



DEPARTMENT OF MATHEMATICS

UNIT-V Z-TRANSFORM

Convolution Theorem:

(1) Using convolution theorem find the inverse z-transform of $\left(\frac{z}{z-a}\right)^2$.

Soln:

$$\begin{aligned} z^{-1} \left[\frac{z}{z-a} \right]^2 &= z^{-1} \left[\frac{z}{z-a} * \frac{z}{z-a} \right] \\ &= z^{-1} \left[\frac{z}{z-a} \right] * z^{-1} \left[\frac{z}{z-a} \right] \\ &= a^n * a^n \\ &= \sum_{m=0}^n a^m a^{n-m} \\ &= a^n \sum_{m=0}^n 1 \\ &= a^n [1+1+\dots+(n+1)] \\ &= (n+1) a^n \end{aligned}$$

(2) Find the inverse z-transform of $\frac{z^2}{(z+q)^2}$ using convolution theorem.

(3) Find the inverse z-transform of $\frac{z^2}{(z-1)(z-3)}$ using convolution theorem.



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soln:
$$Z^{-1} \left[\frac{Z^2}{(Z-1)(Z-3)} \right] = Z^{-1} \left[\frac{Z}{Z-1} * \frac{Z}{Z-3} \right]$$
$$= Z^{-1} \left[\frac{Z}{Z-1} \right] * Z^{-1} \left[\frac{Z}{Z-3} \right]$$
$$= 1^n * 3^n$$
$$= \sum_{m=0}^n 1^m 3^{n-m}$$
$$= \sum_{m=0}^n 3^{n-m}$$

$$= 3^n + 3^{n-1} + 3^{n-2} + \dots + 3 + 1$$

$$= \frac{3^{n+1} - 1}{3 - 1}$$

$$= \frac{1}{2} [3^{n+1} - 1]$$



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(4) Use convolution theorem to find $z^{-1} \left[\frac{8z^2}{(2z-1)(4z+1)} \right]$

$$\begin{aligned} \text{Soln} \\ z^{-1} \left[\frac{8z^2}{(2z-1)(4z+1)} \right] &= z^{-1} \left[\frac{8z}{2z-1} * \frac{z}{4z+1} \right] \\ &= z^{-1} \left[\frac{8z}{2(z-\frac{1}{2})} * \frac{z}{4(z+\frac{1}{4})} \right] \\ &= z^{-1} \left[\frac{z}{z-\frac{1}{2}} \right] * z^{-1} \left[\frac{z}{z+\frac{1}{4}} \right] \\ &= \left(\frac{1}{2}\right)^n * \left(-\frac{1}{4}\right)^n \\ &= \sum_{m=0}^n \left(\frac{1}{2}\right)^m \left(-\frac{1}{4}\right)^{n-m} \\ &= \sum_{m=0}^n \left(\frac{1}{2}\right)^m \left(-\frac{1}{2}\right)^{n-m} \left(\frac{1}{2}\right)^{n-m} \\ &= \left(\frac{1}{2}\right)^n \sum_{m=0}^n \left(-\frac{1}{2}\right)^{n-m} \\ &= \left(\frac{1}{2}\right)^n \left[\left(-\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^{n-1} + \dots + \left(-\frac{1}{2}\right)^0 \right] \\ &= \left(\frac{1}{2}\right)^n \left[\frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{1 - \left(-\frac{1}{2}\right)} \right] \\ &= \left(\frac{1}{2}\right)^n \left[\frac{1 - \left(-\frac{1}{2}\right)^{n+1}}{\frac{3}{2}} \right] \\ &= \frac{2}{3} \cdot \left(\frac{1}{2}\right)^n \left[1 - \left(-\frac{1}{2}\right)^{n+1} \right] \end{aligned}$$