Convolution Theorem:

(1) Using convolution theorem find the inverse z-Transform of $\left(\frac{z}{z-a}\right)^2$

Soln:

$$z^{-1}\left[\frac{z}{z-a}\right]^2 = z^{-1}\left[\frac{z}{z-a} * \frac{z}{z-a}\right]$$

$$= z^{-1}\left[\frac{z}{z-a}\right] * z^{-1}\left[\frac{z}{z-a}\right]$$

$$= a^n * a^n$$

$$= \sum_{m=0}^{n} a^m a^{n-m}$$

$$= a^n \sum_{m=0}^{n} 1.$$

$$= a^n [1 + 1 + \ldots + (n+1)]$$

$$= (n+1)\, a^n$$

(2) Find the inverse z-Transform of $\frac{z^2}{(z+a)^2}$ using convolution theorem.

(3) Find the inverse z-Transform of $\frac{z^2}{(z-1)(z-3)}$ using convolution theorem.

Soln:

$$Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right] = Z^{-1}\left[\frac{z}{z-1} * \frac{z}{z-3}\right]$$

$$= Z^{-1}\left[\frac{z}{z-1}\right] * Z^{-1}\left[\frac{z}{z-3}\right]$$

$$= 1^n * 3^n$$

$$= \sum_{m=0}^{n} 1^m 3^{n-m}$$

$$= \sum_{m=0}^{n} 3^{n-m}$$

$$= 3^n + 3^{n-1} + 3^{n-2} + \cdots + 3 + 1$$

$$= \frac{3^{n+1} - 1}{3 - 1}$$

$$= \frac{1}{2}\left[3^{n+1} - 1\right]$$

(4) Use convolution theorem to find $z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$

Soln:

$$z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right] = z^{-1}\left[\frac{8z}{2z-1} * \frac{z}{4z+1}\right]$$

$$= z^{-1}\left[\frac{8z}{2(z-1/2)} * \frac{z}{4(z+1/4)}\right]$$

$$= z^{-1}\left[\frac{z}{z-1/2}\right] * z^{-1}\left[\frac{z}{z+1/4}\right]$$

$$= (1/2)^n * (-1/4)^n$$

$$= \sum_{m=0}^{n} (1/2)^m (-1/4)^{n-m}$$

$$= \sum_{m=0}^{n} \left(\frac{1}{2}\right)^m \left(-\frac{1}{2}\right)^{n-m} \left(\frac{1}{2}\right)^{n-m}$$

$$= \left(\frac{1}{2}\right)^n \sum_{m=0}^{n} \left(-\frac{1}{2}\right)^{n-m}$$

$$= \left(\frac{1}{2}\right)^n \left[\left(-\frac{1}{2}\right)^n + \left(-\frac{1}{2}\right)^{n-1} + \cdots + \left(-\frac{1}{2}\right) + 1\right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1-\left(-\frac{1}{2}\right)^{n+1}}{1-\left(-\frac{1}{2}\right)}\right]$$

$$= \left(\frac{1}{2}\right)^n \frac{\left[1-(-1/2)^{n+1}\right]}{3/2}$$

$$= \frac{2}{3}\left(\frac{1}{2}\right)^n \left[1-(-1/2)^{n+1}\right]$$