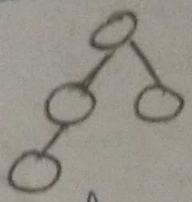
B Tree

1. Balanced m-way tree (Order = m)
2. Generalization of BST tree
   - Node can have more than one key & more than 2 children
3. Maintain sorted data
4. All leaf node must be at same level



B tree of m has following Properties

→ Every node has max m children

→ Min children :- leaf = 0

root = 2

Internal node = $\frac{m}{2}$

→ Every node has max (m-1) keys

Min keys - root node = 1

all other nodes = $(\frac{m}{2}) - 1$

Example.

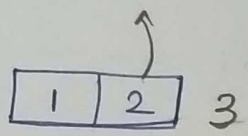
Construct the B tree of order 3 by inserting from 1 to 10

Solution

Order $m = 3$

Max children = 3.

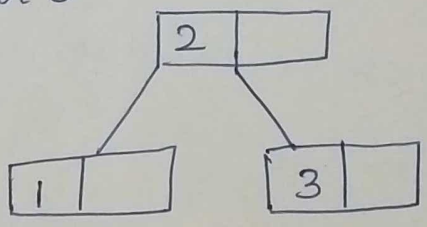
Max key = $m-1 = 2$



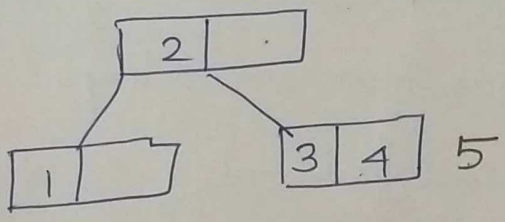
Middle value 1

level up

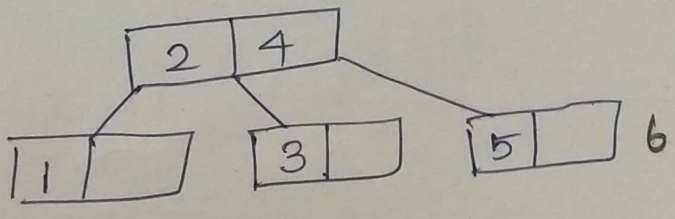
Insert 3



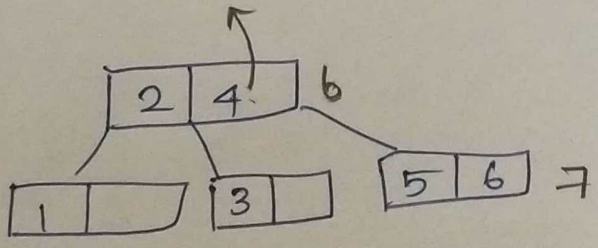
Insert 4



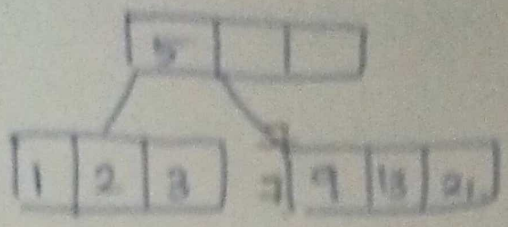
Insert 5



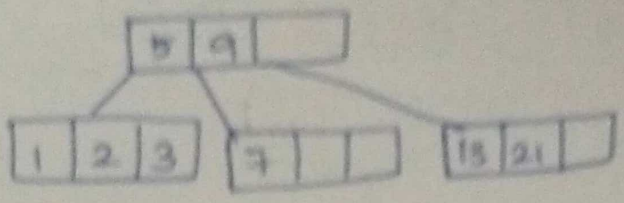
Insert 6



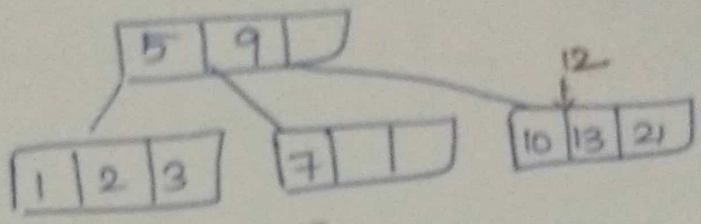
Insert 2



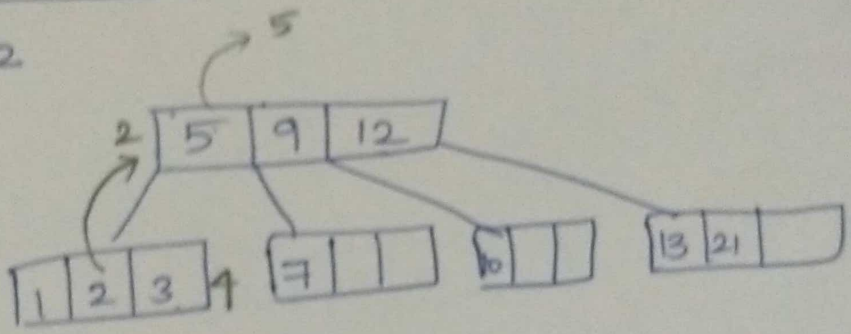
Insert 7



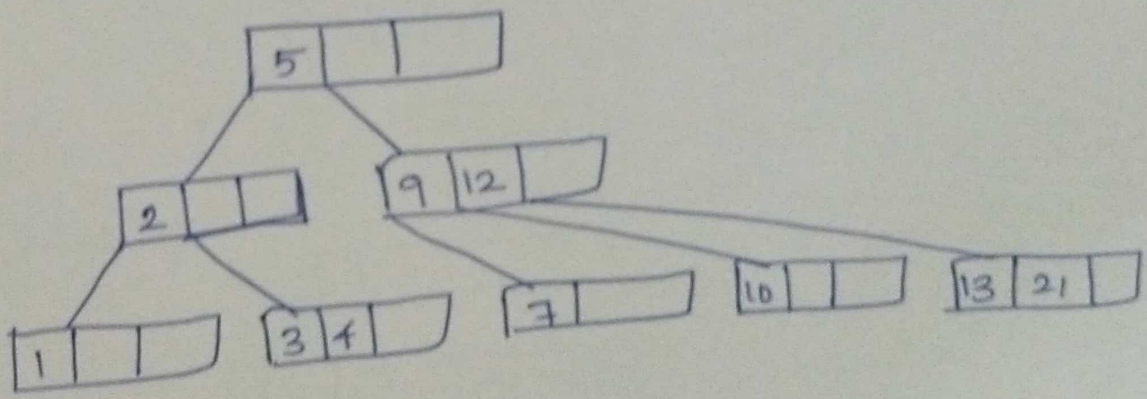
Insert 10



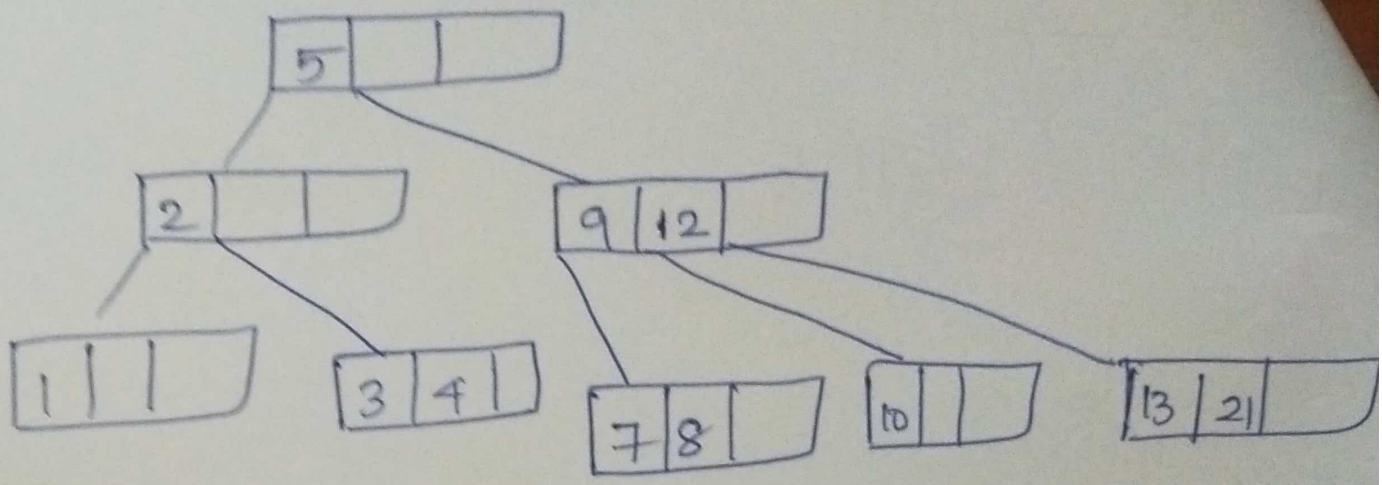
Insert 12



Insert 4.



Insert 8



Build a B-tree of Order 4 with following set of data

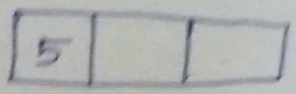
5, 3, 21, 9, 1, 13, 2, 7, 10, 12, 4, 8

m = 4

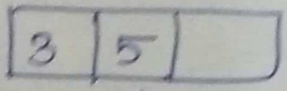
Max key = m-1 = 3

Max children = 4

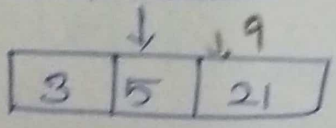
Insert 5



Insert 3

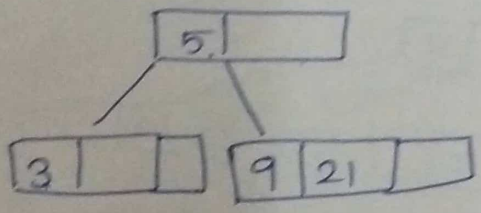


Insert 21

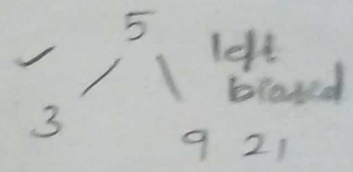


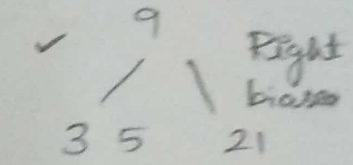
Split → one level go up

Insert 9

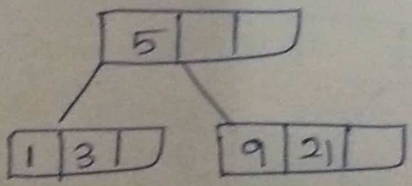


3 5 9 21

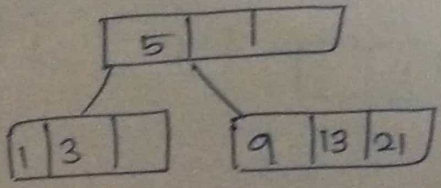




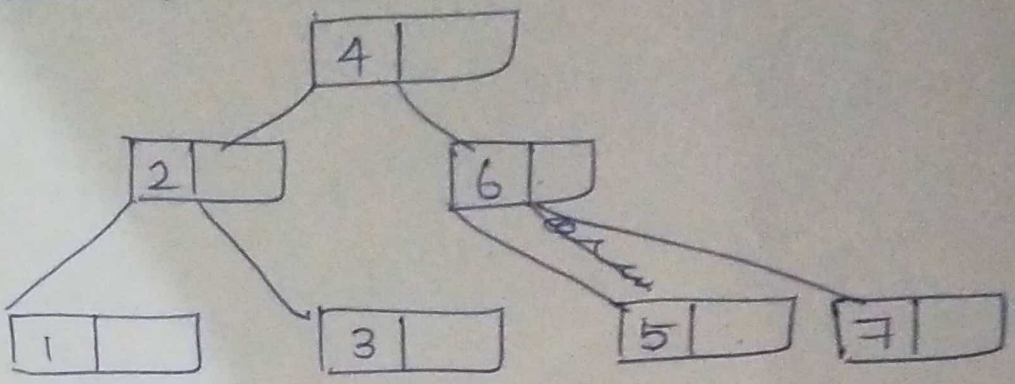
Insert 1



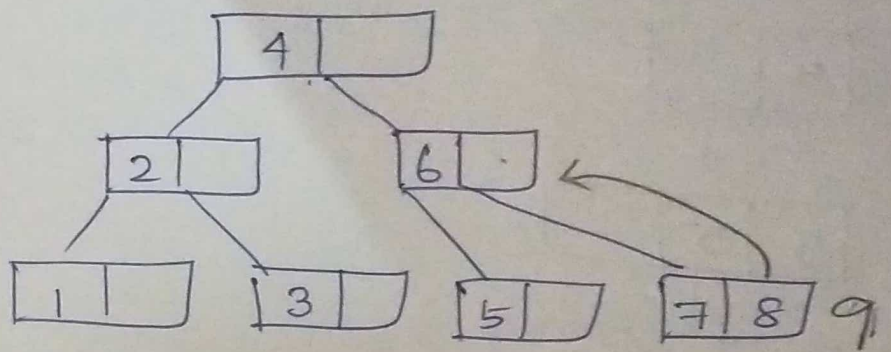
Insert 13



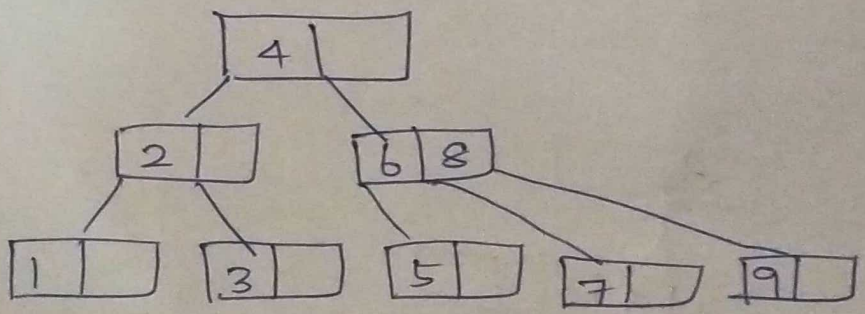
Insert 7



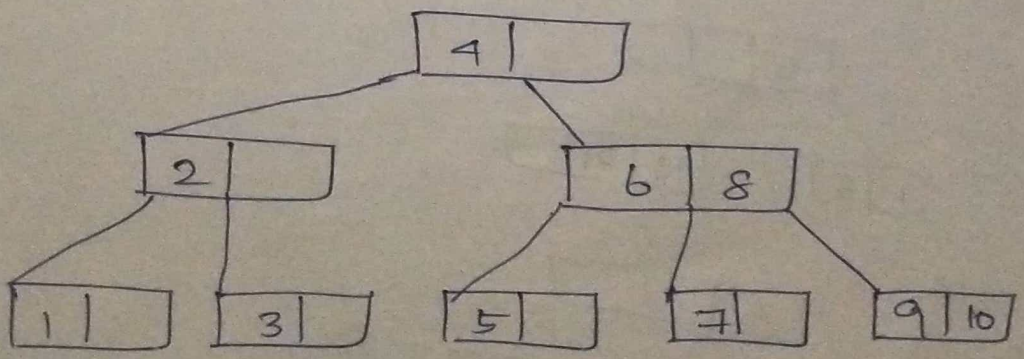
Insert 8



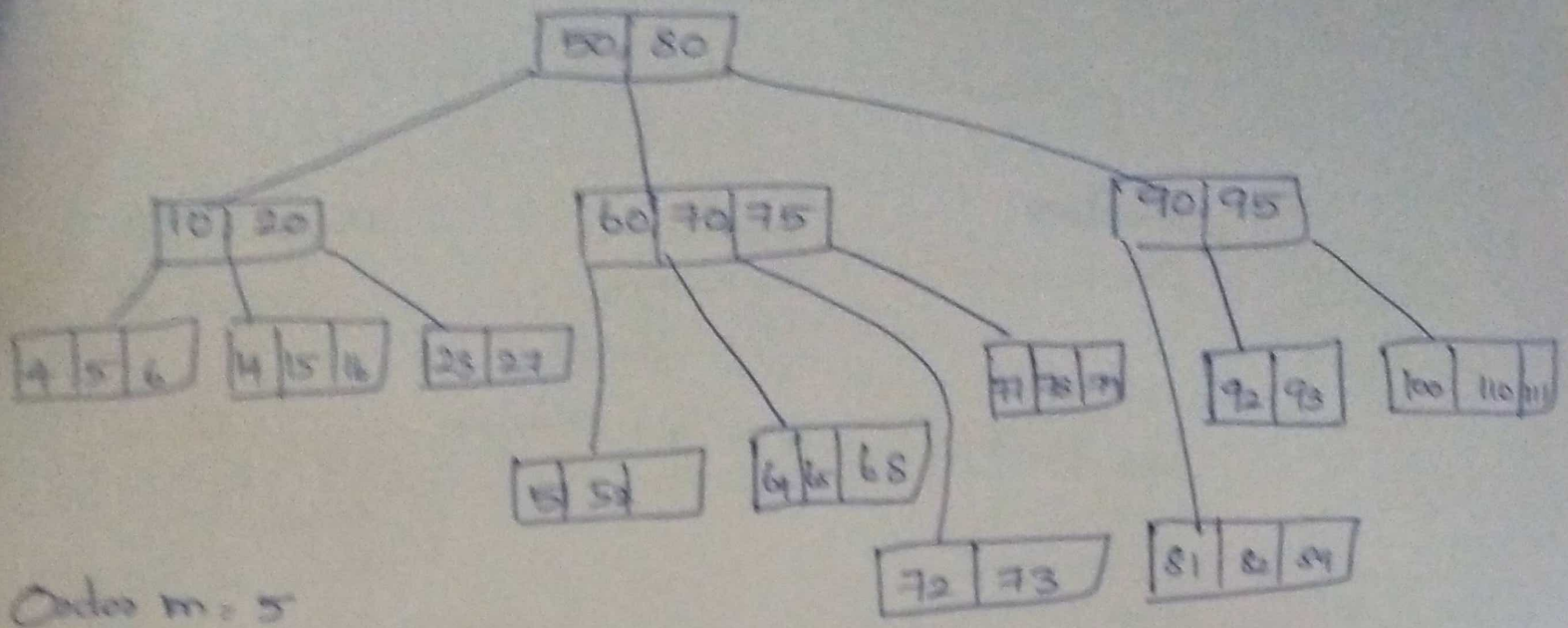
Insert 9



Insert 10



Deletion in B Tree



Order m = 5

Min children = $\frac{m}{2} = \frac{5}{2} = 3$

Max children = m = 5

Min key = $2\left(\frac{m}{2}-1\right) = \frac{5}{2}-1 = 2.5-1 = 1.5 = 2$

Max key = m-1 = 5-1 = 4

Delete - 64, 23, 72, 65, 20, 70, 95, 77, 80, 100, 6, 27, 60, 16, 50

If the target is in leaf node

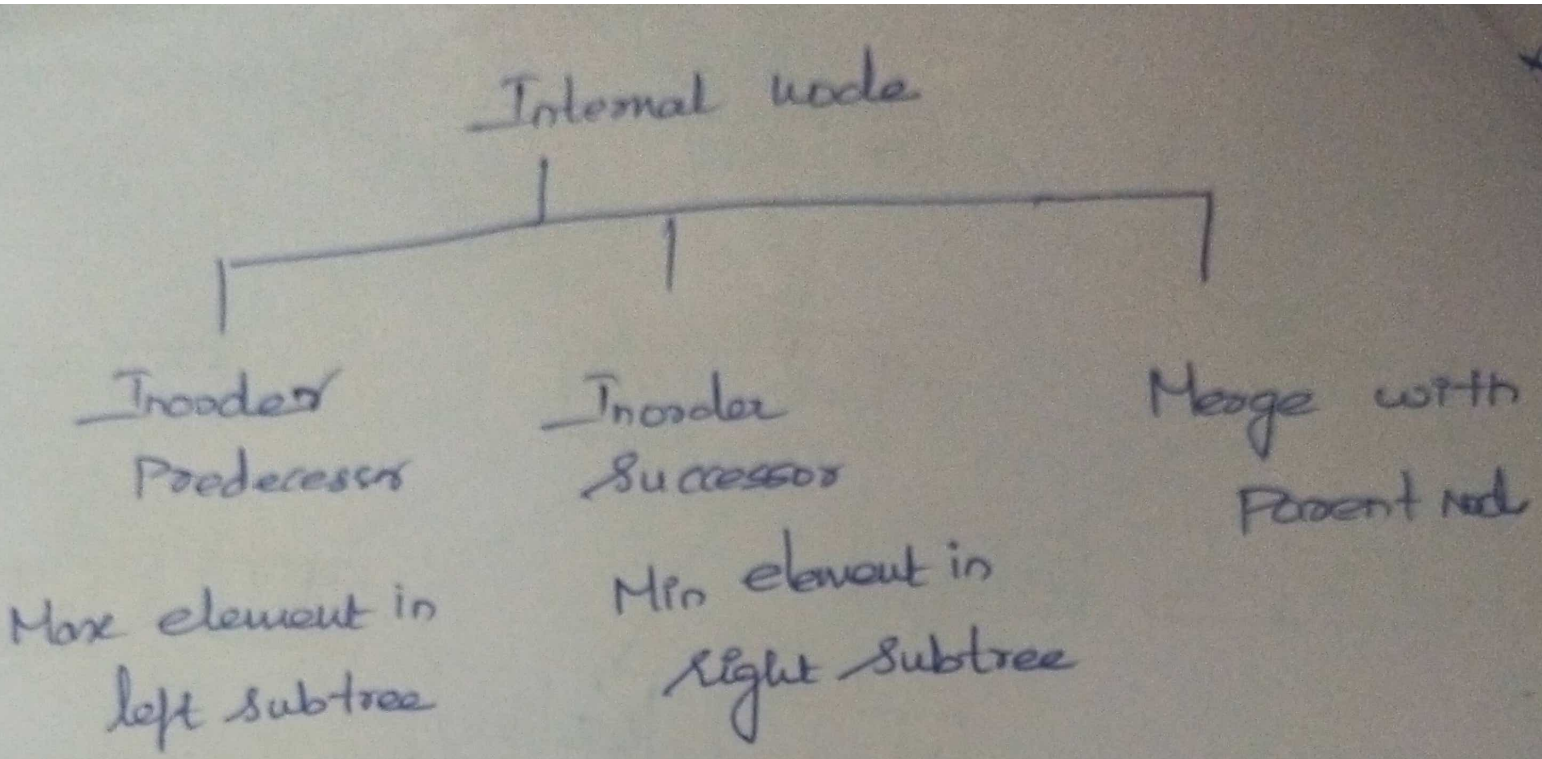
If the target is in internal node

Leaf node

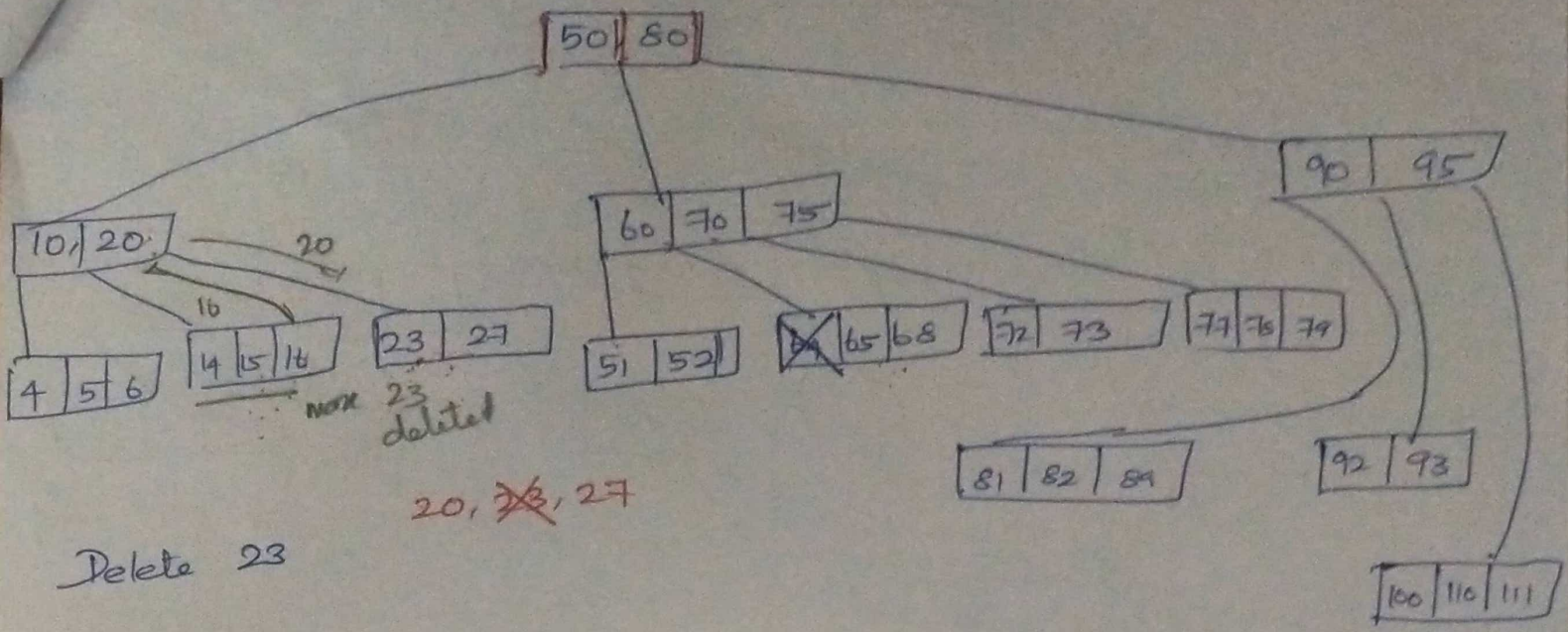
Leaf node contains more than min no of keys

That leaf node contains min no of key

- Borrow from Immediate left node (Max)
- Borrow from Immediate Right node (Min)
- Merge with left or Right with parent node

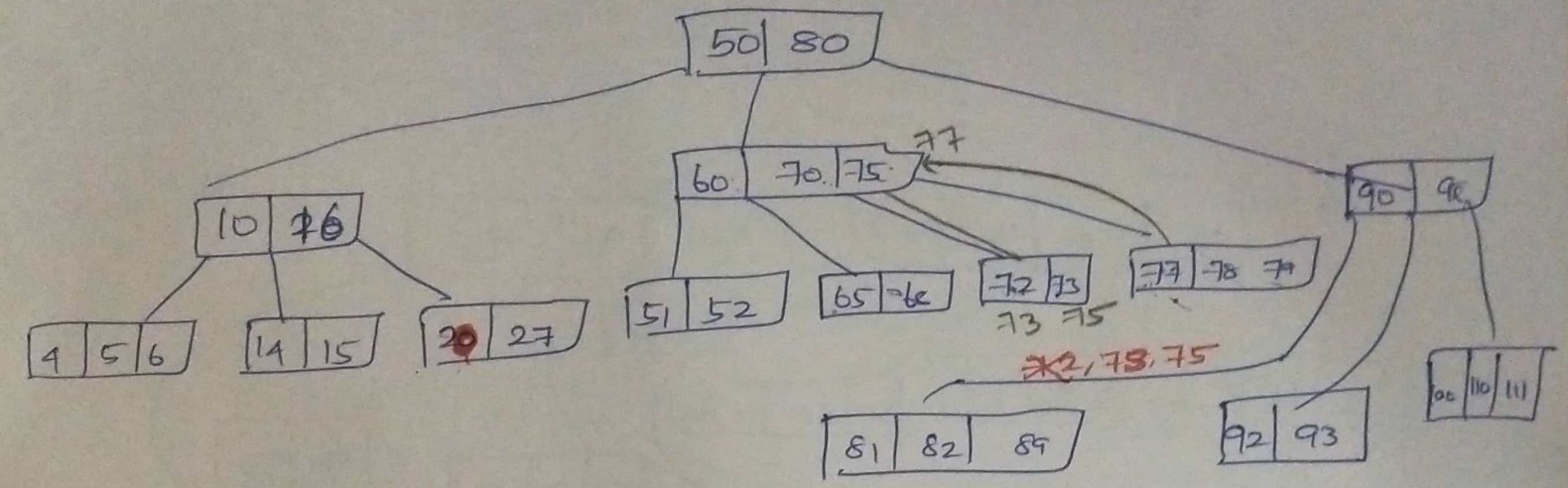

Internal node
Inorder Predecessor
Max element in left subtree
Inorder Successor
Min element in right subtree
Merge with Parent node

lete 64

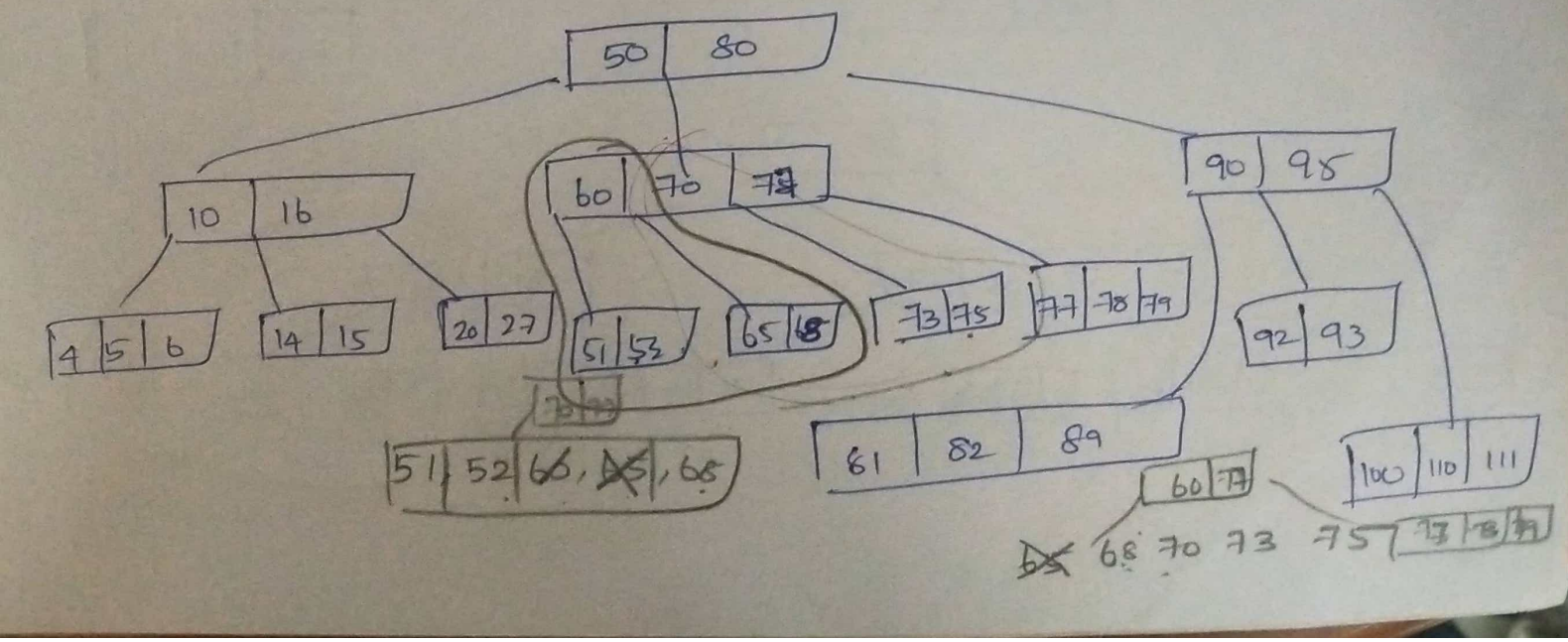


20, ~~23~~, 27

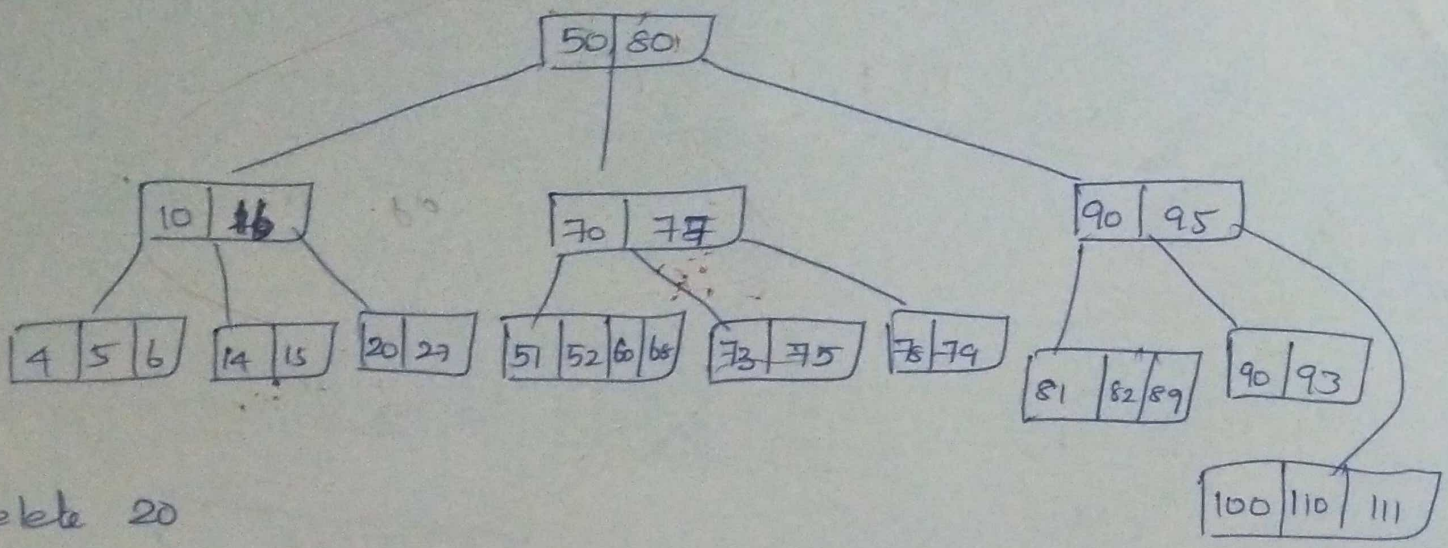
Delete 23



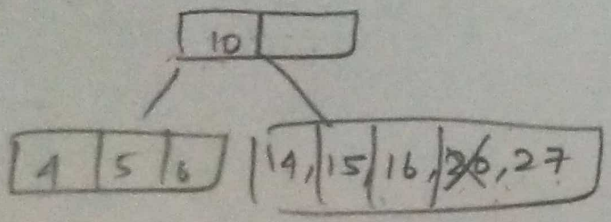
Delete 72

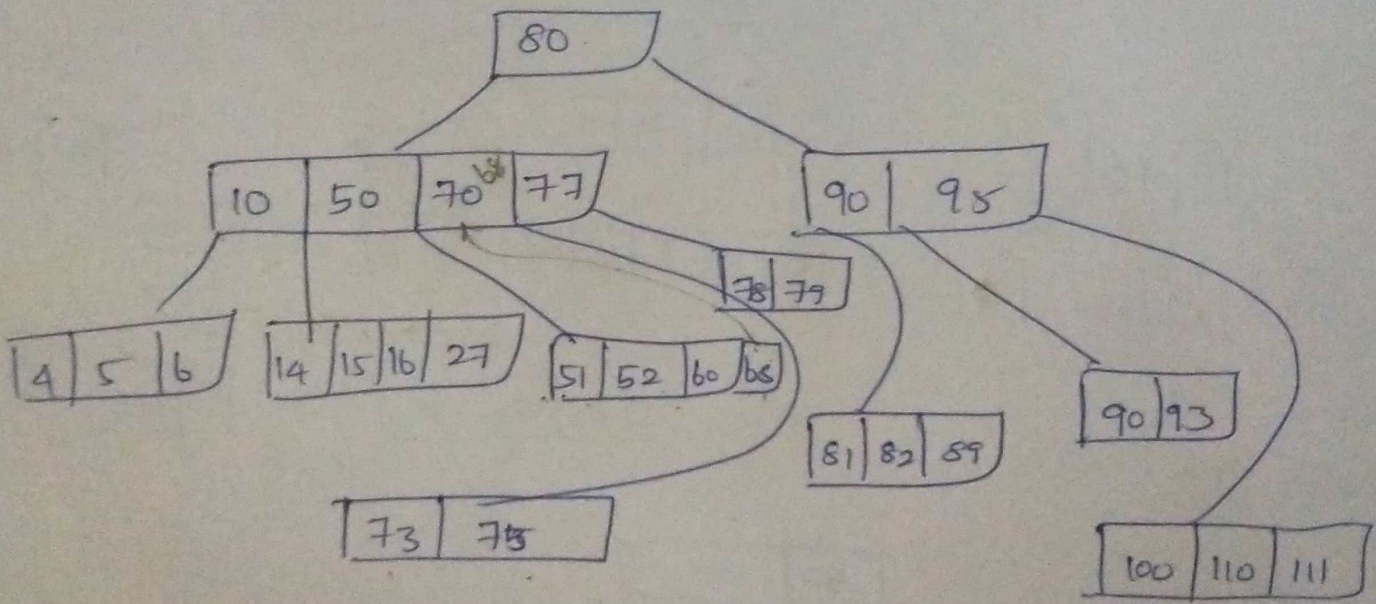


Delete 65

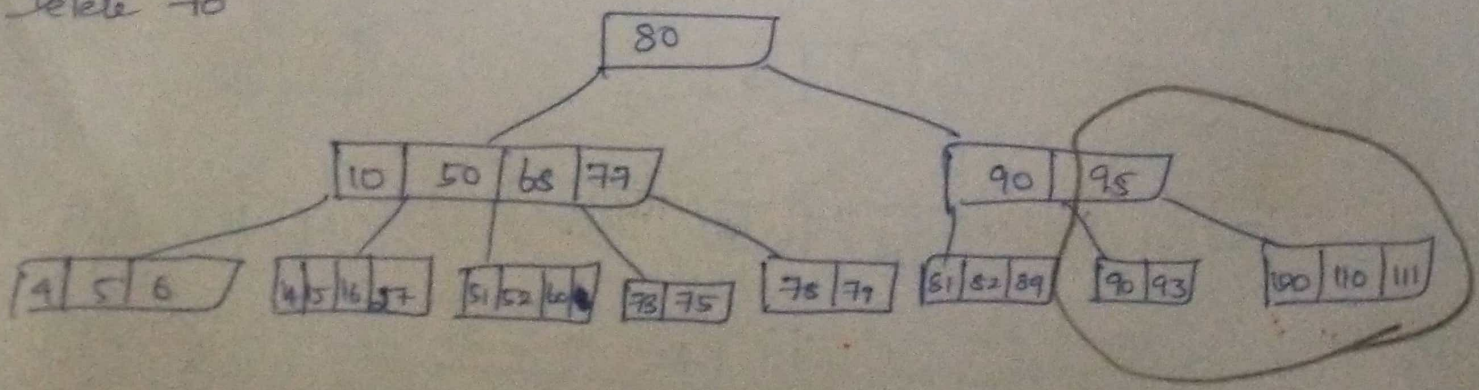


Delete 20

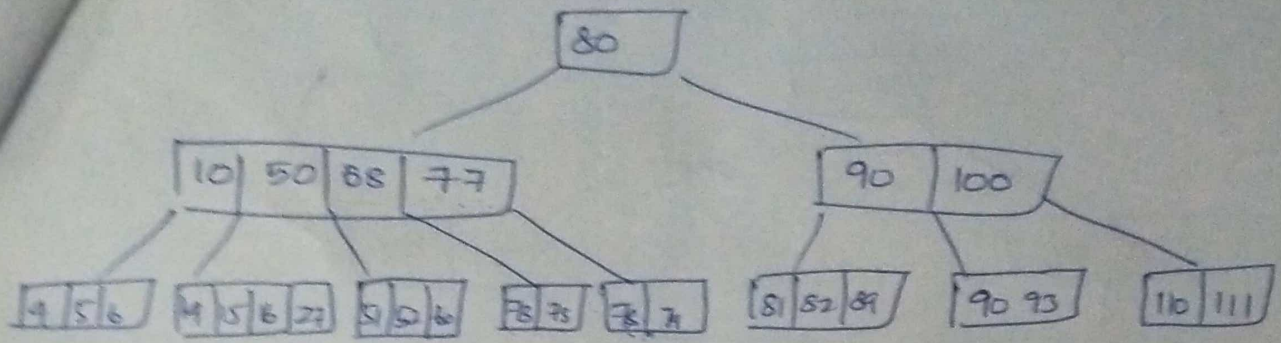




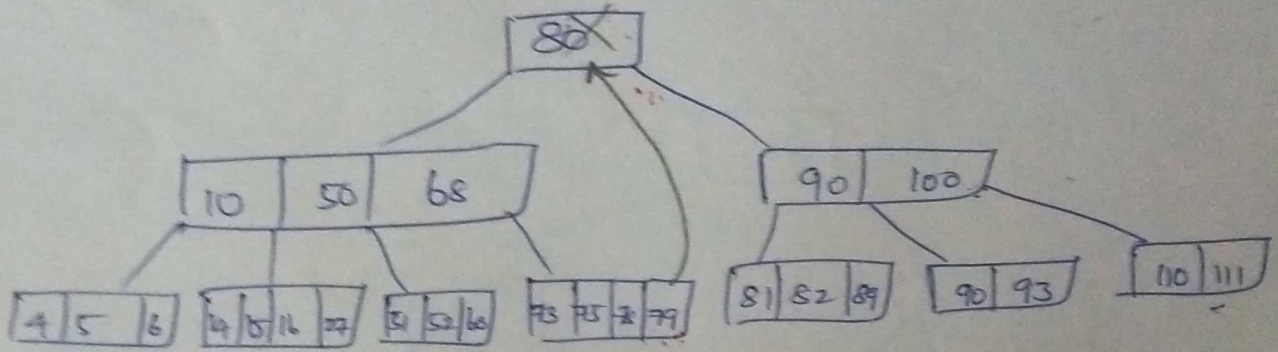
Delete 70



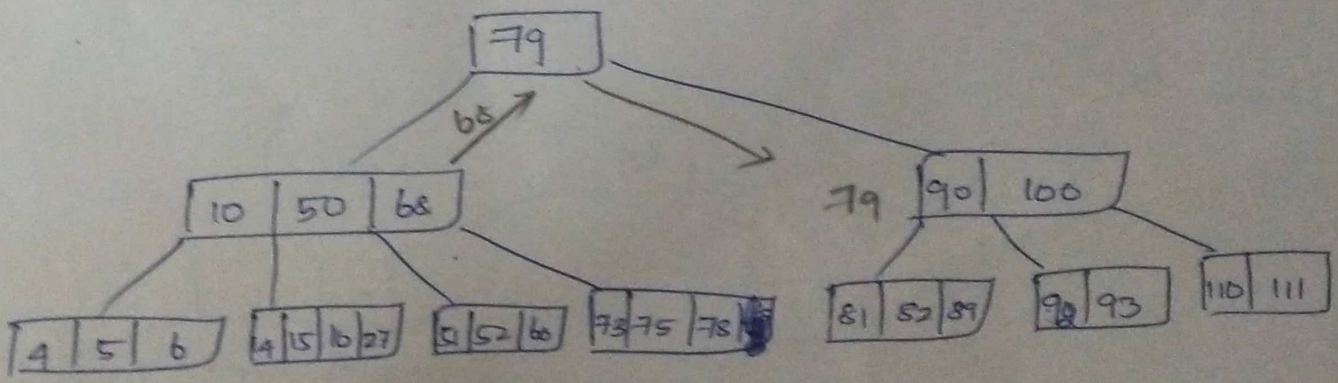
95



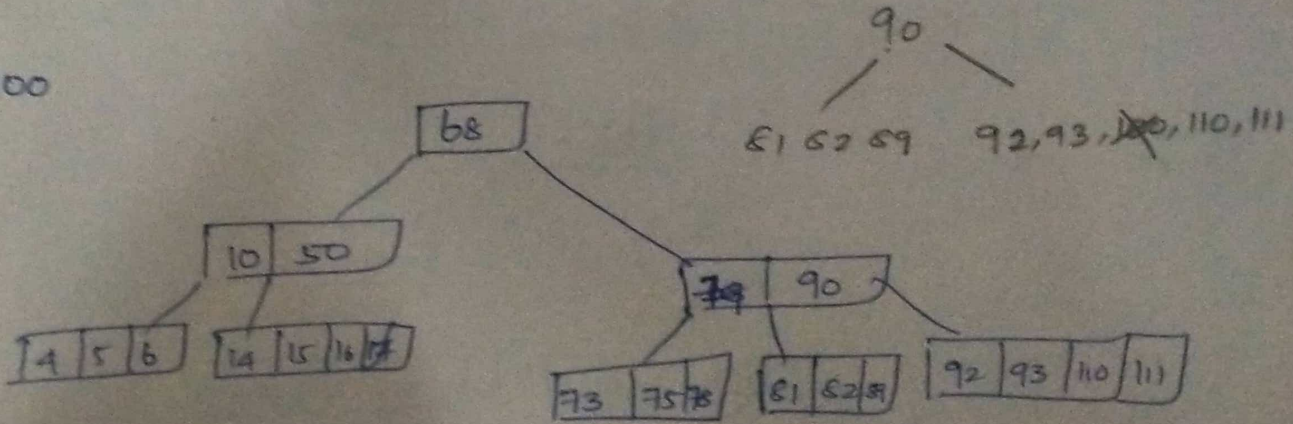
Delete 77



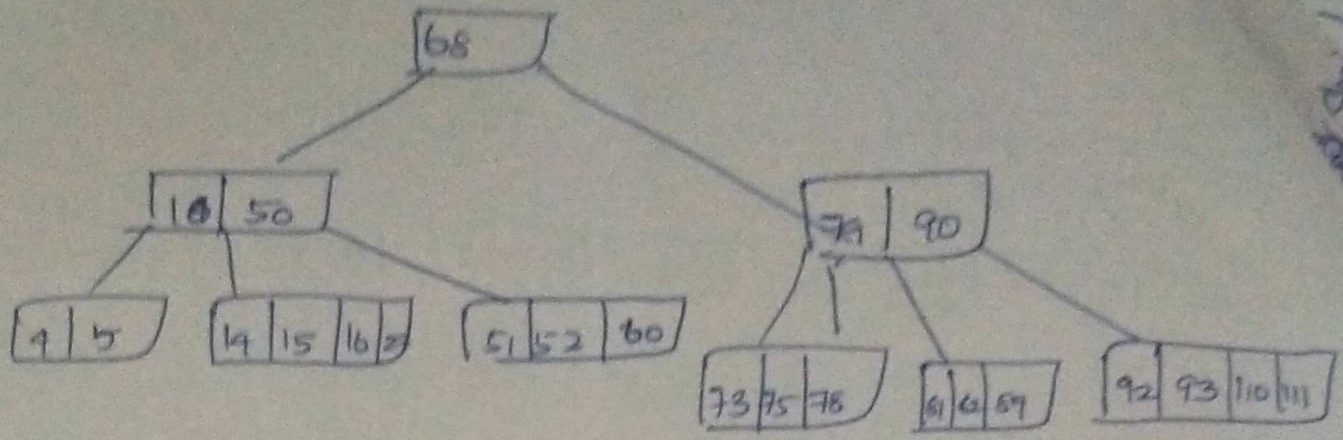
Delete 80



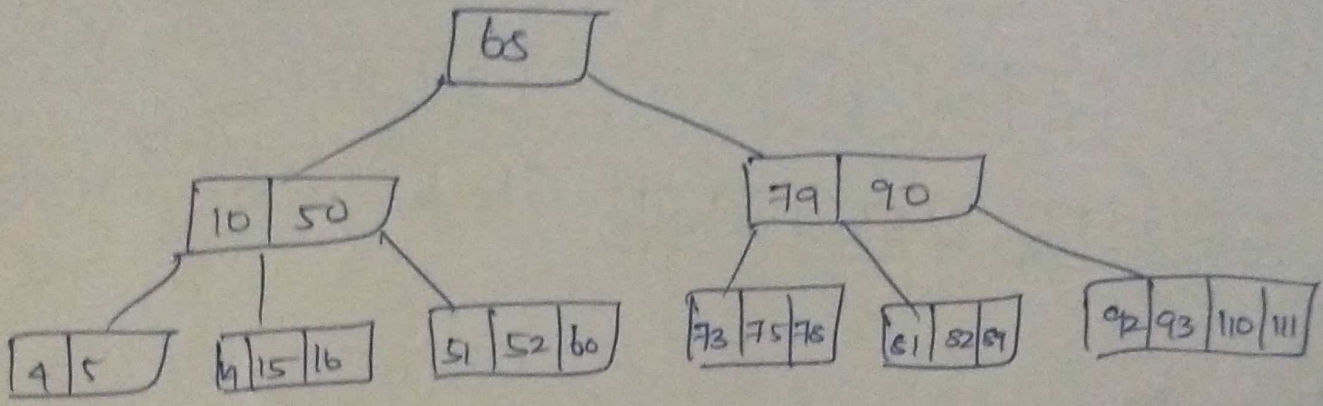
Delete 100



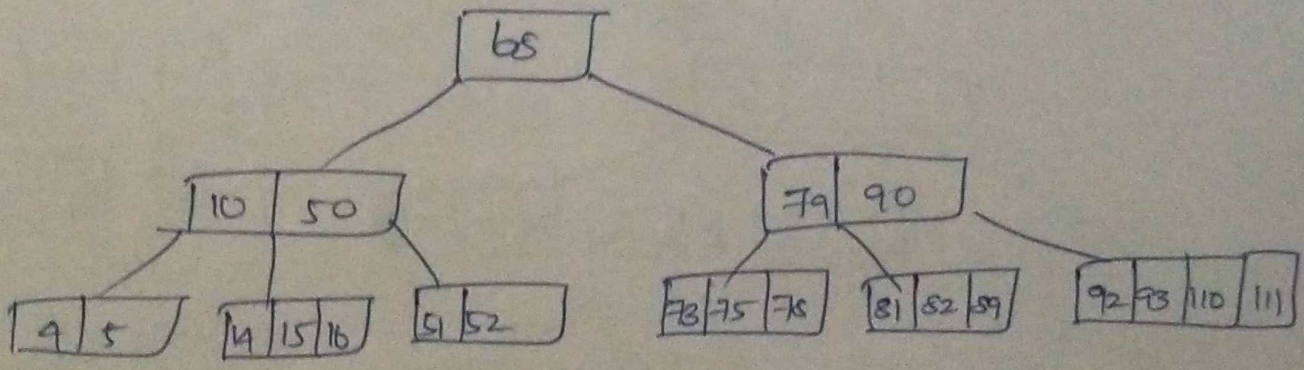
Delete 6



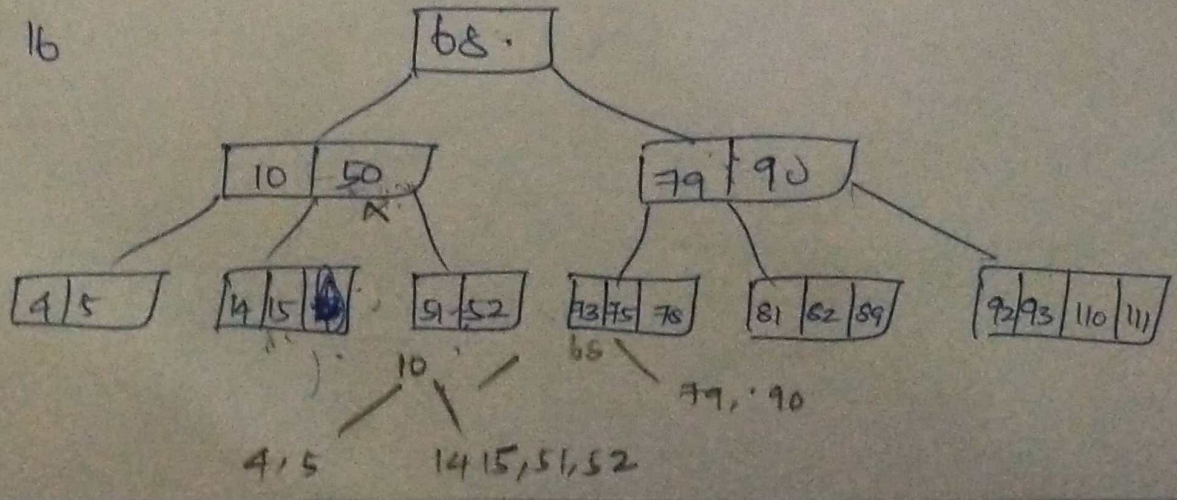
Delete 27



Delete 60

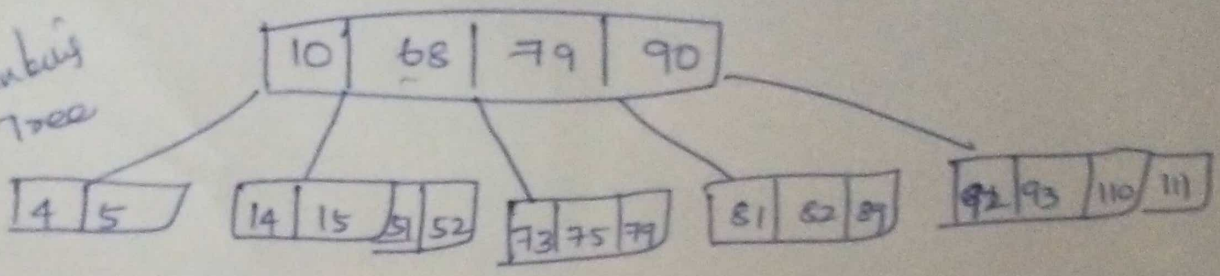


Delete 16



ete 50

Shrinking B Tree



B^+ Tree

Order $m = 4$

Max children $= 4$

Min children $= \frac{m}{2} = 2$

Max key $= m-1 = 4-1 = 3$

Min key $= \left(\frac{m}{2}\right) - 1 = 2-1 = 1$

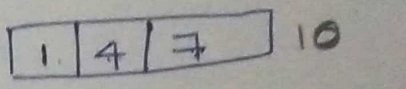
1, 4, 7, 10, 17, 21, 31, 25, 19, 20, 28, 42

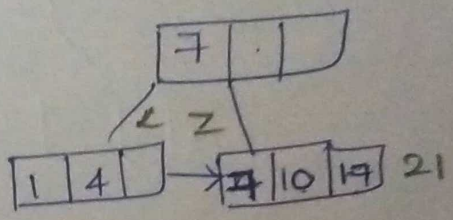
Data is stored in leaf node

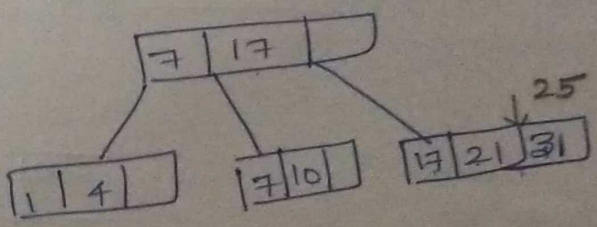
↓

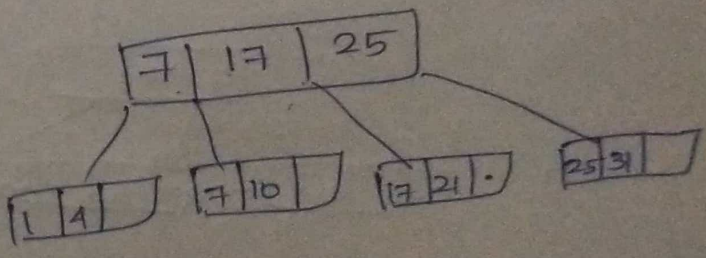
leaf node is linked list

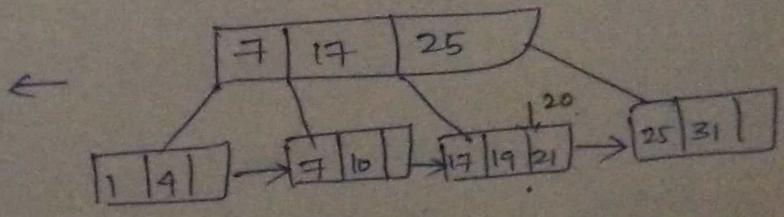
left biased or right biased

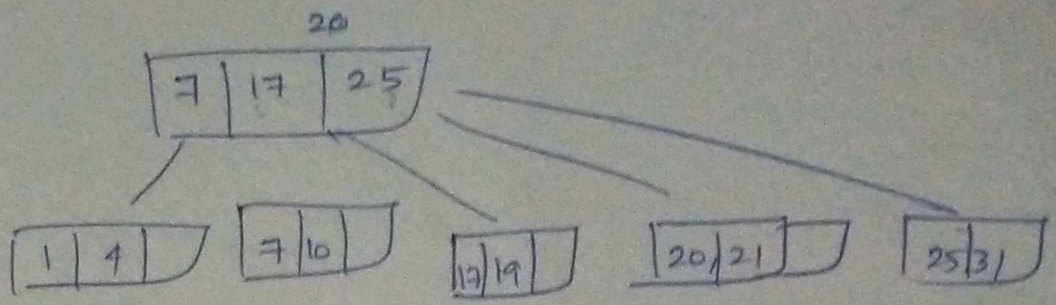


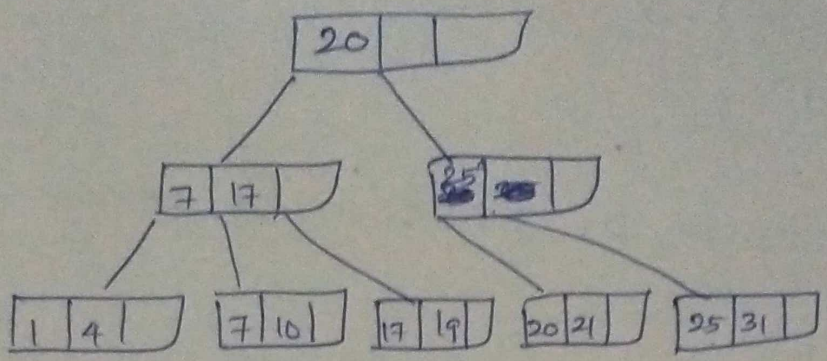


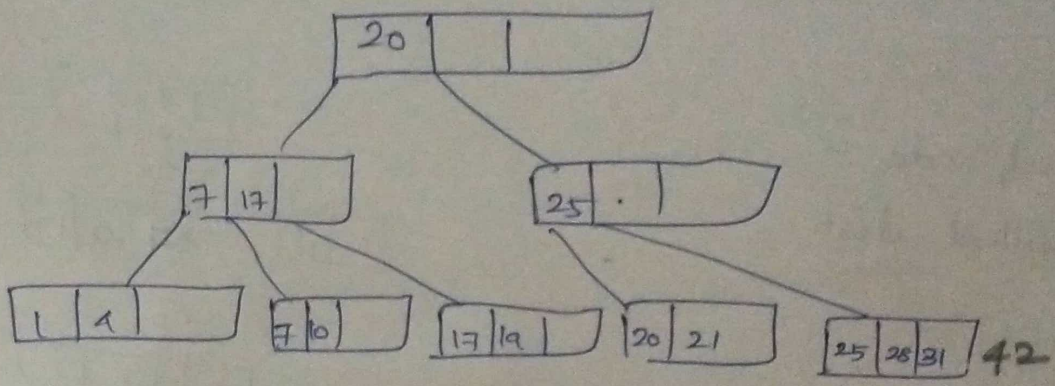




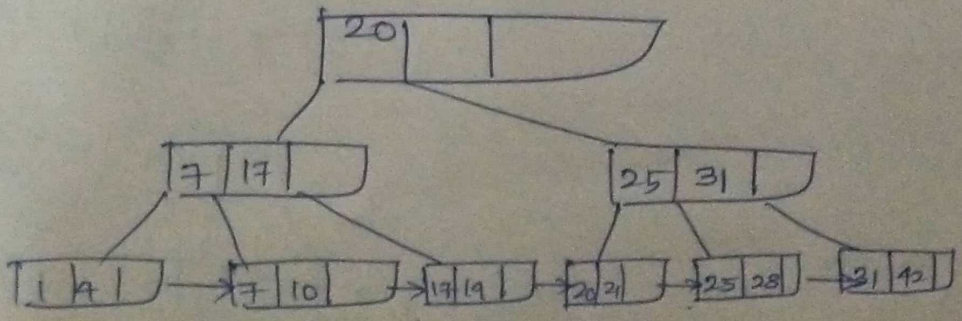



20
7 17 25
1 4
7 10
17 19
20 21
25 31


20
7 17
25
1 4
7 10
17 19
20 21
25 31


20
7 17
25
1 4
7 10
17 19
20 21
25 28 31

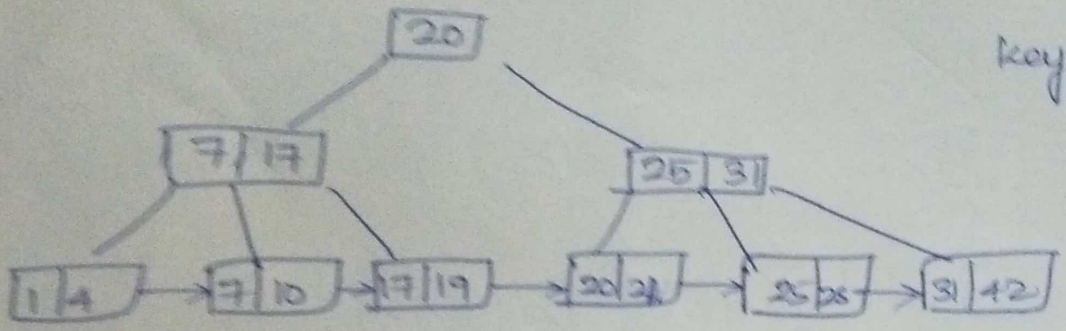
42


20
7 17
25 31
1 4
7 10
17 19
20 21
25 28
31 42

Tree Deletion

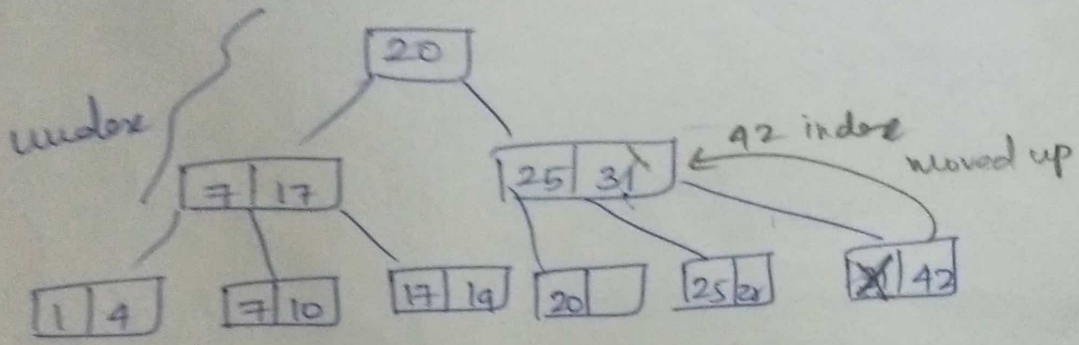
Children
Max = 4
Min = $\frac{m}{2}$ = 2

Key
Max = m-1 = 3
Min $(\frac{m}{2})$ -1 = 1

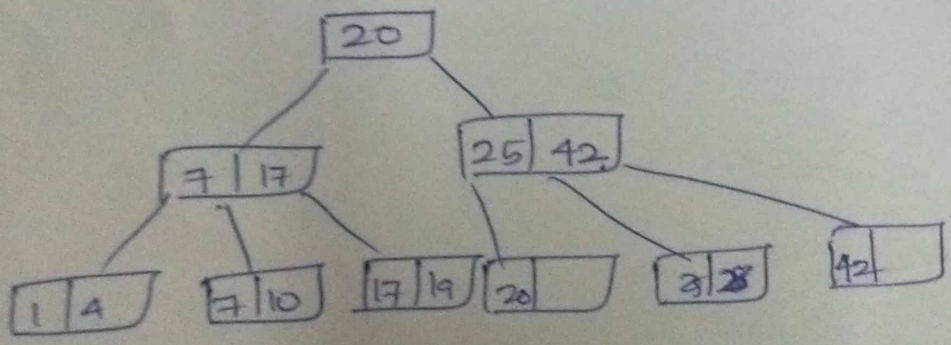


Delete: 21, 31, 20, 10, 7, 25, 42, 4

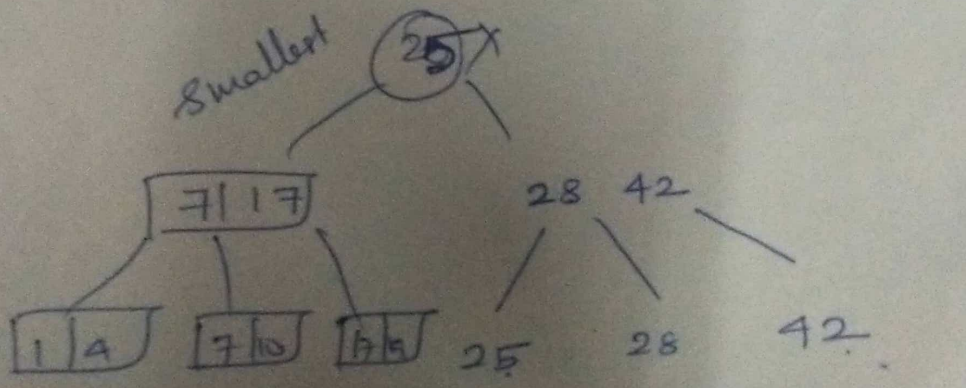
Delete 21



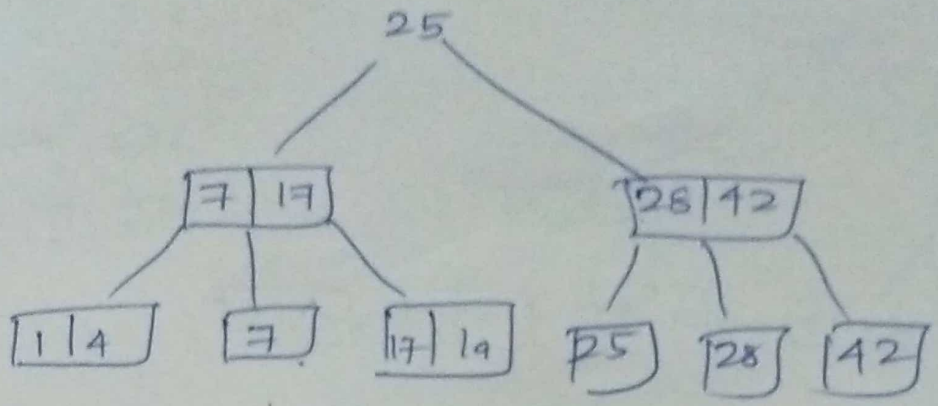
Delete 31



Delete 20



Delete 10



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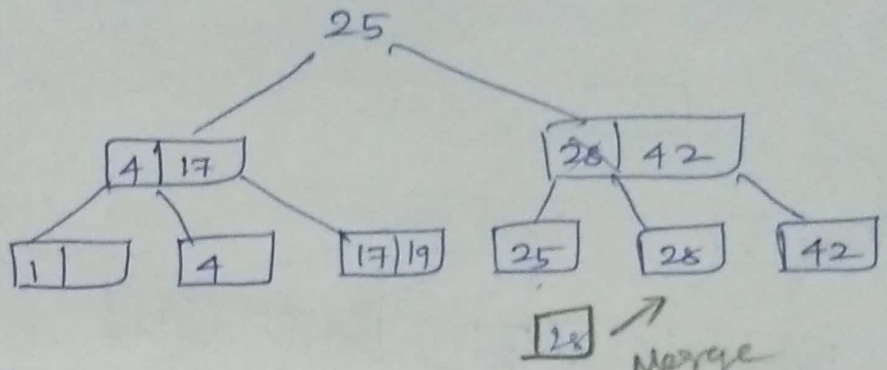

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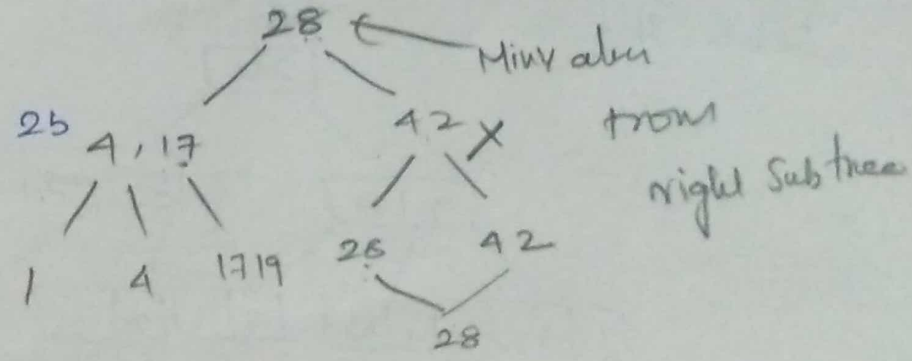

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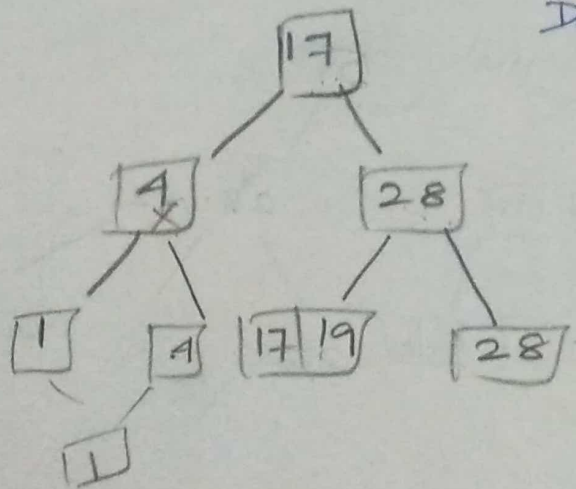

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