SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Steps to design digital filter using bilinear transform technique.

1. From the given specifications, find prewarping analog frequencies using formula $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$.
2. Using the analog frequencies find $H(s)$ of the analog filter.
3. Select the sampling rate of the digital filter, call it T seconds per sample.
4. Substitute $s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$ into the transfer function found in step 2.

Example 5.16 Apply bilinear transformation to $H(s) = \frac{2}{(s+1)(s+2)}$ with $T = 1$ sec and find $H(z)$.

Solution

Given $H(s) = \frac{2}{(s+1)(s+2)}$

Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{2}{(s+1)(s+2)}\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Given $T = 1$ sec

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)}$$

$$= \frac{(1+z^{-1})^2}{6-2z^{-1}}$$

$$= \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

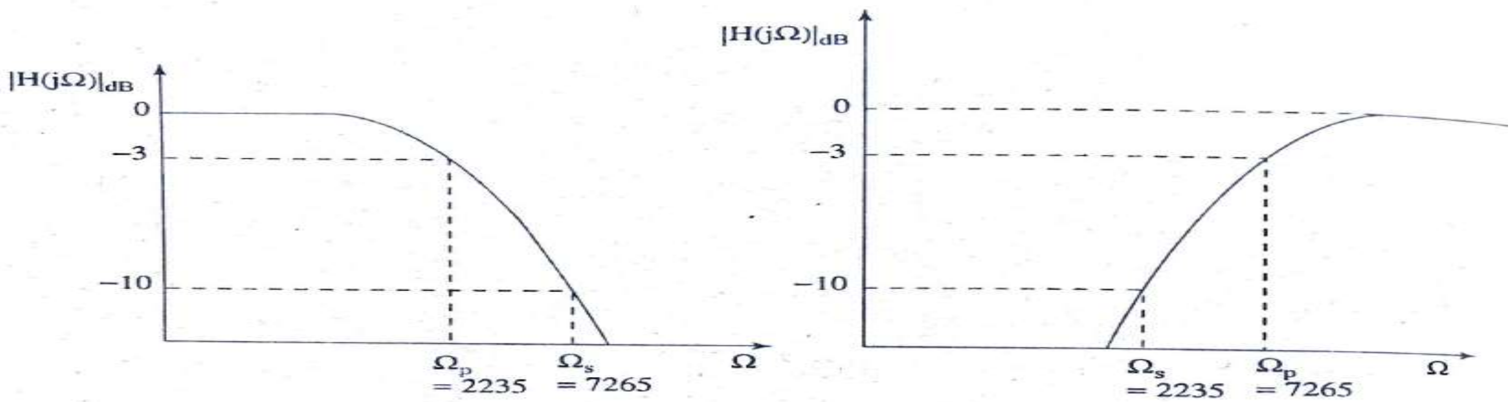
Example 5.17 Using the bilinear transform, design a highpass filter, monotonic in passband with cutoff frequency of 1000 Hz and down 10 dB at 350 Hz. The sampling frequency is 5000 Hz.

Solution

Given $\alpha_p = 3\,\text{dB};\ \omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/sec

$$\alpha_s = 10\,\text{dB}; \quad \omega_s = 2 \times \pi \times 350 = 700\pi \text{ rad/sec}$$

$$T = \frac{1}{f} = \frac{1}{5000} = 2 \times 10^{-4} sec$$



The characteristics are monotonic in both passband and stopband. Therefore, the filter is Butterworth filter.

Prewarping the digital frequencies we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times 10^{-4}}\tan\frac{(2000\pi \times 2\times 10^{-4})}{2}$$
$$= 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$
$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times 10^{-4}}\tan\frac{(700\pi \times 2\times 10^{-4})}{2}$$
$$= 10^4 \tan(0.07\pi) = 2235\text{rad/sec}$$

First we design a lowpass filter for the given specifications and use suitable transformation to obtain transfer function of highpass filter.

The order of the filter

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\frac{7265}{2235}} = \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

Therefore, we take $N = 1$.

The first-order Butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

The highpass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$s \longrightarrow \frac{\Omega_c}{s}$$

$$i.e., \; s \longrightarrow \frac{(7265)}{s}$$

The transfer function of highpass filter

$$H(s) = \frac{1}{s+1}\Big|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Big|_{s=\frac{2}{2\times 10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$

Example 5.18 Determine $H(z)$ that results when the bilinear transformation is applied to $H_a(s) = \dfrac{s^2 + 4.525}{s^2 + 0.692s + 0.504}$

Solution

In bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Assume $T = 1$ sec.

Then

$$H(z) = \frac{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 4.525}{4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 0.692 \times 2 \times \left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.504}$$

$$= \frac{1.4479 + 0.1783z^{-1} + 1.4479z^{-2}}{1 - 1.18752z^{-1} + 0.5299z^{-2}}$$