SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

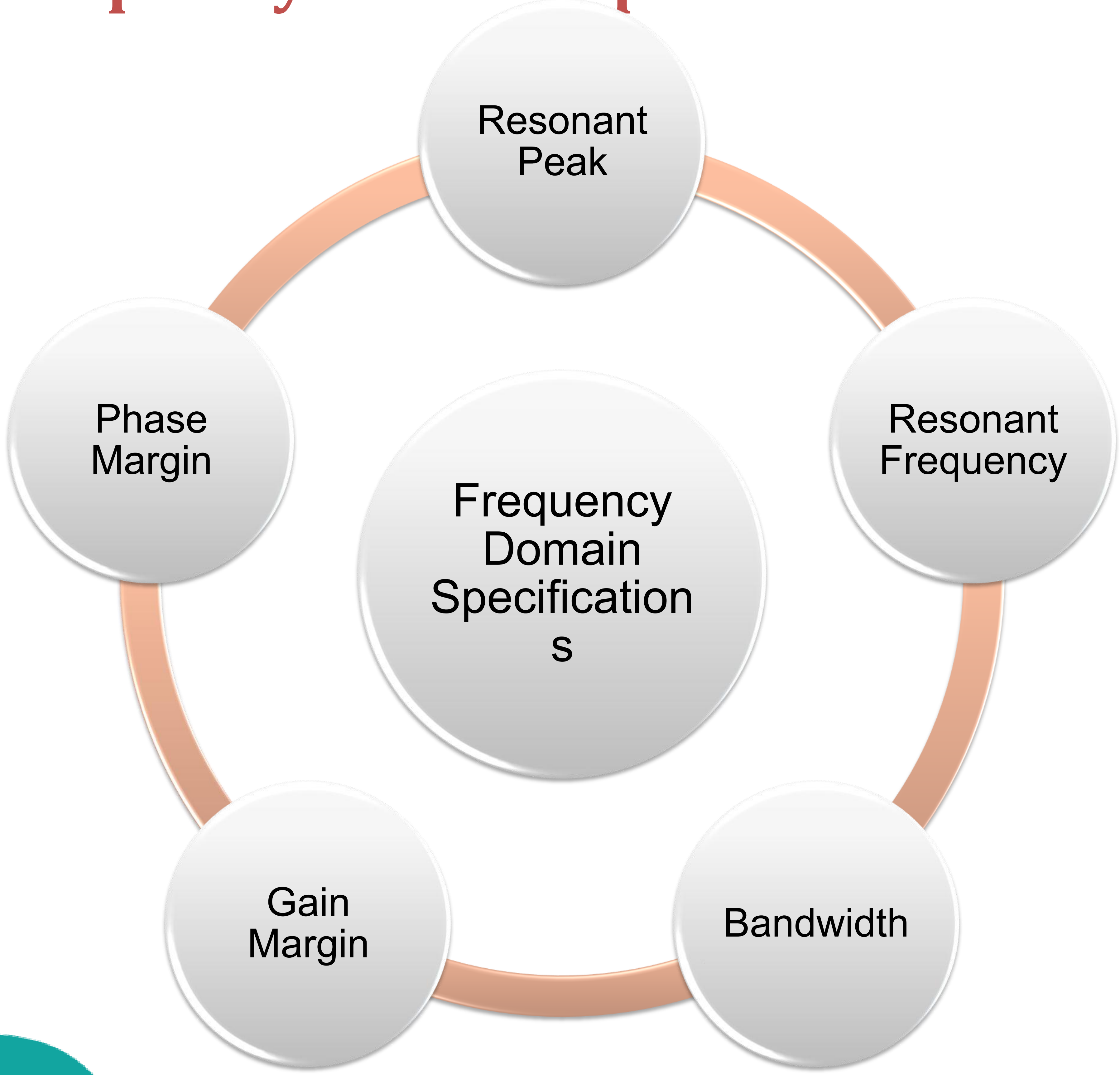
Course Name: Control Systems

III Year : V Semester

Unit II –Frequency Response Analysis

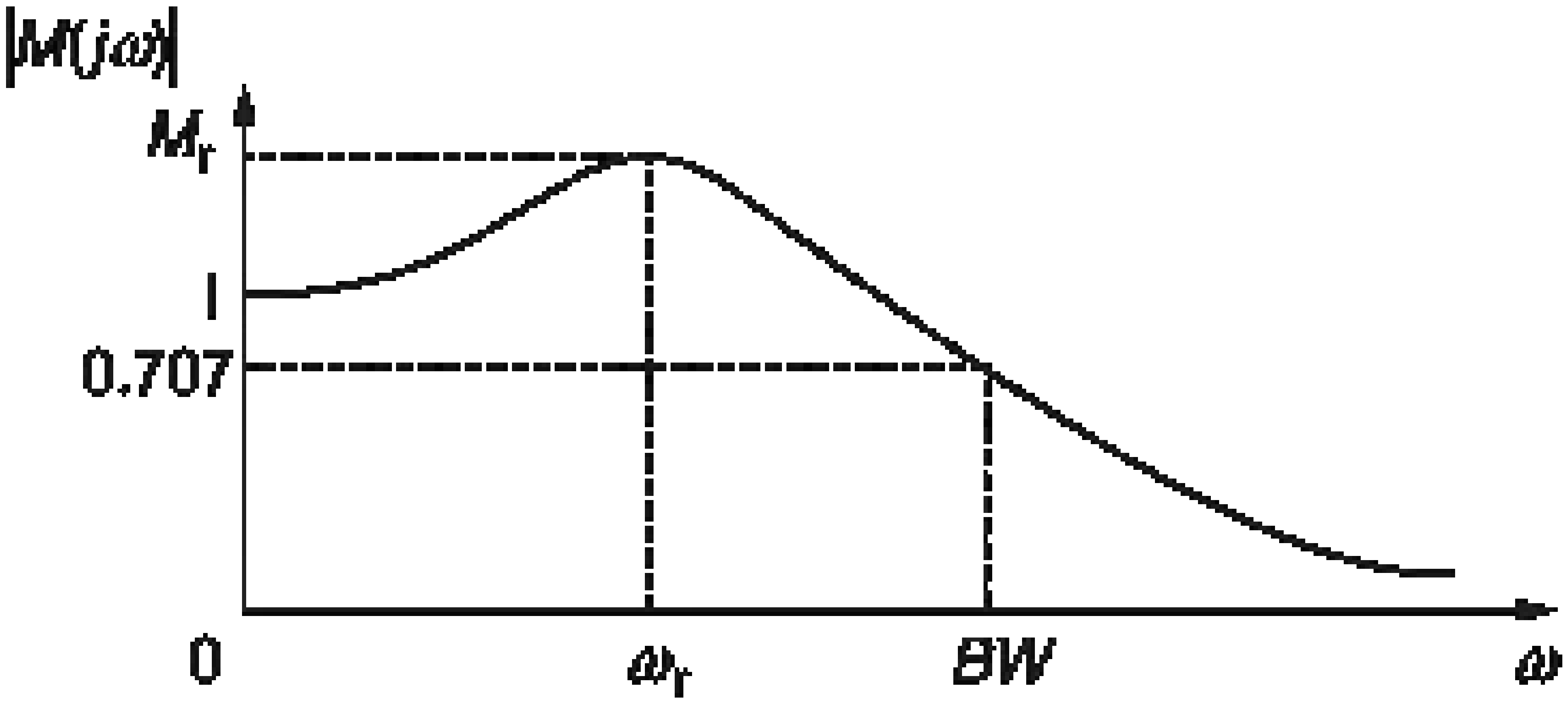
Topic : Frequency Domain Specifications

Frequency Domain Specifications



Frequency Domain Specifications

- The steady state response of a system to a purely sinusoidal input is defined as the frequency response of a system.



Frequency Domain Specifications

- Consider the transfer function of the second order closed loop control system as

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

- Substitute, s=jω in the above equation.

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n(j\omega) + \omega_n^2}$$

- Magnitude of T(jω) is

$$M = |T(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\delta u)^2}}$$

- Phase of T(jω) is

$$\angle T(j\omega) = -tan^{-1}\left(\frac{2\delta u}{1-u^2}\right)$$

Resonant Frequency

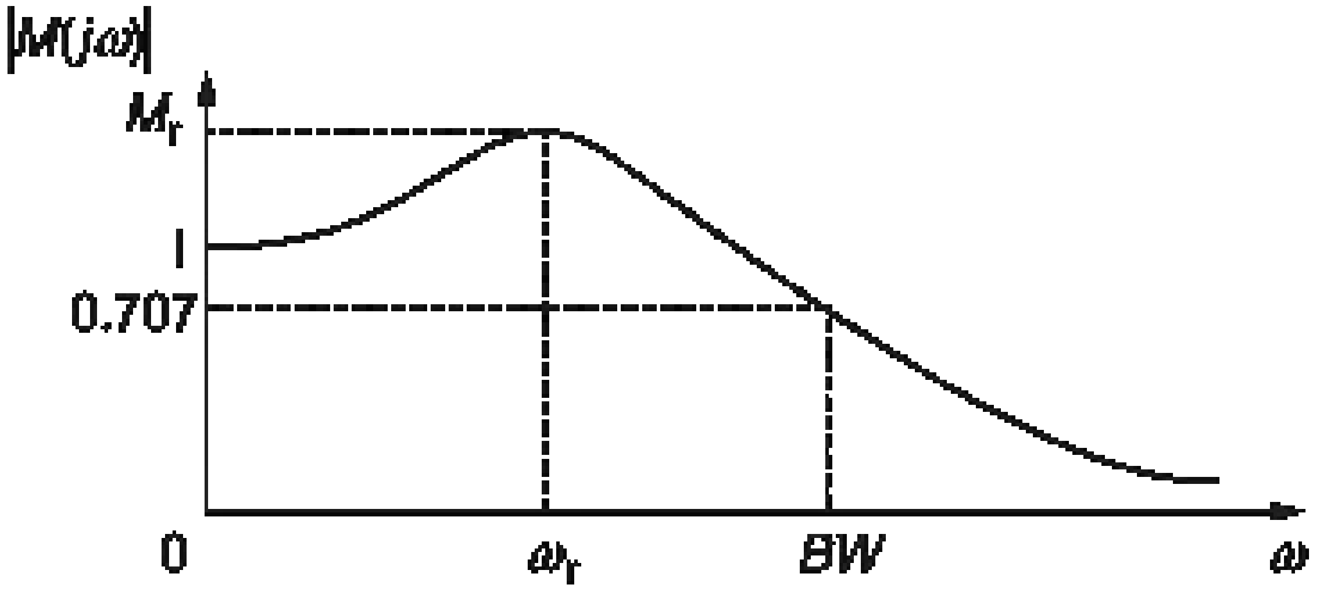
- It is the frequency at which the magnitude of the frequency response has peak value for the first time. It is denoted by ω_r. At $\omega=\omega_r$, the first derivate of the magnitude of T(jω) is zero.

$$\frac{dM}{du} = -\frac{1}{2}\left[\left(1-u^2\right)^2+\left(2\delta u\right)^2\right]^{\frac{-3}{2}}\left[2\left(1-u^2\right)\left(-2u\right)+2\left(2\delta u\right)\left(2\delta\right)\right]$$

Substitute, $u = u_r$ and $\frac{dM}{du} == 0$ in the above equation.

$$u_r = \sqrt{1-2\delta^2}$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\delta^2}}$$



Resonant Peak

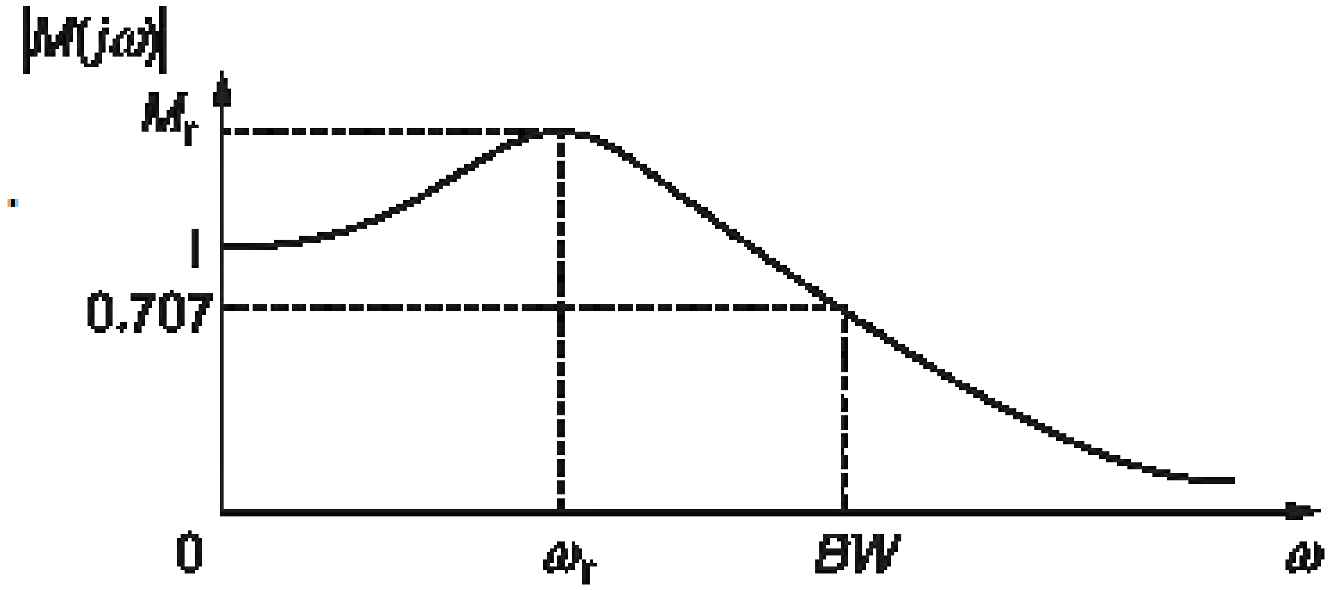
- It is the peak (maximum) value of the magnitude of T(jω). It is denoted by M_r.
- At $u=u_r$, the Magnitude of T(jω) is –

$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2 + (2\delta u_r)^2}}$$

Substitute, $u_r = \sqrt{1-2\delta^2}$ and $1-u_r^2 = 2\delta^2$ in the above equation.

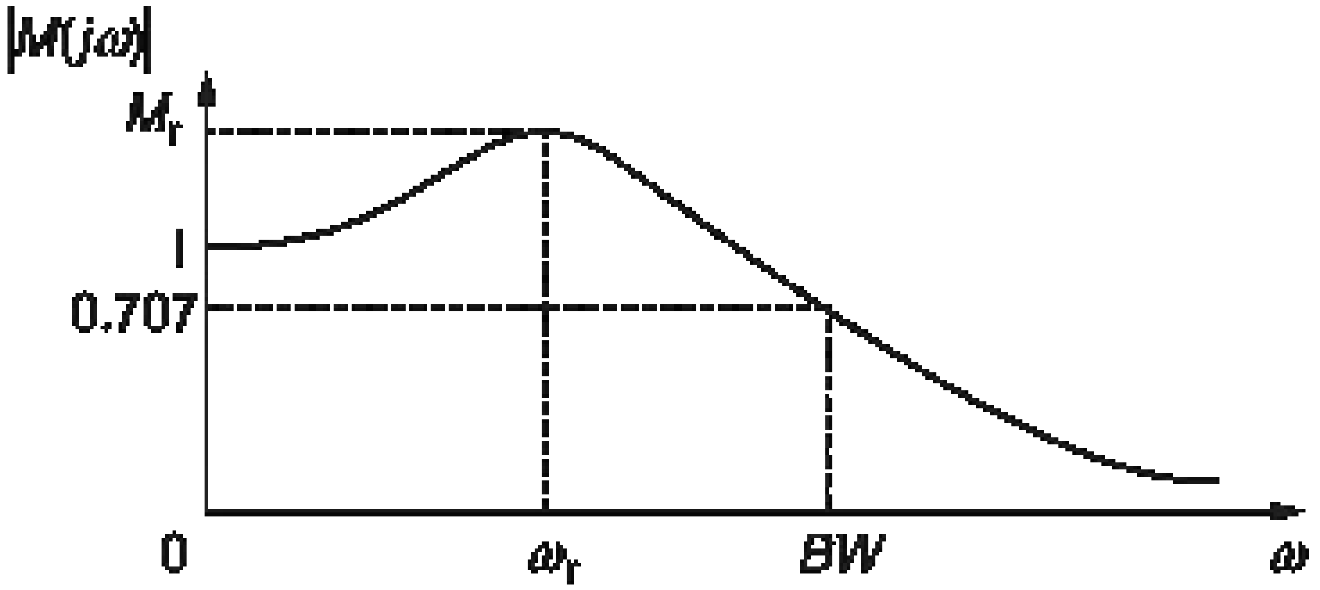
$$M_r = \frac{1}{\sqrt{(2\delta^2)^2 + (2\delta\sqrt{1-2\delta^2})^2}}$$

$$\Rightarrow M_r = \frac{1}{2\delta\sqrt{1-\delta^2}}$$



Bandwidth

- It is the range of frequencies over which, the magnitude of T(jω) drops to 70.7% from its zero frequency value.
- At 3-dB frequency, the magnitude of T(jω) will be 70.7% of magnitude of T(jω) at ω=0.
- i.e., at $\omega=\omega_b$ M=0.707(1)=1/√2



$$\Rightarrow M = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\delta u_b)^2}}$$

$$\Rightarrow \omega_b = \omega_n \sqrt{1 - 2\delta^2 + \sqrt{(2 - 4\delta^2 + 4\delta^4)}}$$