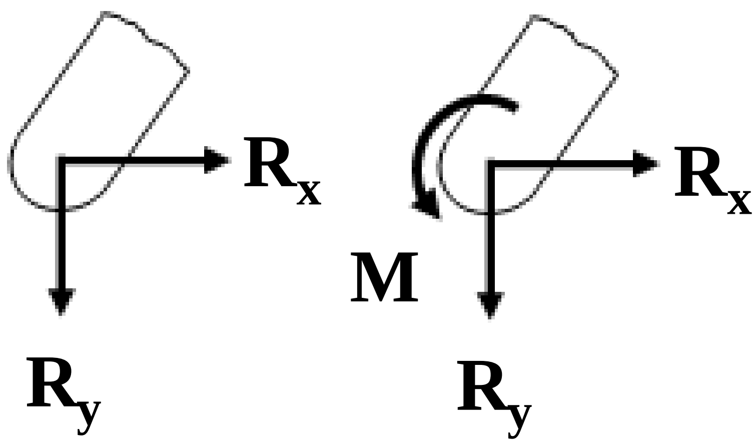


Rigid Link



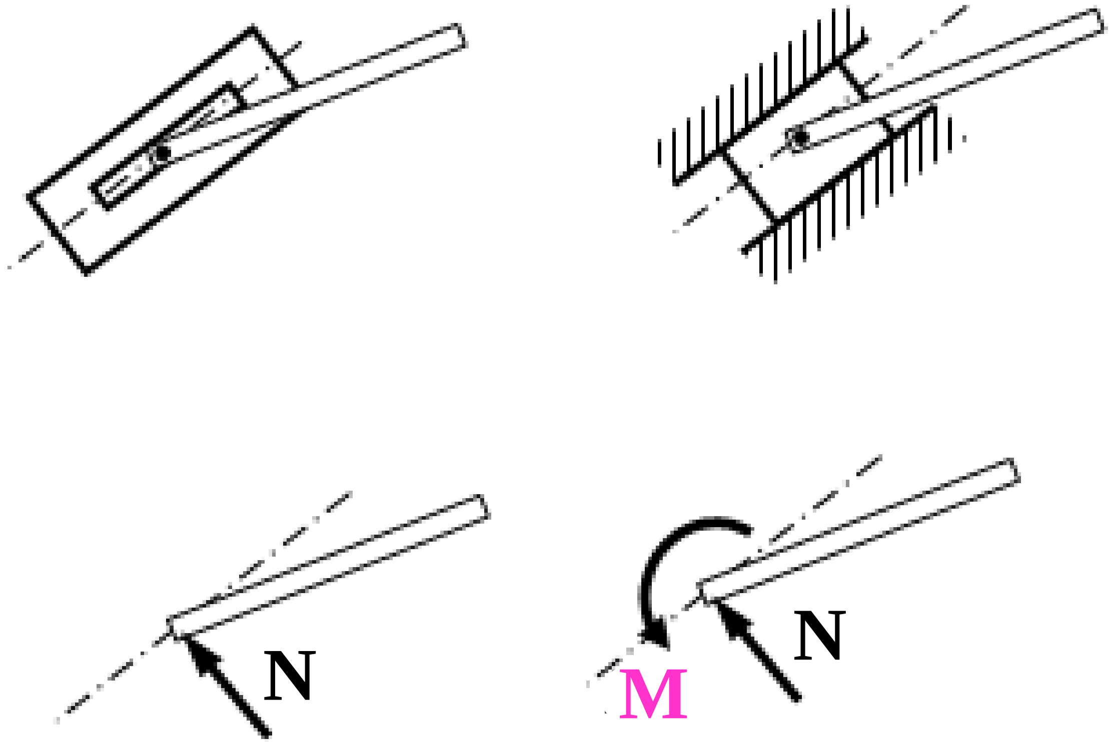
Smooth Pin





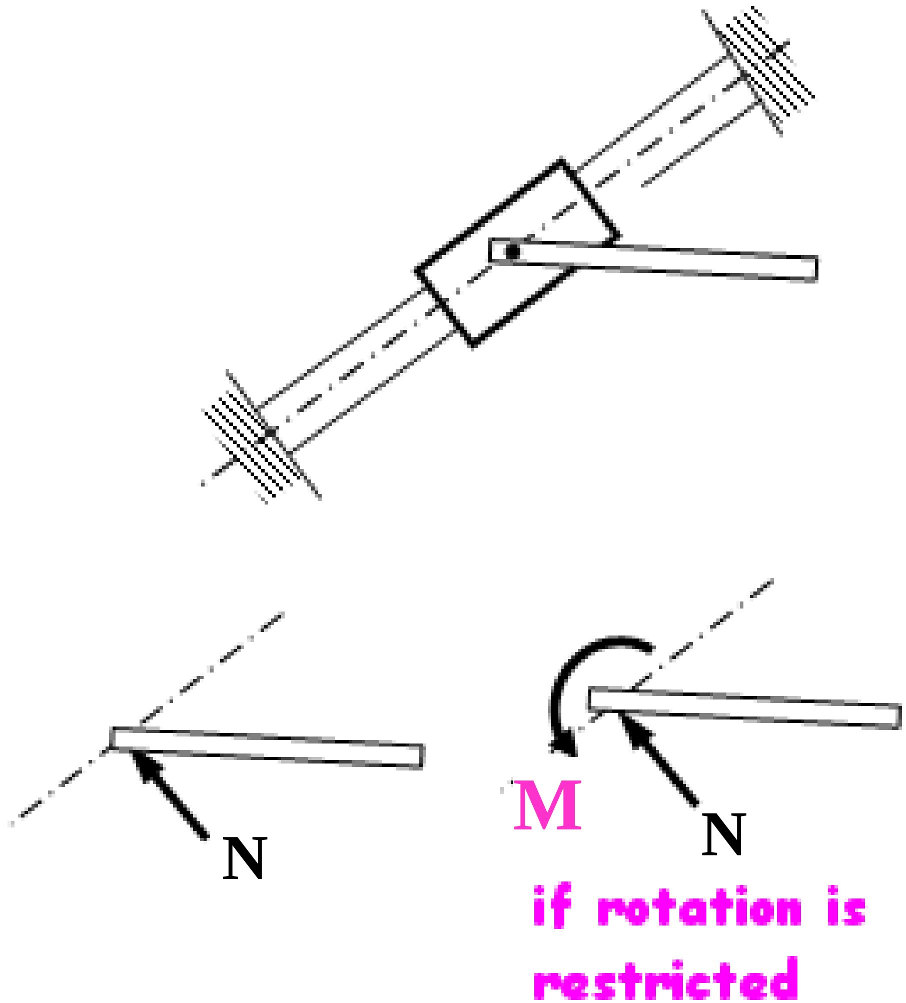
if rotation is restricted

Pin or Block in Smooth Slot

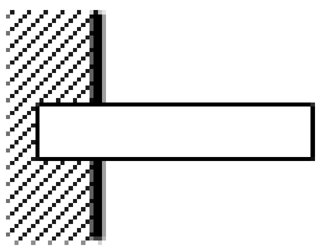


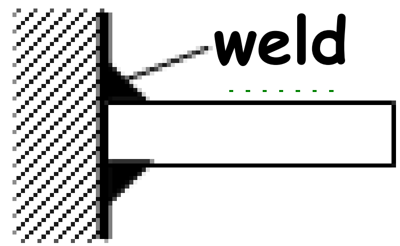
if rotation is restricted

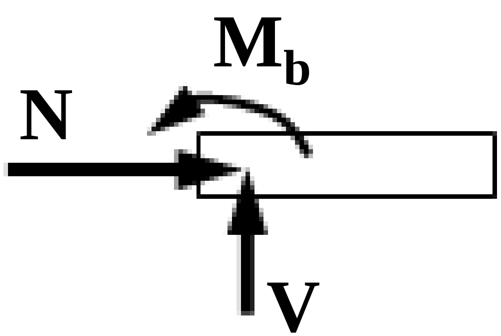
Collar on Smooth Bar



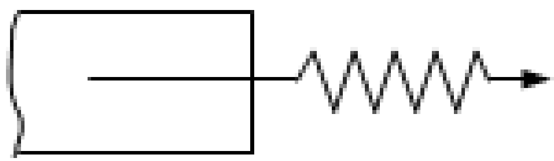
Fixed Support

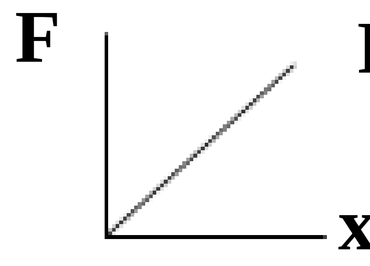






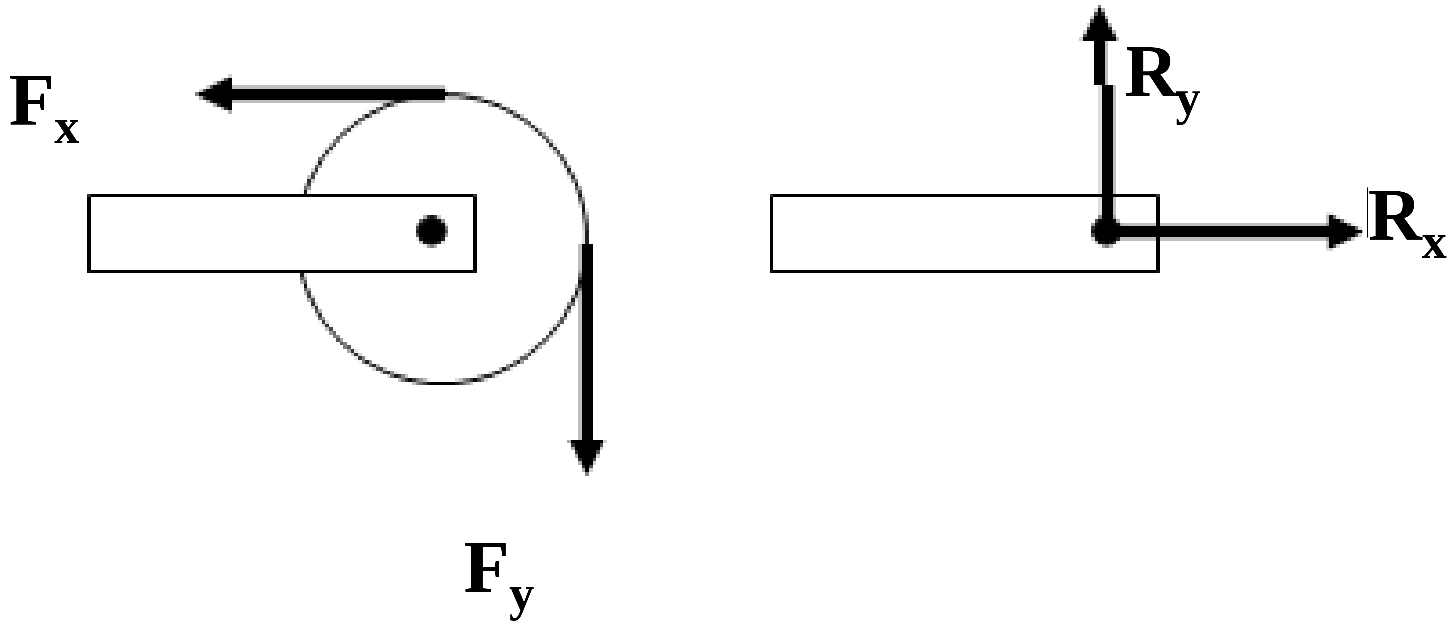
Spring Force





$$F = kx$$

Ideal Pulley



Equations of Equilibrium in Two Dimensional Case

If all the forces acting on the rigid body are planar and all the couples are perpendicular to the plane of the body, equations of equilibrium become two dimensional.

$$\vec{R} = \Sigma\vec{F} = R_x\vec{i} + R_y\vec{j} = 0 \qquad R_x = \Sigma F_x = 0 \qquad R_y = \Sigma F_y = 0$$

$$\Sigma\vec{M} = \Sigma M_z\vec{k} = 0$$

or in scalar form,

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

At most three unknowns can be determined.

Alternative Equations of Equilibrium

In two dimensional problems, in alternative to the above set of equations, two more sets of equations can be employed in the solution of problems.

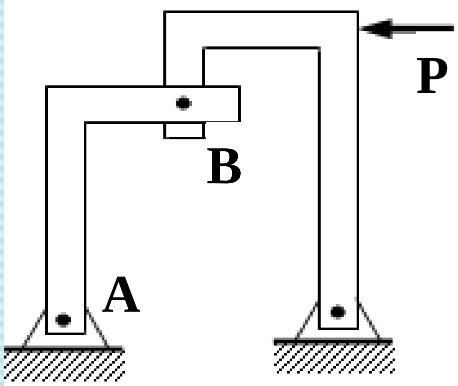
$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

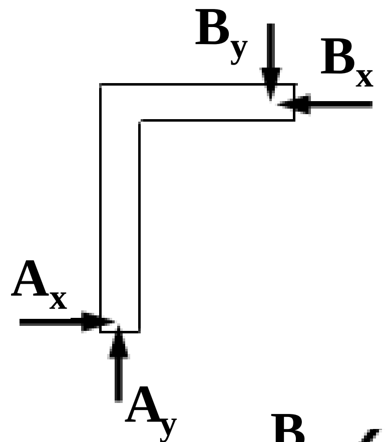
$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

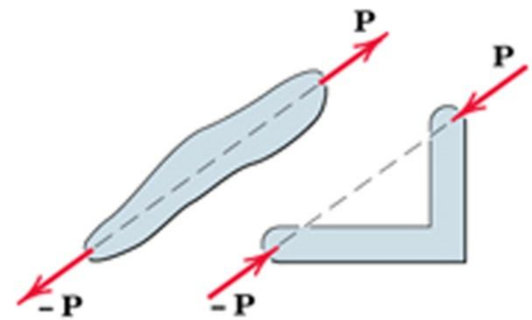
Points A, B and C in the latter set cannot lie along the same line, if they do, trivial equations will be obtained.

Two-Force Member

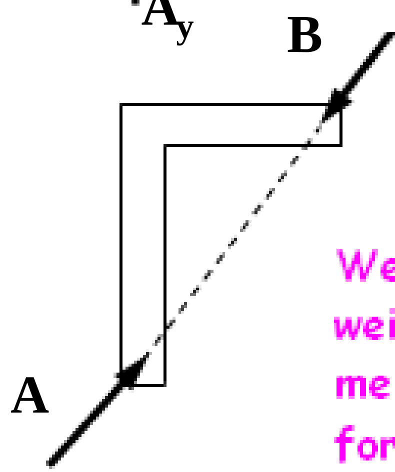
Members which are subjected to only two forces are named as "**two force members**". Forces acting on these members are equal in magnitude, opposite in direction and are directed along the line joining the two points where the forces are applied.





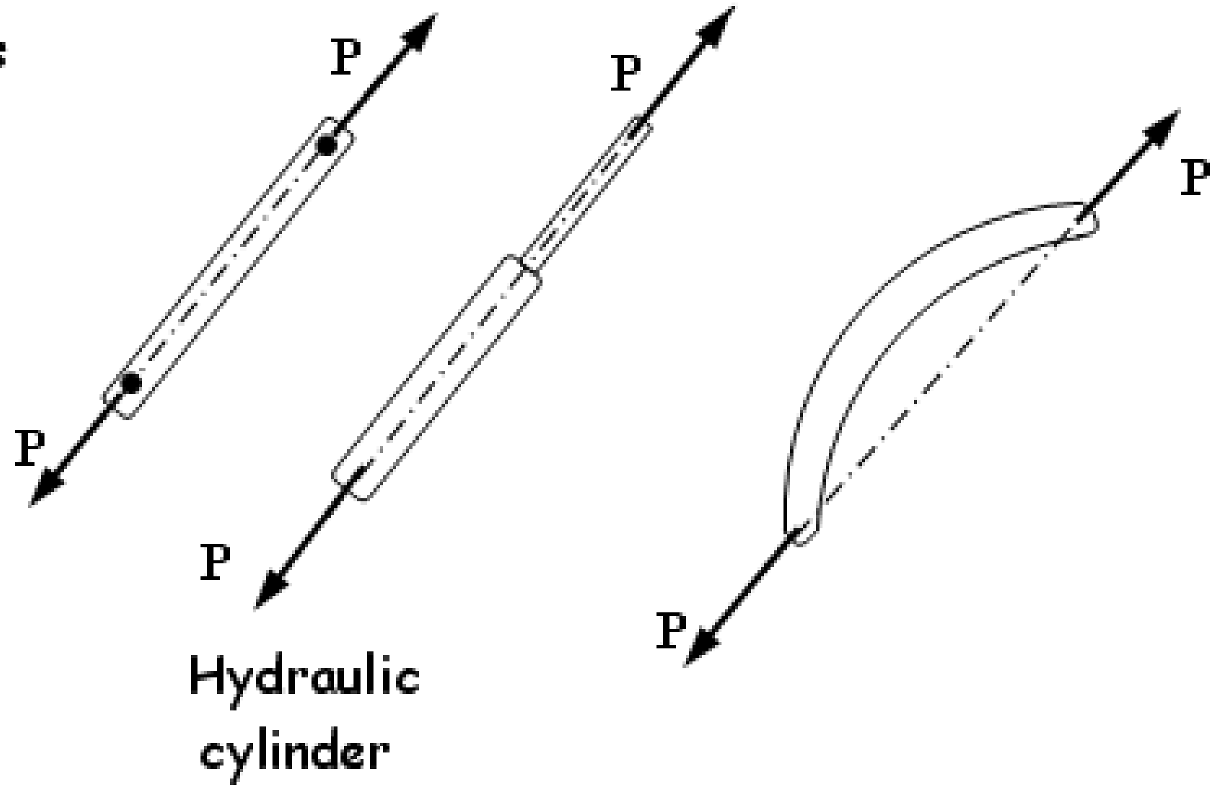


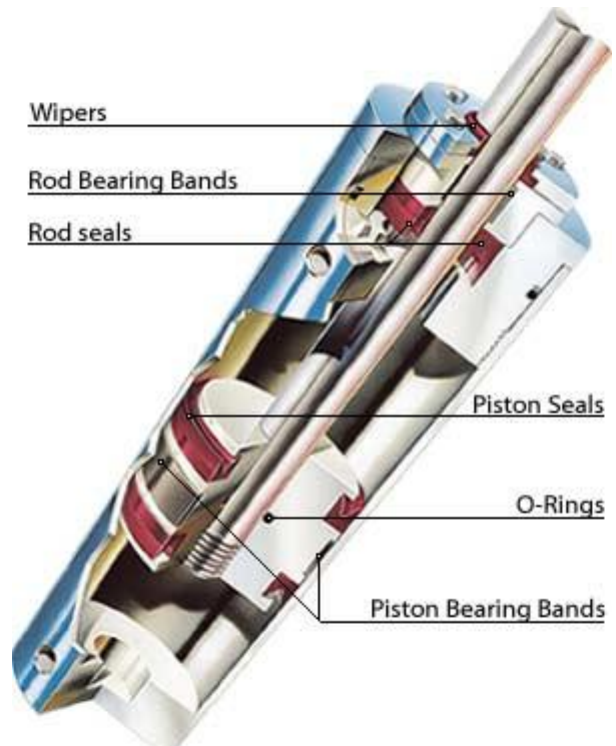
Two-force members



Weight is neglected. If weight is considered, the member will not be a two force member!

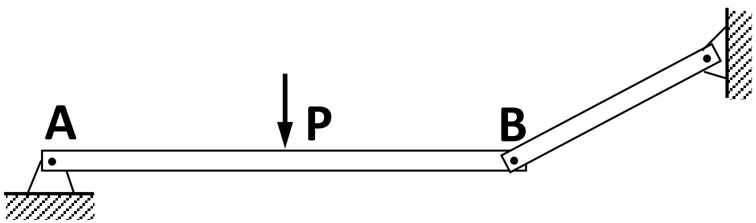
Examples of two force members

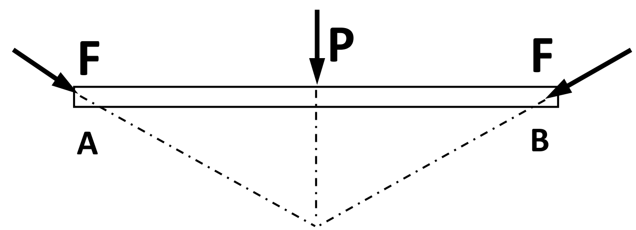


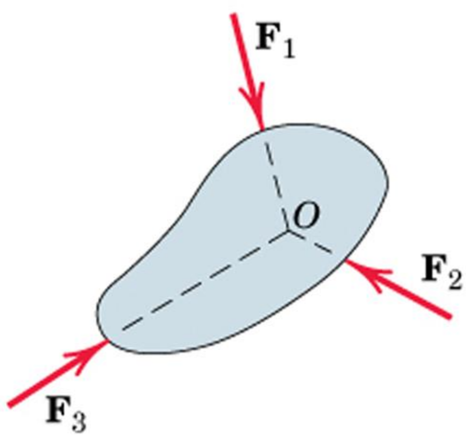

Wipers
Rod Bearing Bands
Rod seals
Piston Seals
O-Rings
Piston Bearing Bands

Three-Force Member

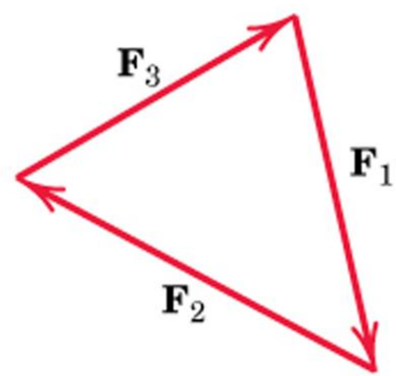
In rigid bodies acted on by only three forces, the lines of action of the forces must be concurrent; otherwise the body will rotate about the intersection point of the two forces due to the third force which is not concurrent. If the forces acting on the body are parallel, then the point of concurrency is assumed to be in infinity.







(*a*) Three-force member



(*b*) Closed polygon satisfies $\Sigma\mathbf{F} = \mathbf{0}$

FREE BODY DIAGRAM – FBD

A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies. A system may be a single body or a combination of connected bodies. Once we decide which body or combination of bodies to analyze, we then treat this body or combination as a single body isolated from all surrounding bodies. This isolation is accomplished by means of the **free body diagram**, which is a schematic representation of the isolated system treated as a single body.

The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. If appreciable body forces are present, such as gravitational attraction (weight) or magnetic attraction, then these forces must also be shown on the free body diagram of the isolated system.

Internal forces acting on the body are not considered since according to Newton's third law of action – reaction, they will cancel out. The principle of transmissibility permits the treatment of forces as sliding vectors as far as their external effects on a rigid body are concerned.

The procedure for drawing a free body diagram which isolates a body or system consists of the following steps:

1) If there exists, identify the two force members in the problem.

2) Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

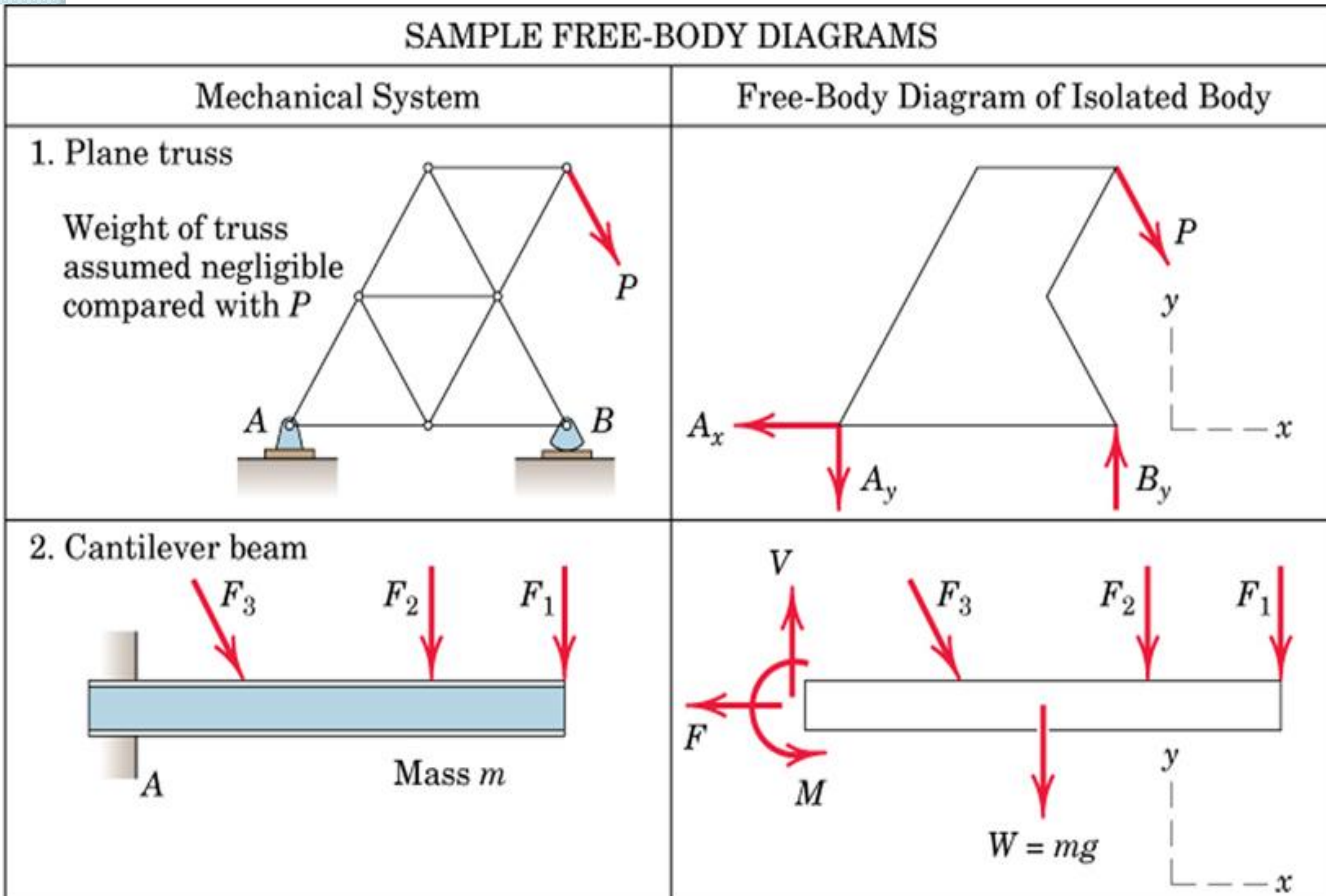
3) Isolate the chosen system by drawing a diagram which represents its *complete external boundary*. This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed. This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.

4) If not given with the problem, select a coordinate system which appropriately suits with the given forces and/or dimensions. If necessary show some of the dimensions on the FBD.

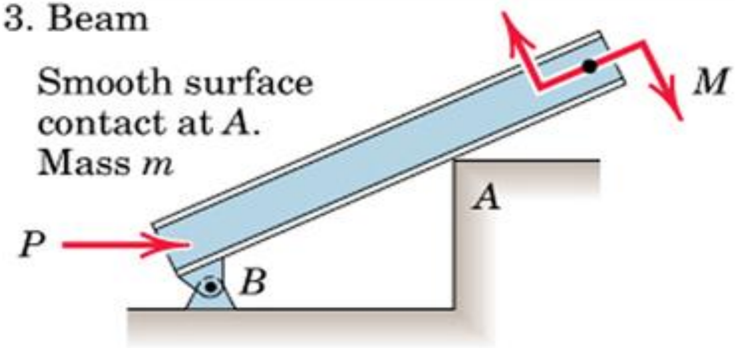
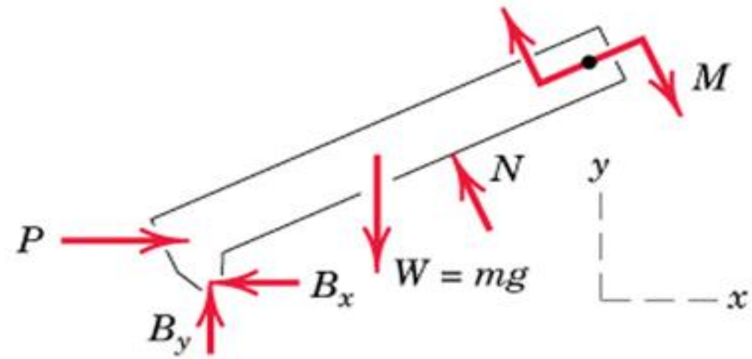
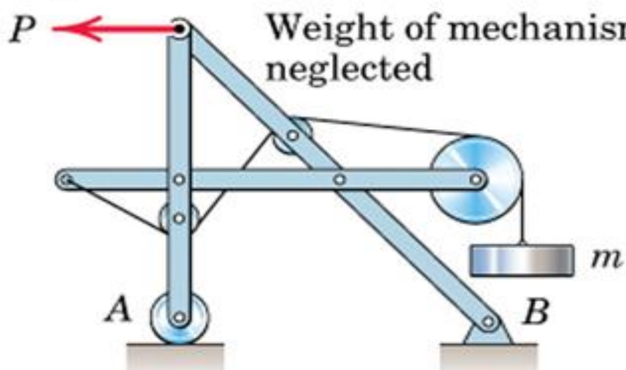
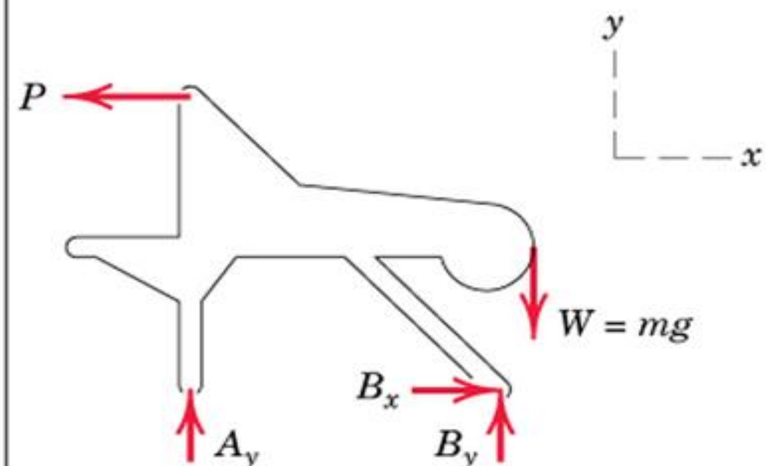
5) Identify all forces which act on the isolated system applied by removing the contacting or attracting bodies, and represent them in their proper positions on the diagram. Represent all known forces by vector arrows, each with its proper magnitude, direction and sense. Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by a letter. If the sense of the vector is also unknown, you must arbitrarily assign a sense.

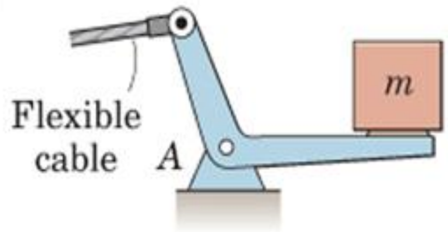
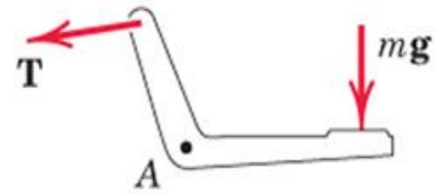
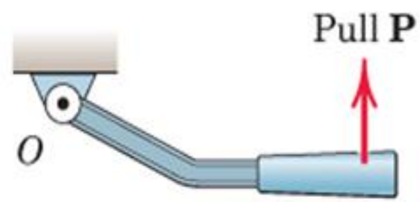
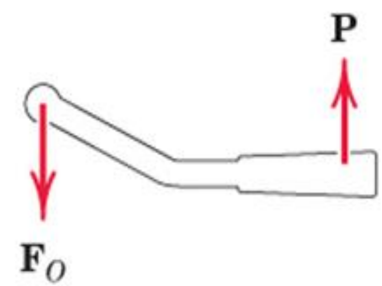
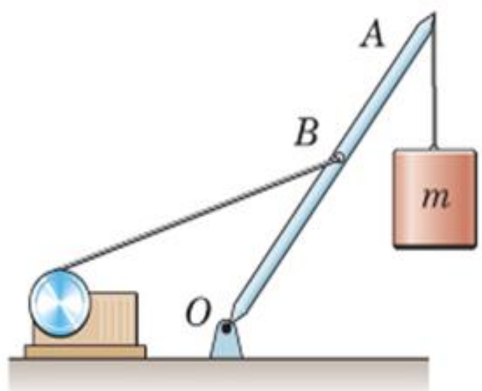
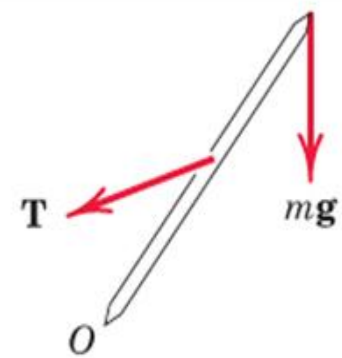
The subsequent calculations with the equilibrium equations will yield a positive quantity if the correct sense was assumed and a negative quantity if the incorrect sense was assumed. Instead of a force with unknown sense, its appropriate components can be used. In this way, there will be two forces with known directions but unknown magnitudes.

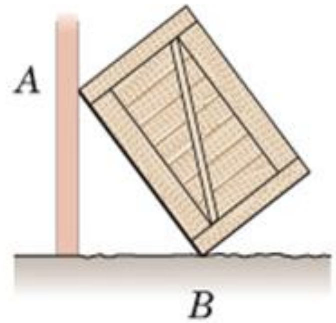
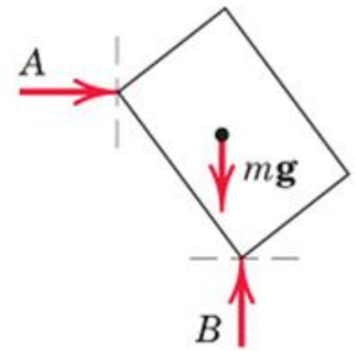
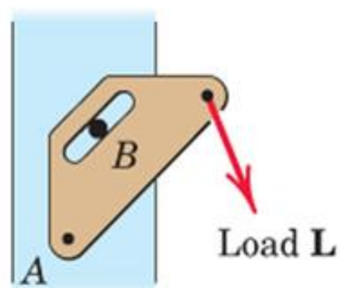
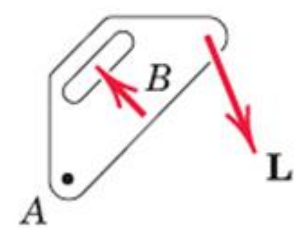
6) Write the equations of equilibrium and solve for the unknowns. Check for dimensional homogeneity.

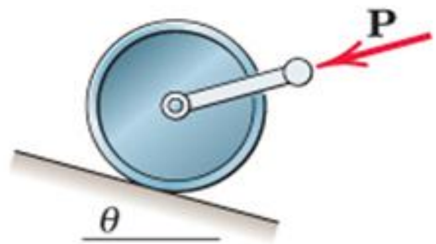
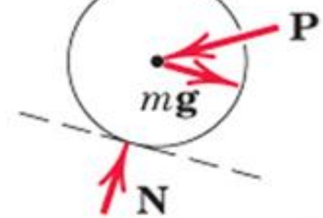
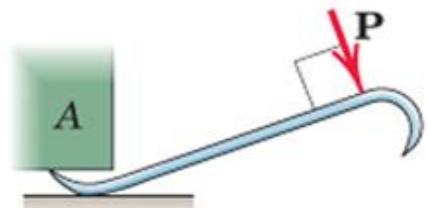
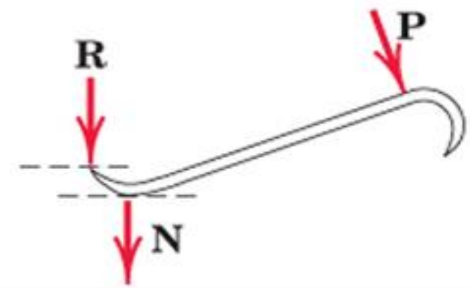
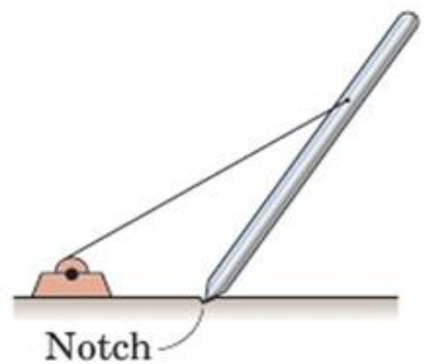
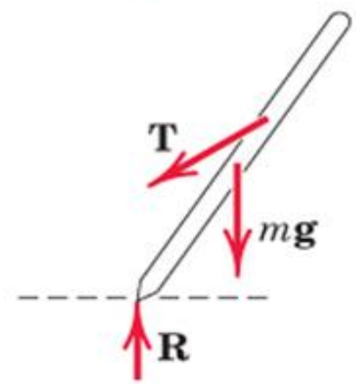

SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
A
Mass m
V
F_3
F_2
F_1
F
M
W = mg
y
x

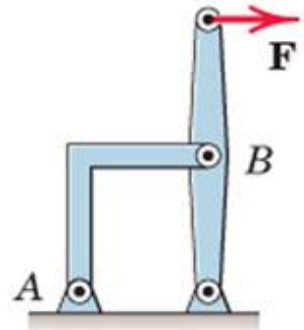
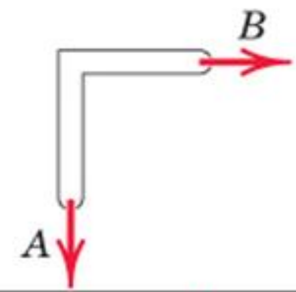
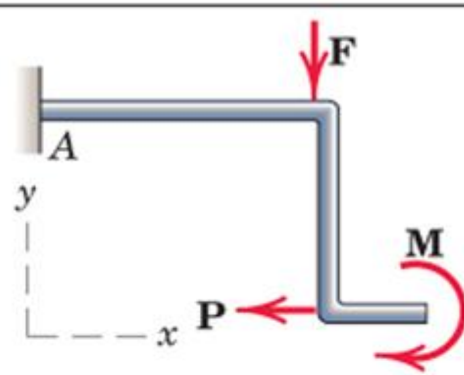
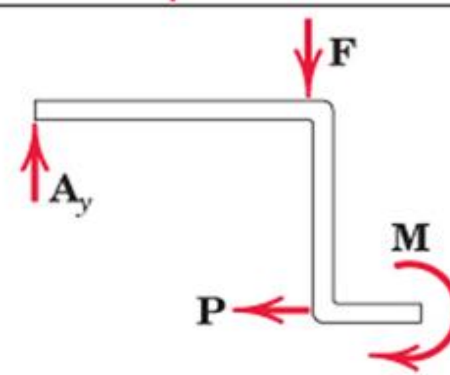
SAMPLE FREE-BODY DIAGRAMS

Mechanical System	Free-Body Diagram of Isolated Body
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

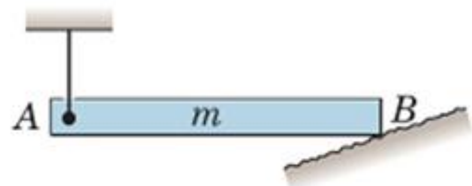
	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		

	Body	Incomplete FBD
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

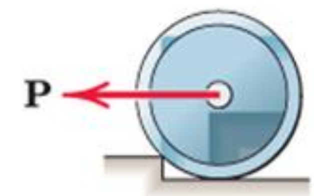
	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		

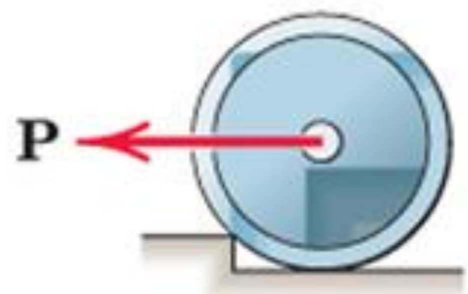
	Body	Wrong or Incomplete FBD
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.

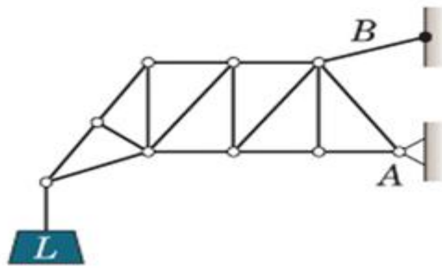


2. Wheel of mass m on verge of being rolled over curb by pull **P**.

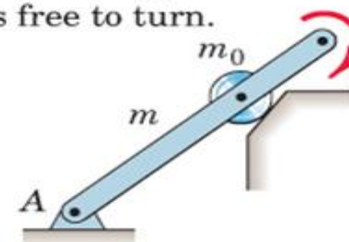


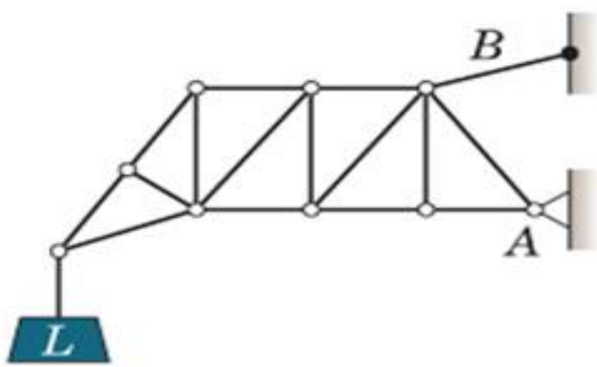


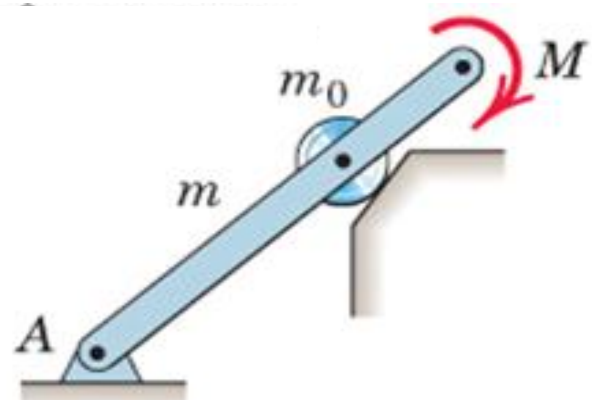
3. Loaded truss supported by pin joint at A and by cable at B.



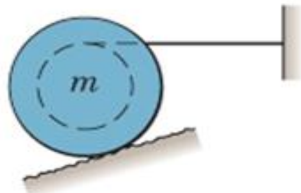
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



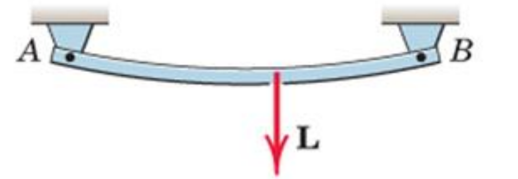


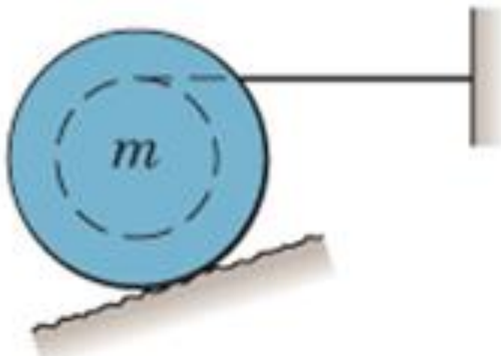


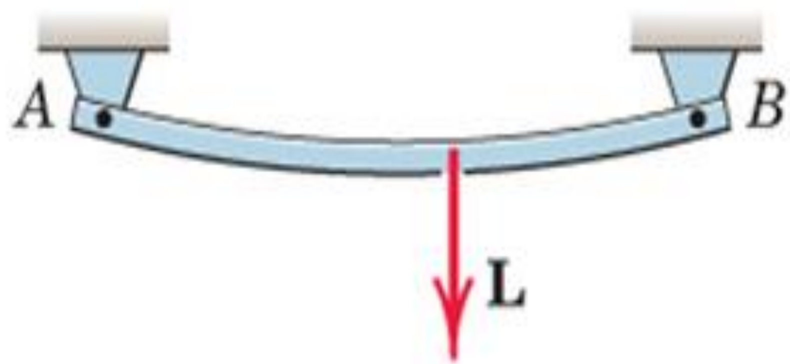
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.



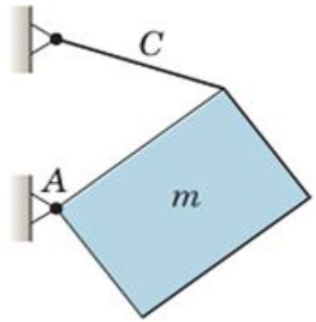
6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.







7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.