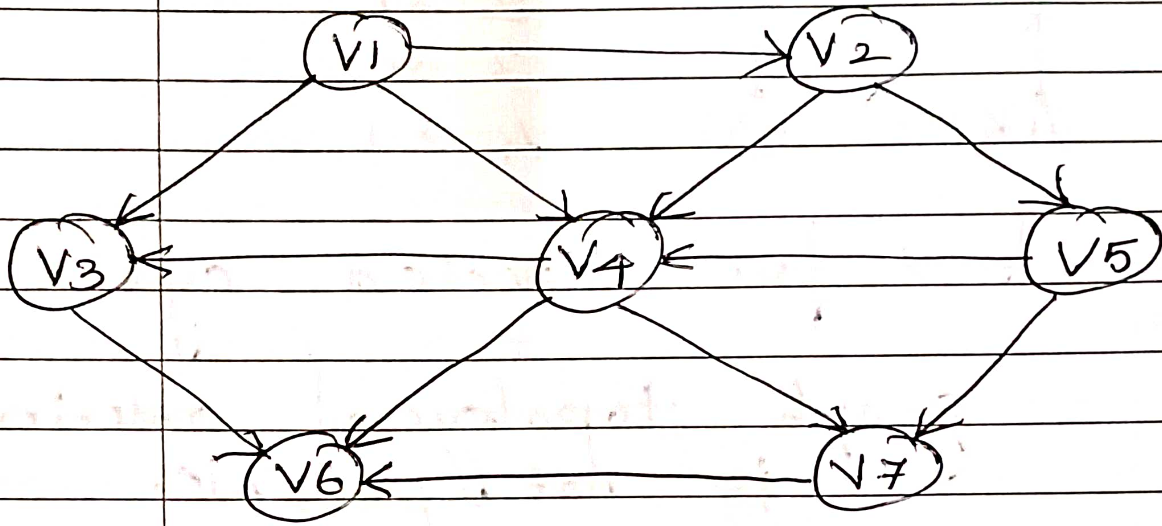
Topological Ordering



Indegree of $V_1 = 0$

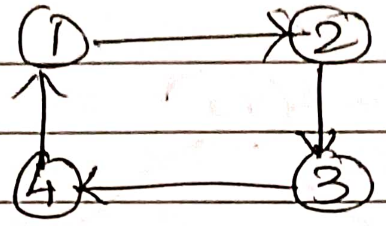
$V_2 = 1$ $V_6 = 3$

$V_3 = 2$ $V_7 = 2$

$V_4 = 3$

$V_5 = 1$

Vertex	1	2	3	4	5	6	7
	Indegree	Before		Dequeue			
V_1	0	0	0	0	0	0	0
V_2	1	$\boxed{0}$	0	0	0	0	0
V_3	2	1	1	1	$\boxed{0}$	0	0
V_4	3	2	1	$\boxed{0}$	0	0	0
V_5	1	1	$\boxed{0}$	0	0	0	0
V_6	3	3	3	3	2	1	0
V_7	2	2	2	1	0	$\boxed{0}$	0
Enqueue	V_1	V_2	V_5	V_4	V_3, V_7	V_7	V_6
Dequeue	V_1	V_2	V_5	V_4	V_3	V_7	V_6



1 → 1

2 → 1

3 → 1

4 → 1

→ for directed cyclic graph topological ordering is not possible.

Vertices

V_1

V_2

V_3

V_4

__/__/__

TOPOLOGICAL SORTING

The topological sort algorithm takes a directed graph as input and returns an array of nodes where each node appears before all nodes it points to.

(or)

Topological sort is a linear ordering of the vertices in such a way that if there is an edge in the DAG (Directed Acyclic Graph) from vertex 'v', then 'u' comes before 'v' in the ordering.

It is important to note that

1) Topological sorting is possible if and only if the graph is Directed Acyclic Graph.
2.) There may exist multiple different topological orderings for a given directed acyclic graph.
3) Remove self loops and parallel loops from DAG if there are any.

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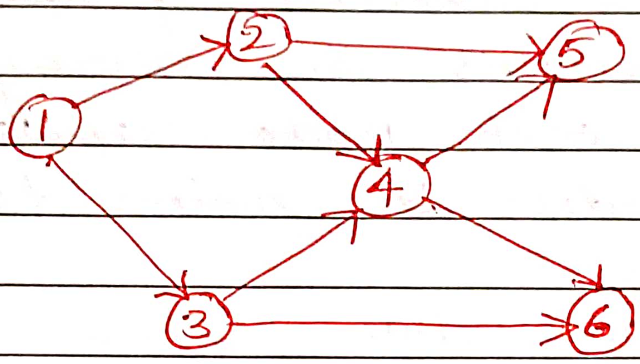
Consider the following Directed Acyclic Graph (DAG)



1) Find the in-degree for all the vertices

Indegree (1) = 0

Indegree (2) = 1

Indegree (3) = 1

Indegree (4) = 2

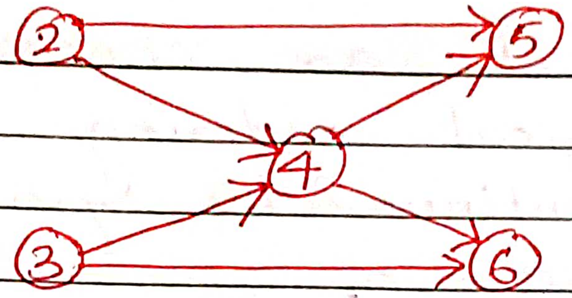
Indegree (5) = 2

Indegree (6) = 2

- choose the vertex with indegree '0' as the starting vertex and add it to topological ordering
- Remove that vertex & outgoing edges from the graph.

Topological ordering

1



Now the change in indegree is as follows.

(2) = 0

(3) = 0

(4) = 2

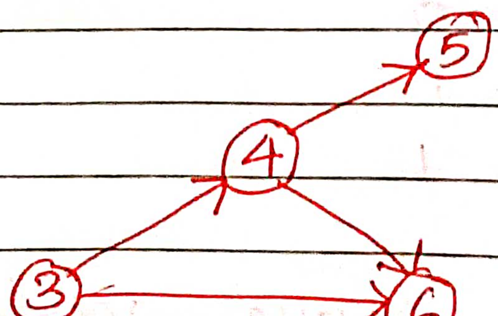
(5) = 2

(6) = 2.

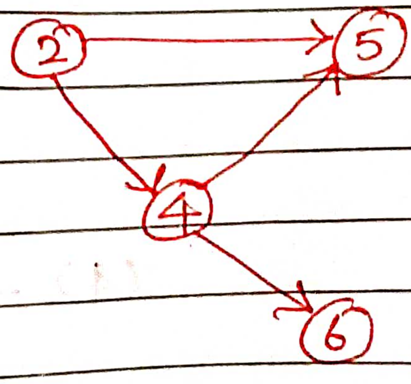
choose either (2) or (3) as the next source vertex & continue as above.

Topological ordering

1, 2 (or) 1, 3



(or)



Here, I am choosing the topological ordering & going to continue the process.

Indegree of

(3) = 0

(4) = 2

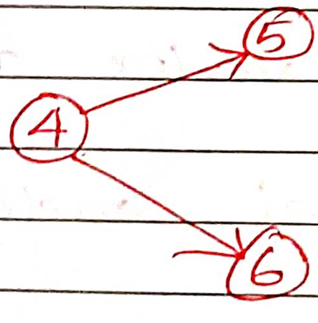
(5) = 1

(6) = 2

choose (3) as next source.

Topological ordering

1, 2, 3.



Indegree of

(4) = 0

(5) = 1

(6) = 1

(4) → next source vertex.

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Topological ordering

1, 2, 3, 4

5

6

Indegree of (5) = 0
(6) = 0

(5) or (6) can be chosen as the next source.

Topological ordering

1, 2, 3, 4, 5, 6 (or) 1, 2, 3, 4, 6, 5

Other possible topological orderings are

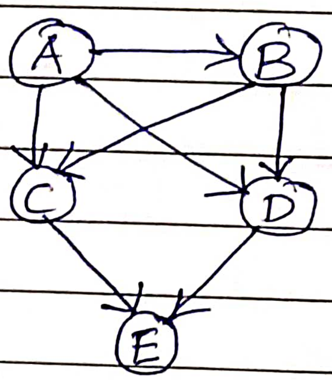
1, 3, 2, 4, 5, 6

(or)

1, 3, 2, 4, 6, 5

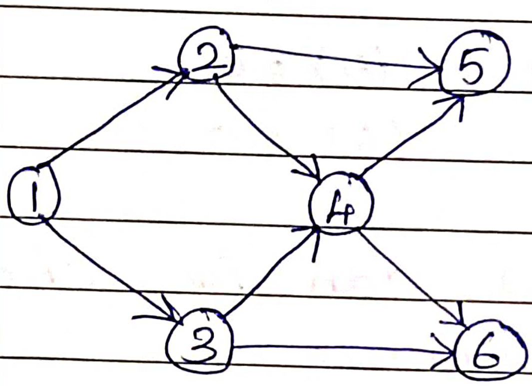
Examples

Topological ordering



A → B → C → D → E

A → B → D → C → E



1 → 2 → 3 → 4 → 5 → 6

1 → 2 → 3 → 4 → 6 → 5

1 → 3 → 2 → 4 → 5 → 6

1 → 3 → 2 → 4 → 6 → 5