PROPERTIES OF Z-TRANSFORM

(i) Linear property:

$$Z[af(n)+bg(n)] = aF(z)+bG(z)$$

$$Z[af(n)+bg(n)] = \sum_{n=0}^{\infty} (af(n)+bg(n))z^{-n}.$$

$$= \sum_{n=0}^{\infty} af(n)z^{-n} + \sum_{n=0}^{\infty} bg(n)z^{-n}$$

$$= aF(z)+bG(z)$$

(ii) First shifting Theorem:

If $Z[f(t)] = F(z)$ then $Z[e^{-at}f(t)] = F[ze^{aT}]$

$$Z[e^{-at}f(t)] = F[e^{-anT}f(nT)]$$

$$= \sum_{n=0}^{\infty} e^{-anT} f(nT)z^{-n}.$$

$$= \sum_{n=0}^{\infty} f(nT)(ze^{aT})^{-n}$$

$$= F(ze^{aT}).$$

Result:

(i) $Z[e^{-at} f(t)] = F(z)_{z \to z e^{aT}}$

(ii) $Z[e^{at} f(t)] = F(z)_{z \to z e^{-aT}}$

(iii) Second shifting theorem:

(i) $Z[f(n+1)] = z F(z) - z f(0)$

(iv) Scaling in Z-Transform (or) multiplication by a^n.

$Z[a^n f(n)] = \{F(z)\}_{z \to z/a}$

(v) Differentiation in Z-domain:

$Z[n f(n)] = -z \frac{d}{dz} [\{F(z)\}]$

1) Find $z[e^{-iat}]$

$$z[e^{-iat}] = z[1 \cdot e^{-iat}]$$

$$= [z(1)]_{z \to ze^{+iaT}}$$

$$= \left[\frac{z}{z-1}\right]_{z \to ze^{iaT}}$$

$$= \frac{ze^{iaT}}{ze^{iaT} - 1}$$

2) Find Z-Transform of cos at & sin at

$z[\cos at]$ & $z[\sin at]$

$$z[e^{iat}] = z[1 \cdot e^{iat}]$$

$$= [z(1)]_{z \to ze^{-iaT}}$$

$$= \left[\frac{z}{z-1}\right]_{z \to ze^{-iaT}}$$

$$= \frac{ze^{-iaT}}{ze^{-iaT} - 1}$$

Divide by e^{-iaT}

$$= \frac{z}{z - e^{iaT}}$$

$$= \frac{z}{z - [\cos aT + i \sin aT]}$$

$$= \frac{z}{(z - \cos aT) - i \sin aT}$$

$$z[\cos at + i \sin at] = \frac{z[z - \cos aT + i \sin aT]}{(z - \cos aT)^2 + (\sin aT)^2} \quad \left[\text{taking conjugate}\right]$$

$$\Rightarrow z[\cos at] = \frac{z[z - \cos aT]}{(z - \cos aT)^2 + (\sin aT)^2} = \frac{z[z - \cos aT]}{z^2 - 2z\cos aT + 1}$$

$$\Rightarrow z[\sin at] = \frac{z \sin aT}{z^2 - 2z\cos aT + 1}$$

) $Z[n^2+2n+3]$

$$Z[n^2+2n+3] = Z[n^2] + 2Z[n] + Z[3]$$

$$= Z[n^2] + 2Z[n] + 3[Z(1)]$$

$$= \frac{Z[Z+1]}{(Z-1)^3} + 2\cdot\frac{Z}{(Z-1)^2} + 3\cdot\frac{Z}{Z-1}$$

$$= \frac{Z^2+Z+2Z[Z-1]+3Z[Z-1]^2}{(Z-1)^3}$$

$$= \frac{Z^2+Z+2Z^2-2Z+3Z^3+3Z-6Z^2}{(Z-1)^3}$$

$$= \frac{3Z^3-3Z^2+2Z}{(Z-1)^3} = \frac{Z[3Z^2-3Z+2]}{(Z-1)^3}$$