SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Example 5.20 Realize the second order digital filter $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

Solution

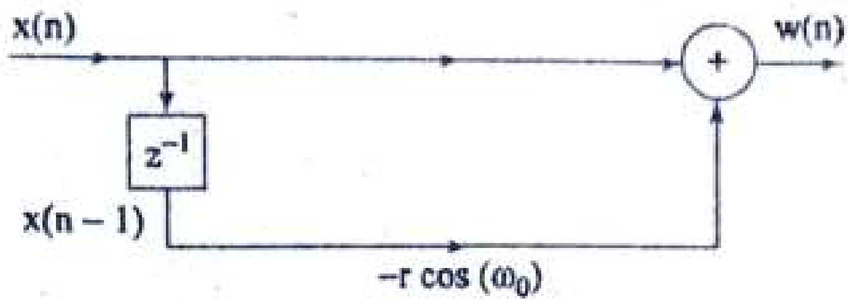
Let

$$x(n) - r\cos(\omega_0)x(n-1) = w(n) \tag{5.106}$$

then

$$y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + w(n) \tag{5.107}$$

Realizing Eq.(5.106) we get



Realizing Eq.(5.107) we obtain

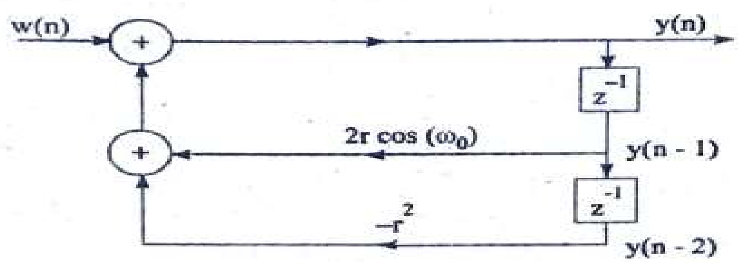
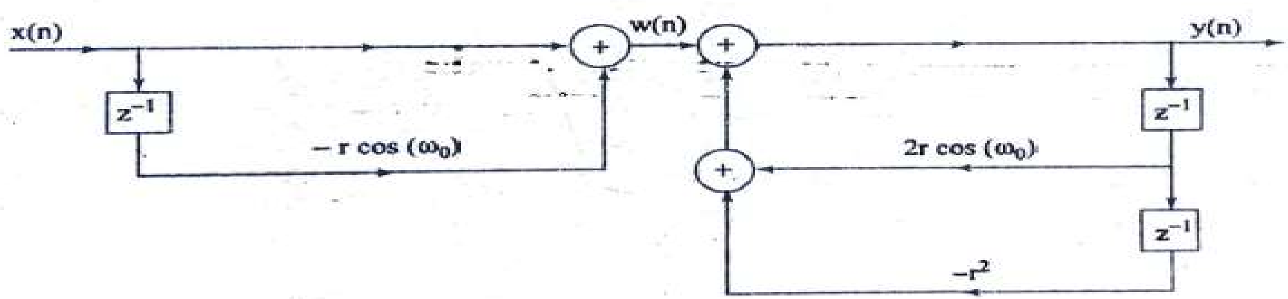


Fig. 5.32 Realization of Equation 5.107

If we combine both figures, we obtain the realization of the second order digital filter as shown in Fig. 5.33.



Example 5.21 Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Solution

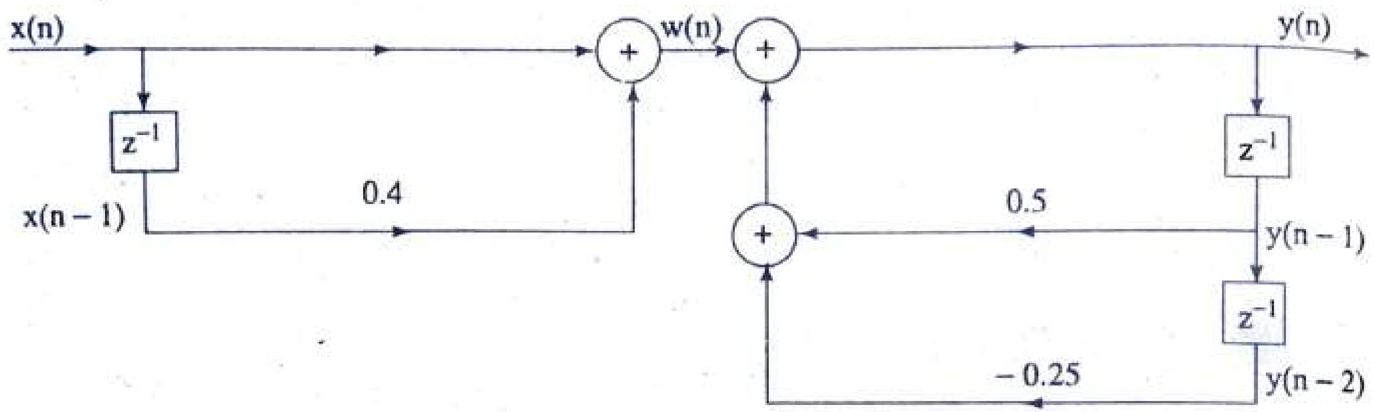
Let

$$x(n) + 0.4x(n-1) = w(n) \qquad (5.108)$$

then

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \qquad (5.109)$$

Realizing Eq. (5.108) and Eq. (5.109) and combining we get



5.14.2 *Direct form II realization*

Consider the difference equation of the form

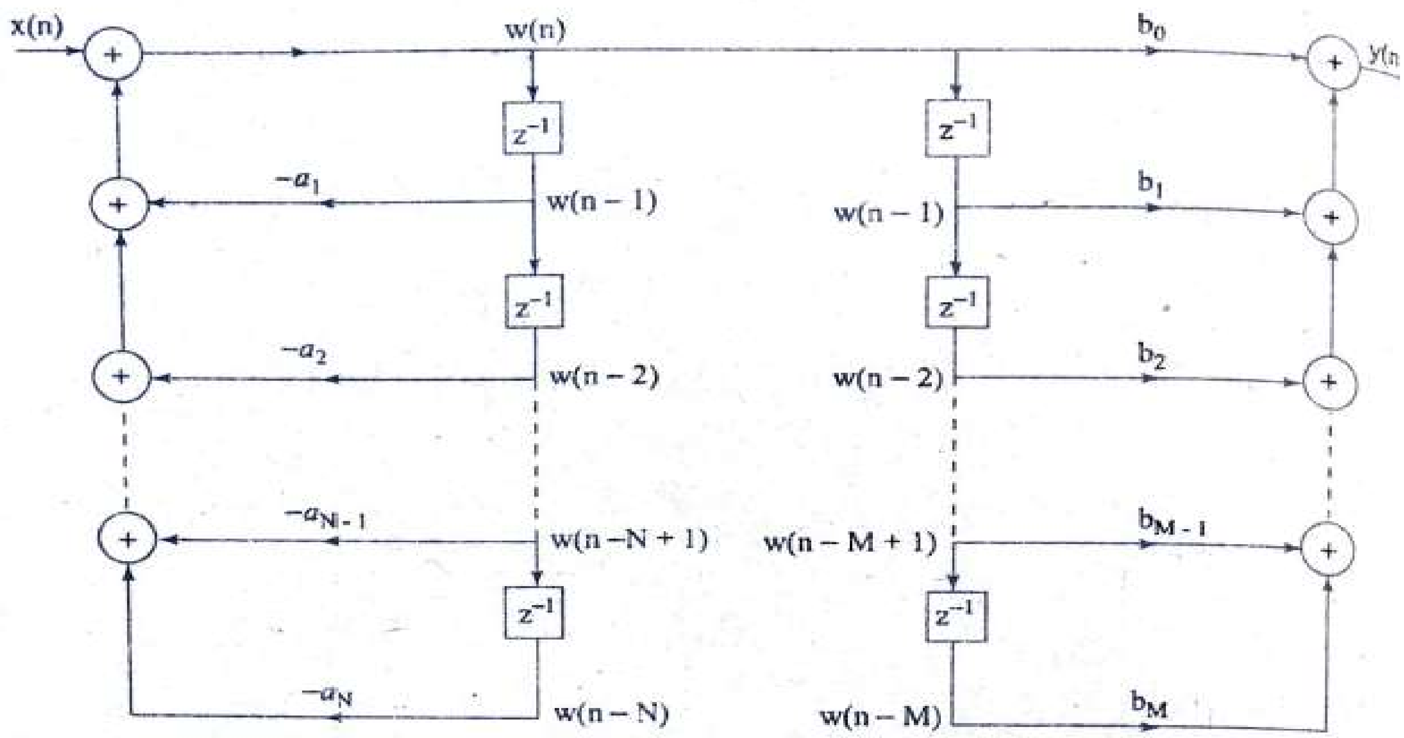
$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad (5.110)$$

The system function of above difference equation can be expressed as

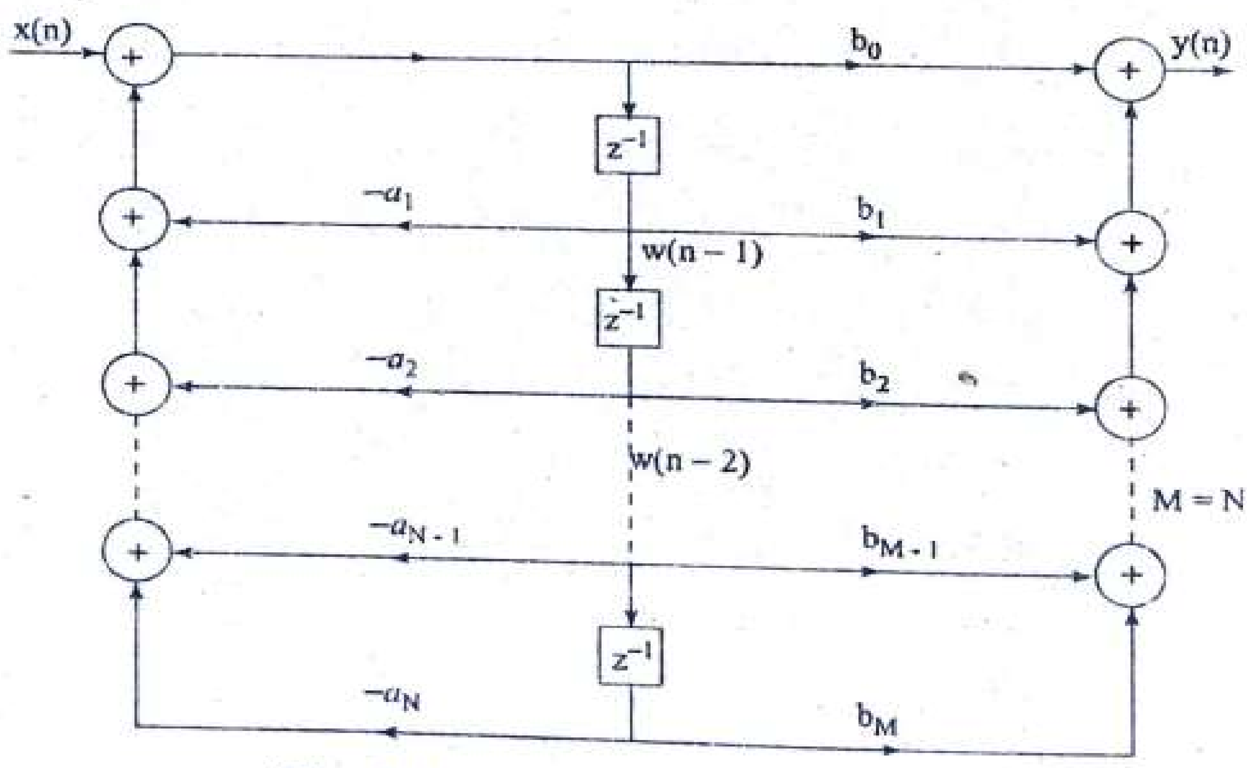
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad (5.111)$$

Let $\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$ where

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum^{N} a_k z^{-k}}$$



x(n)
w(n)
b_0
y(n)
z^{-1}
$-a_1$
w(n – 1)
b_1
$-a_2$
w(n – 2)
b_2
$-a_{N-1}$
w(n –N + 1)
w(n – M + 1)
b_{M-1}
$-a_N$
w(n – N)
w(n – M)
b_M



x(n)
y(n)
b_0
z^{-1}
$-a_1$
b_1
w(n − 1)
z^{-1}
$-a_2$
b_2
w(n − 2)
M = N
$-a_{N-1}$
b_{M-1}
z^{-1}
$-a_N$
b_M

Example 5.22 Realize the second order system $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$ in direct form II.

Solution

Given $y(n) = 2r\cos(\omega_0)y(n-1) - r^2y(n-2) + x(n) - r\cos(\omega_0)x(n-1)$

The system function

$$\frac{Y(z)}{X(z)} = \frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.116}$$

Let $\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$

where
$$\frac{Y(z)}{W(z)} = 1 - r\cos(\omega_0)z^{-1} \tag{5.117a}$$

and
$$\frac{W(z)}{X(z)} = \frac{1}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}} \tag{5.117b}$$

From Eq. (5.117a) we obtain $Y(z) = W(z) - r\cos(\omega_0)z^{-1}W(z)$ which gives us

$$y(n) = w(n) - r\cos(\omega_0)w(n-1) \tag{5.118a}$$

and from Eq. (5.117b) we have $W(z) = X(z) + 2r\cos(\omega_0)z^{-1}W(z) - r^2z^{-2}W(z)$ which gives us

$$w(n) = x(n) + 2r\cos(\omega_0)w(n-1) - r^2w(n-2) \tag{5.118b}$$

We realize Eq. (5.118a) and Eq. (5.118b) and combine them to get the direct form II realization.

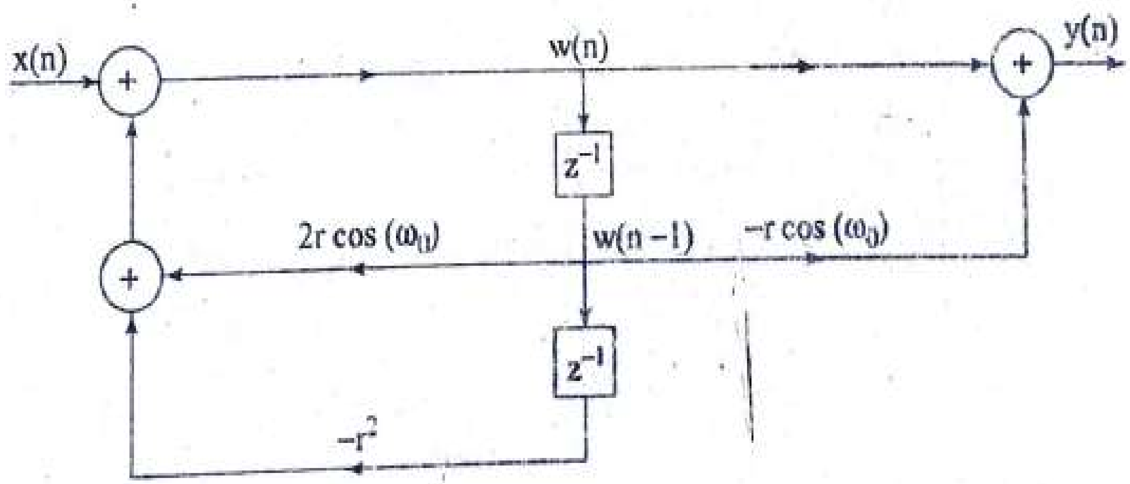


Fig. 5.39 Direct form II realization of example (5.21)

Example 5.23 Determine the direct form II realization for the following system $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

(AU ECE May'07)

Solution

The system function is given by

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} \quad (5.119)$$

Let

$$\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2}$$

$$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$$

Then

$$y(n) = 0.7w(n) - 0.252w(n-2) \tag{5.120}$$

Similarly let $\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$

$$W(z) = X(z) - 0.1z^{-1}W(z) + 0.72z^{-2}W(z)$$

then $w(n) = x(n) - 0.1w(n-1) + 0.72w(n-2)$ (5.121)

If we realize Eq. (5.120) and Eq. (5.121) and combine them we get direct form II realization of the system shown in Fig. 5.40.

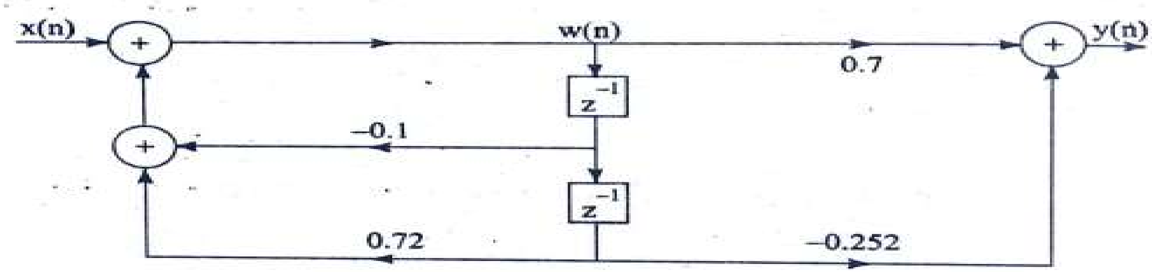


Fig. 5.40 Direct form II realization of example (5.23)

Example 5.25 Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade form.

Solution

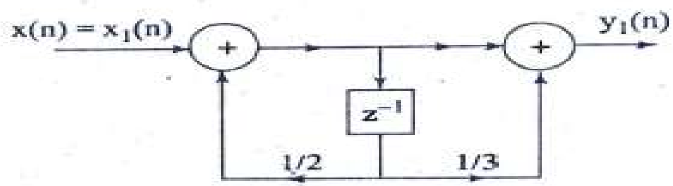
From the difference equation we obtain

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

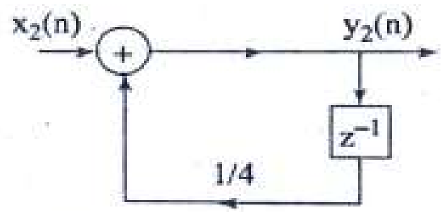
$$= \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} = H_1(z)H_2(z)$$

where $H_1(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$ and $H_2(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$.

$H_1(z)$ can be realized in direct form II as



Similarly, $H_2(z)$ can be realized in direct form II as



Cascading the realization of $H_1(z)$ and $H_2(z)$ we have

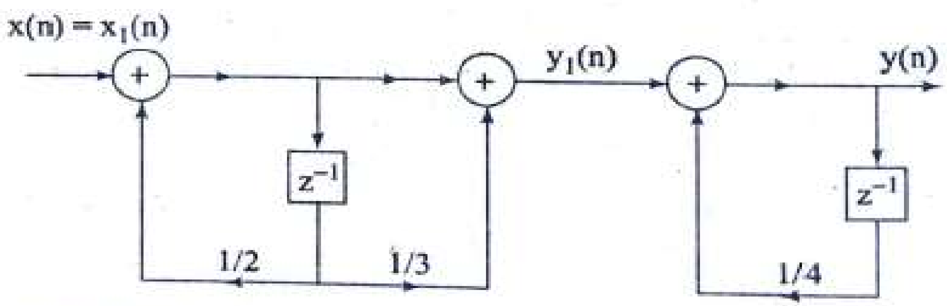


Fig. 5.48 Cascade realization of Example 5.25

5.14.6 *Parallel form structure*

A parallel form realization of an IIR system can be obtained by performing a partial expansion of

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - p_k z^{-1}} \qquad (5.123)$$

where $\{p_k\}$ are the poles

The Eq. (5.123) can be written as

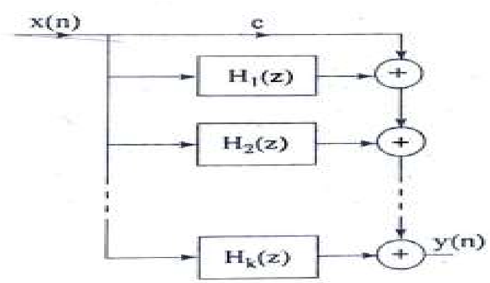
$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \ldots + \frac{c_N}{1 - p_N z^{-1}} \qquad (5.124)$$

$$H(z) = \frac{Y(z)}{X(z)} = c + H_1(z) + H_2(z) + \ldots + H_N(z) \qquad (5.125)$$

Now

$$Y(z) = cX(z) + H_1(z)X(z) + H_2(z)X(z) + \ldots + H_N(z)X(z) \qquad (5.126)$$

The Eq. (5.126) can be realized in parallel form as shown in Fig. 5.49.



Example 5.26 Realize the system given by difference equation $y(n) = -0.1\, y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in parallel form.

Solution

The system function of the difference equation is

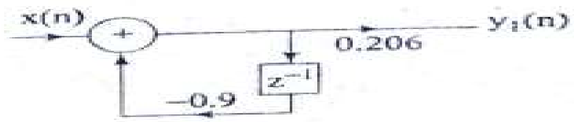
$$H(z) = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

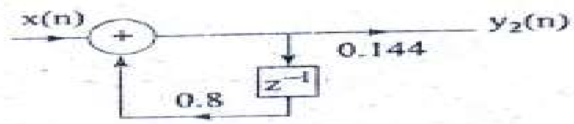
$$= 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$

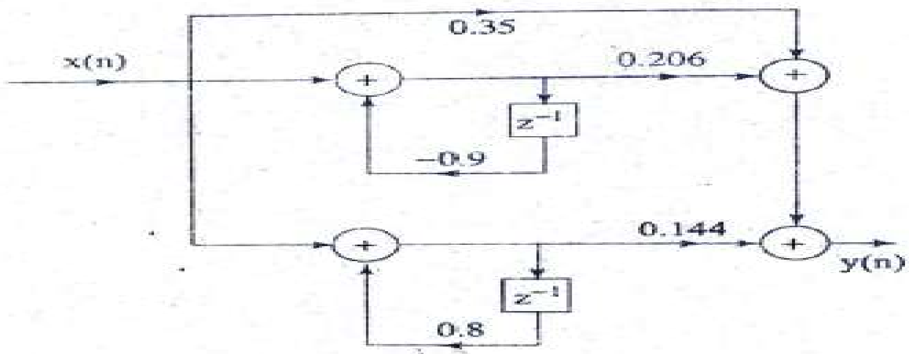
$H_1(z)$ can be realized in direct form II as



$H_2(z)$ can be realized in direct form II as



Now the realization of $H(z)$ is shown in Fig. 5.50.



Example 5.27 Obtain the direct form I, direct form II, cascade and paralle realization for the system $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6$ $1) + 0.6x(n-2)$

Solution

Direct form I

Let $3x(n) + 3.6x(n-1) + 0.6x(n-2) = w(n)$

$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$

By inspection, The direct form I realization is shown in Fig. 5.51.

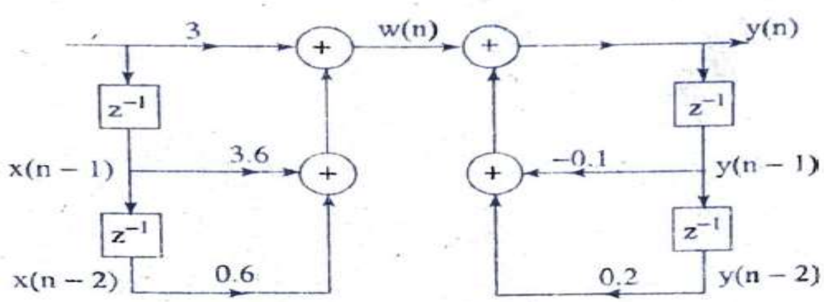


Fig. 5.51 Direct form I realization of example 5.27

Direct form II

From the given difference equation we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

The above system function can be realized in direct form II as shown in Fig. 5.52.

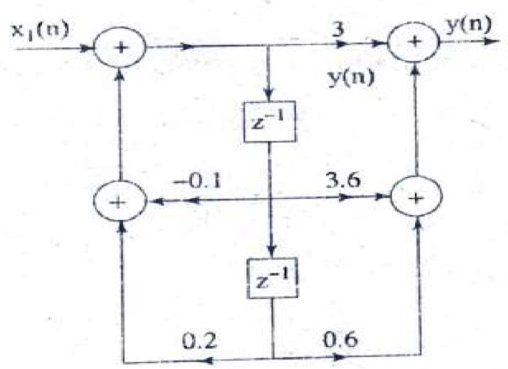


Fig. 5.52 Direct form II realization of example 5.27

Cascade form

$$\text{we have} \quad \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

$$\text{Let} \quad H_1(z) = \frac{3 + 0.6z^{-1}}{1 + 0.5z^{-1}} \quad \text{and}$$

$$H_2(z) = \frac{1 + z^{-1}}{1 - 0.4z^{-1}}$$

Now we realize $H_1(z)$ and $H_2(z)$ and cascade both to get realization of $H(z)$

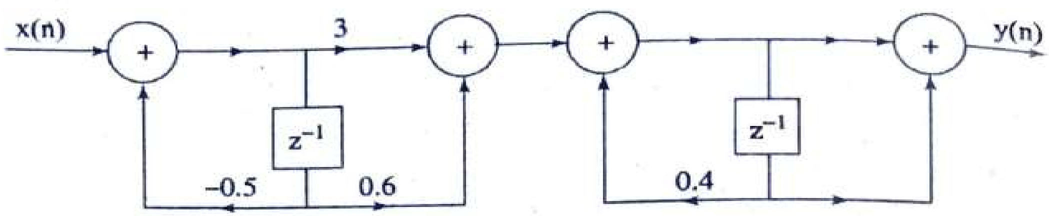


Fig. 5.53 Cascade form realization of example 5.27

Parallel form

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$
$$= -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$
$$= c + H_1(z) + H_2(z)$$

$$-0.2z^{-2} + 0.1z^{-1} + 1 \;\overline{\big)\, 0.6z^{-2} + 3.6z^{-1} + 3}\quad -3$$
$$0.6z^{-2} - 0.3z^{-1} - 3$$
$$3.9z^{-1} + 6$$

$$\rightarrow \quad H(z) = -3 + \frac{3.9z^{-1} + 6}{1 + 0.1z^{-1} - 0.2z^{-2}}$$
$$= -3 + \frac{A}{1 - 0.4z^{-1}} + \frac{B}{1 + 0.5z^{-1}}$$

where $A = 7, B = -1$

Now we realize $H(z)$ in parallel form as shown in Fig. 5.54.

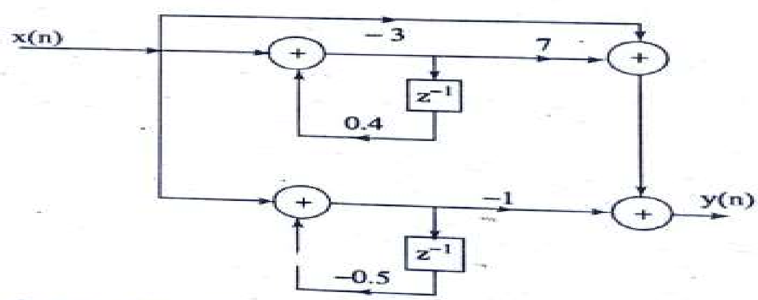


Fig. 5.54 Parallel form realization of example 5.27