SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB301-ANALOG AND DIGITAL COMMUNICATION

III YEAR/ V SEMESTER

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UNIT 5 – INFORMATION THEORY AND ERROR CONTROL CODING

TOPIC – MUTUAL INFORMATION-CHANNEL CAPACITY

MUTUAL INFORMATION

- **Mutual information** measures the amount of information that can be obtained about one random variable by observing another.
- It is important in communication where it can be used to maximize the amount of information shared between sent and received signals.
- The mutual information denoted by I (X, Y) of a channel is defined by:

$$I(X;\ Y) = H(X) - H\left(\frac{X}{Y}\right)\ b/symbol$$

- Since H (X) represents the uncertainty about the channel input before the channel output is observed and H (X/Y) represents the uncertainty about the channel input after the channel output is observed, the mutual information I (X; Y) represents the uncertainty about the channel input that is resolved by observing the channel output.

MUTUAL INFORMATION

- **Properties of Mutual Information I (X; Y)**
- **I (X; Y) = I(Y; X)**
- **I (X; Y) ≥ 0**
- **I (X; Y) = H (Y) – H (Y/X)**
- **I (X; Y) = H(X) + H(Y) – H(X,Y)**

- The Entropy corresponding to mutual information [i.e. I (X, Y)] indicates a measure of the information transmitted through a channel. Hence, it is called **'Transferred information'**.

CHANNEL CAPACITY

- The channel capacity represents the maximum amount of information that can be transmitted by a channel per second.
- To achieve this rate of transmission, the information has to be processed properly or coded in the most efficient manner.
- **Channel Capacity per Symbol C_S:** The channel capacity per symbol of a discrete memoryless channel (DMC) is defined as

$$C_S = \max_{\{P(x_i)\}} I(X;Y) b/symbol$$

Where the maximization is over all possible input probability distributions $\{P(x_i)\}$ on X.

- **Channel Capacity per Second C:** I f 'r' symbols are being transmitted per second, then the maximum rate od transmission of information per second is 'r C_S'. this is the channel capacity per second and is denoted by C (b/s) i.e.

$$C = rC_S \; b/s$$

CAPACITIES OF SPECIAL CHANNELS

- **Lossless Channel:** For a lossless channel, H (X/Y) = 0 and I (X; Y) = H (X).
- Thus the mutual information is equal to the input entropy and no source information is lost in transmission.

$$C_S = \max_{\{P(x_i)\}} H(X) = \log_2 m$$

Where m is the number of symbols in X.

- **Deterministic Channel:** For a deterministic channel, H (Y/X) = 0 for all input distributions P (x_i) and I (X; Y) = H (Y).
- Thus the information transfer is equal to the output entropy. The channel capacity per symbol will be

$$C_S = \max_{\{P(x_i)\}} H(Y) = \log_2 n$$

where n is the number of symbols in Y.

CAPACITIES OF SPECIAL CHANNELS

- **Noiseless Channel:** since a noiseless channel is both lossless and deterministic, we have I (X; Y) = H (X) = H (Y) and the channel capacity per symbol is

$$C_S = \log_2 m = \log_2 n$$

- **Binary Symmetric Channel:** For the BSC, the mutual information is

$$I(X;Y) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)$$

And the channel capacity per symbol will be

$$C_S = 1 + p\log_2 p + (1-p)\log_2(1-p)$$

THANK YOU