SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

Course Name: Control Systems

III Year : V Semester

Unit II –Time Response Analysis

Topic : Routh-Hurwitz Criterion

Introduction

- Routh's stability criterion allows the determination of whether there are any roots of the characteristic equation with positive real parts and, if there are, the number of these roots without actually finding the roots.
- The necessary but not sufficient condition for a characteristic equation

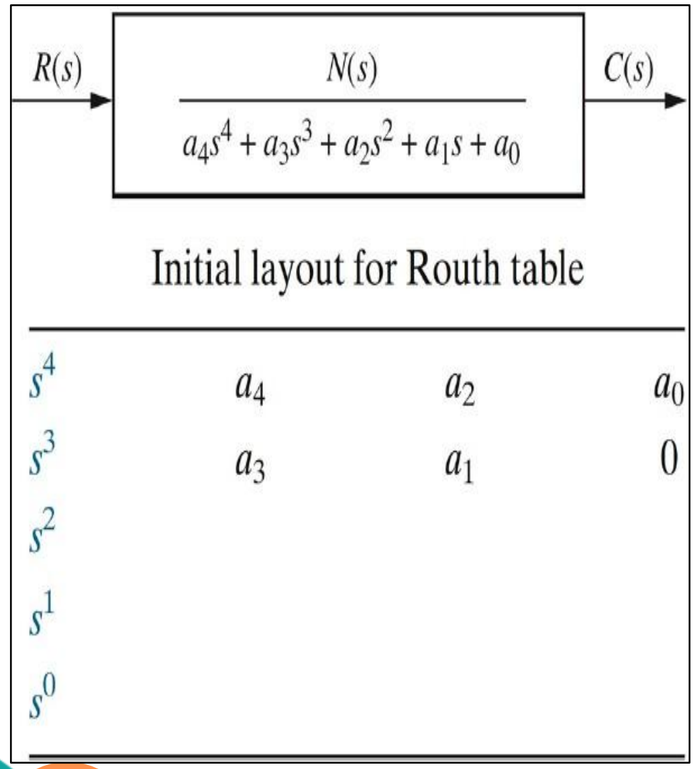
$$D(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = 0$$

- to have all its roots with negative real parts is that all of the coefficients a_i must exist and have the same sign.
- If the characteristic equation fails to meet the above condition, then the system is not stable.

Introduction

- If Hurwitz condition is satisfied, then Routh's stability criterion must be used to determine the stability of the system.
- To be able to apply Routh's criterion, Routh's array must be constructed.
- Begin by labeling the rows with powers of s from the highest power of the denominator polynomial to s0
- List in the first row every other coefficient starting with the one of the highest power of s.
- List in the second row coefficients that were skipped in the first row.
- Complete the rest of the table.

Routh-Hurwitz Criterion



Initial layout for Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

Routh array
(How to compute the third row)

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

$$b_1 = \frac{a_{n-2}a_{n-1} - a_n a_{n-3}}{a_{n-1}}$$

$$b_2 = \frac{a_{n-4}a_{n-1} - a_n a_{n-5}}{a_{n-1}}$$

$\vdots$

txp_fig

Classification of Stability

Routh-Hurwitz criterion

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.

- The necessary and sufficient condition for a characteristic equation to have all its roots with negative real parts is that the elements of the first column of the Routh's array to have the same sign.
- If the elements of the first column have different signs, then the number of sign changes is equal to the number of roots with positive real parts.