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Department of Biomedical Engineering

Course Name: Control Systems

III Year : V Semester

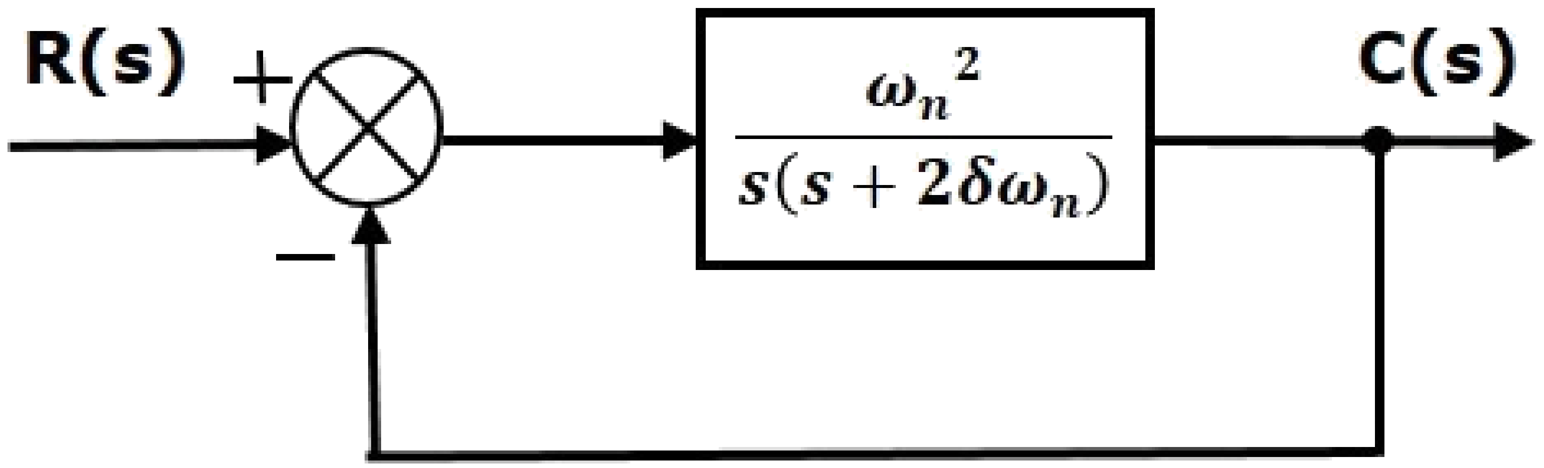
Unit II –Time Response

Topic : Second Order System

Introduction

Consider the following block diagram of the closed loop control system.

- Here, an open loop transfer function, $\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$ is connected with a unity negative feedback. The system is called as second order system



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

Second Order Response

- Depending on the value of damping ratio, second order system can be classified into:
  1. Undamped system, $\zeta = 0$
  2. Underdamped system, $0 < \zeta < 1$
  3. Critically damped system, $\zeta = 1$
  4. Overdamped system, $\zeta > 1$
- The characteristic equation is given by,

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$

Second Order Response

- The roots of characteristic equation is given by,

$$s = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$$

- The roots are imaginary when ζ = 0
- The roots are real and equal when ζ = 1
- The roots are real and unequal when ζ >1
- The roots are complex conjugate when 0 < ζ < 1

Undamped System

- Step Response of undamped second order system:

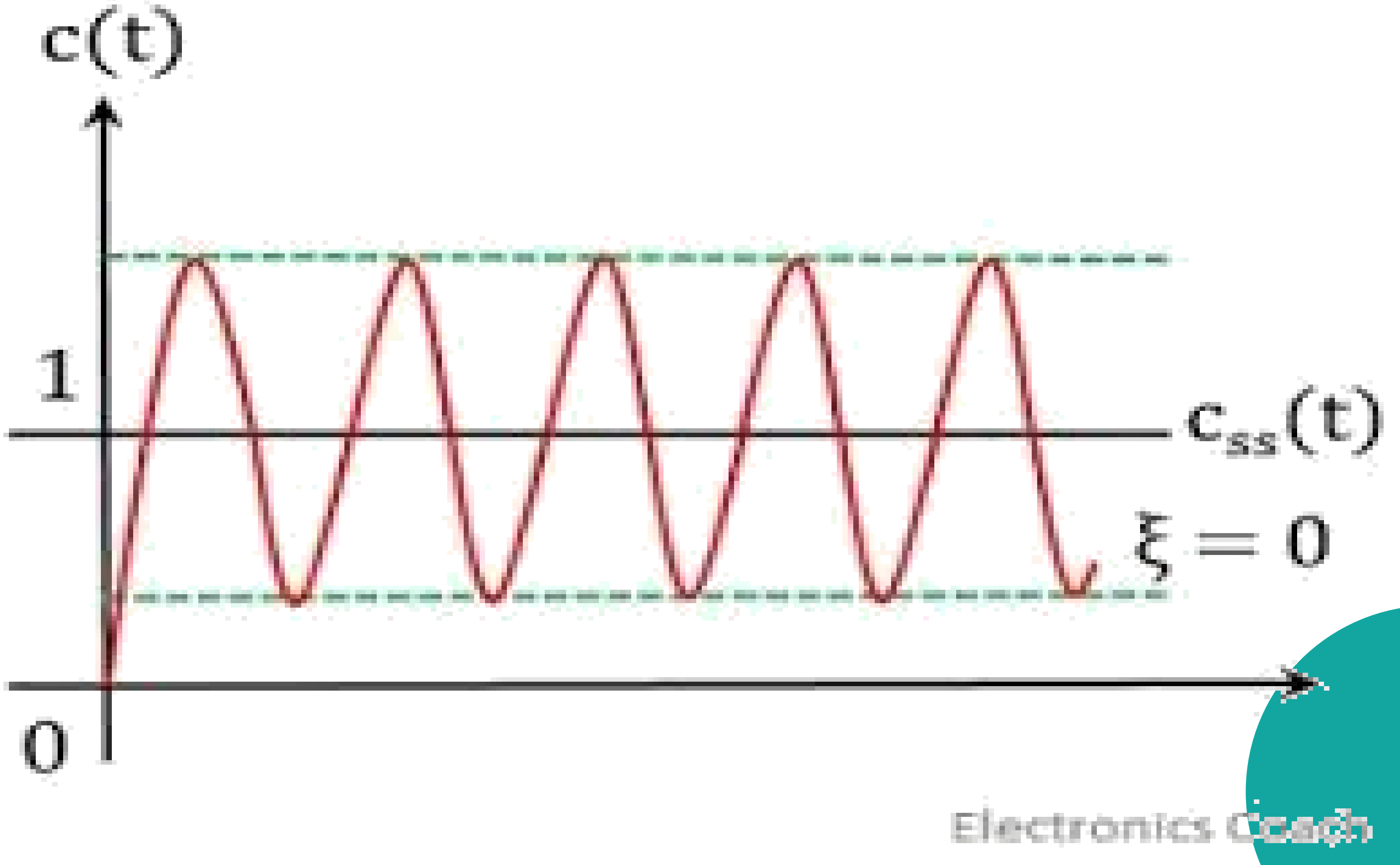
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When ζ = 0

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

- The response of the system for unit step input is

$$c(t) = (1 - \cos(\omega_n t))$$



Under damped System

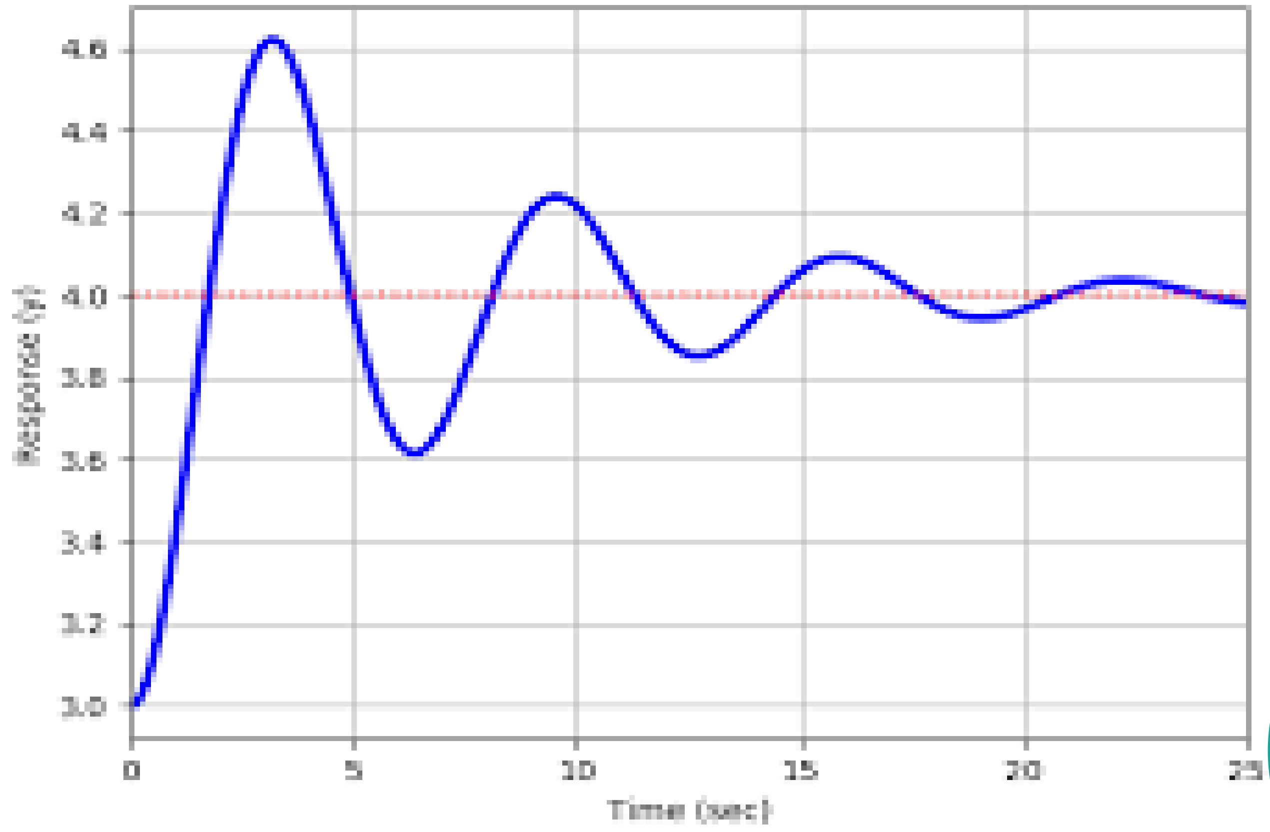
- Step Response of underdamped second order system:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When $0 < \zeta < 1$
- The response of the system for unit step input is

$$\frac{C(s)}{R(s)} = \frac{A}{s} + \frac{(Bs + C)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$c(t) = \left(1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)\right)$$



Critically damped System

- Step Response of underdamped second order system:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- When ζ = 1:
- The response of the system for unit step input is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$$c(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})$$

Second Order Response