SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Example 5.31 Design a Chebyshev lowpass filter with the specifications $\alpha_p = 1\,\text{dB}$ ripple in the passband $0 \le \omega \le 0.2\pi$, $\alpha_s = 15\,\text{dB}$ ripple in the stopband $0.3\pi \le \omega \le \pi$, using (a) bilinear transformation (b) Impulse invariance.

Solution

Given data $\alpha_p = 1\,\text{dB}; \omega_p = 0.2\pi; \alpha_s = 15\,\text{dB}; \omega_s = 0.3\pi$.

Prewarped frequency values: Since we intend to employ the bilinear transformation method, we must prewarp these frequencies. The prewarped values are given by (Assume $T = 1$ sec);

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\frac{0.2\pi}{2} = 0.65$$
$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2\tan\frac{0.3\pi}{2} = 1.02$$

Value of N

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} \geq \frac{\cosh^{-1}\sqrt{\frac{10^{1.5}-1}{10^{0.1}-1}}}{\cosh^{-1}\frac{1.02}{0.65}} = 3.01$$

Let us take $N = 4$.

Axis of the ellipse

We know $\varepsilon = \sqrt{10^{0.1\alpha_p}-1} = 0.508$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 4.17$$

$$a = \Omega_p\left[\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right] = 0.65\left[\frac{(4.17)^{1/4}-(4.17)^{-1/4}}{2}\right]$$

$$= 0.237$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.65 \left[\frac{(4.17)^{1/4} + (4.17)^{-1/4}}{2}\right]$$

$$= 0.6918$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} k = 1, 2, 3, 4$$

$\phi_1 = 112.5°; \phi_2 = 157.5°, \phi_3 = 202.5°; \phi_4 = 247.5°$.

The poles are

$$s_k = a\cos\phi_k + jb\sin\phi_k; \quad k = 1, 2, 3, 4$$

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = 0.237\cos 112.5° + j0.6918\sin 112.5°$$
$$= -0.0907 + j0.639$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = 0.237\cos 157.5° + j0.6918\sin 157.5°$$
$$= -0.2189 + j0.2647$$

$$s_3 = a\cos\phi_3 + jb\sin\phi_3 = 0.237\cos 202.5° + j0.6918\sin 202.5°$$

$$s_3 = a\cos\phi_3 + jb\sin\phi_3 = 0.237\cos 202.5^\circ + j0.6918\sin 202.5^\circ$$
$$= -0.2189 - j0.2647$$
$$s_4 = a\cos\phi_4 + jb\sin\phi_4 = 0.237\cos 247.5^\circ + j0.6918\sin 247.5^\circ$$
$$= -0.0907 - j0.639$$

The denominator polynomial of

$$H(s) = [(s + 0.0907)^2 + (0.639)^2][(s + 0.2189)^2 + (0.2647)^2]$$
$$= (s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.118)$$

As N is even, the numerator of $H(s) = \dfrac{(0.4165)(0.118)}{\sqrt{1+\varepsilon^2}} = 0.04381$.

The transfer function $H(s) = \dfrac{0.04381}{(s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.1180)}$.

The z-transform of the digital filter

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.1180)}\Bigg|_{s=2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$\because T = 1\,\text{sec}$$

$$= \frac{0.04381(1+z^{-1})^4}{\{4(1-z^{-1})^2+0.1814\times 2(1-z^{-2})+0.4165(1+z^{-1})^2\}\{4(1-z^{-1})^2+0.4378\times 2(1-z^{-2})+0.1180(1+z^{-1})^2\}}$$

$$= \frac{0.04381(1+z^{-1})^4}{(4.7794-7.1668z^{-1}+4.0538z^{-2})(4.9936-7.764z^{-1}+3.2424z^{-2})}$$

$$= \frac{0.001836(1+z^{-1})^4}{(1-1.499z^{-1}+0.8482z^{-2})(1-1.5548z^{-1}+0.6493z^{-2})}$$

(b) Impulse Invariance Method

Given data $\omega_p = 0.2\pi;\ \omega_s = 0.3\pi;\ \alpha_p = 1\,\text{dB};\ \alpha_s = 15\,\text{dB}.$

The Analog frequency ratio $\dfrac{\Omega_s}{\Omega_p} = \dfrac{\omega_s}{\omega_p} = \dfrac{0.3\pi}{0.2\pi} = 1.5$

$(\because\ \omega = \Omega T$ and $T = 1\,\text{sec})$

Value of *N*

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.5}-1}{10^{0.1}-1}}}{\cosh^{-1}(0.5)} = 3.2$$

Rounding the value of N to a higher value, we get $N = 4$.

Axis of ellipse

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}; \quad k = 1, 2, \ldots N$$

$$\phi_1 = 112.5^\circ; \phi_2 = 157.5^\circ; \phi_3 = 202.5^\circ$$

$$\phi_4 = 247.5^\circ$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = \sqrt{10^{0.1} - 1} = 0.508$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 4.17$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.2\pi \left[\frac{4.17^{1/4} - 4.17^{-1/4}}{2}\right] = 0.229$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.2\pi \left[\frac{4.17^{1/4} + 4.17^{-1/4}}{2}\right] = 0.67$$

The poles of the filter

$$s_k = a\cos\phi_k + jb\sin\phi_k; \quad k = 1, 2, \ldots 4$$

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = -0.0876 + j0.619$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = -0.2115 + j0.2564$$

$$s_3 = a\cos\phi_3 + jb\sin\phi_3 = -0.2115 - j0.2564$$

$$s_4 = a\cos\phi_4 + jb\sin\phi_4 = -0.0876 - j0.619$$

The denominator polynomial of

$$H(s) = \{(s + 0.0876)^2 + (0.619)^2\}\{(s + 0.215)^2 + (0.2564)^2\}$$
$$= (s^2 + 0.175s + 0.391)(s^2 + 0.423s + 0.11)$$

For N even

The numerator of $H(s) = \dfrac{(0.391)(0.11)}{\sqrt{1+\varepsilon^2}} = 0.03834.$

$$H(s) = \frac{0.03834}{(s^2 + 0.175s + 0.391)(s^2 + 0.423s + 0.11)}$$

$$= \frac{A}{s - (-0.0876 + j0.619)} + \frac{A^*}{s - (-0.0876 - j0.619)}$$

$$+ \frac{B}{s - (-0.2115 + j0.2564)} + \frac{B^*}{s - (-0.2115 - j0.2564)}$$

Solving for A, A^*, B, B^* and using

$$A = -0.0413 + j0.0814$$
$$B = 0.0413 - j0.2166$$

Impulse invariant transform

$$\text{i.e.,} \quad \sum_{k=1}^{N} \frac{c_k}{s - p_k} = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

we can obtain

$$H(z) = \frac{-0.083 - 0.0245z^{-1}}{1 - 1.49z^{-1} + 0.839z^{-2}} + \frac{0.083 + 0.0238z^{-1}}{1 - 1.56z^{-1} + 0.655z^{-2}}$$

Example 5.32 Design a Butterworth filter using the impulse variance method for the following specifications

$$0.8 \leq |H(e^{j\omega})| \leq 1 \qquad 0 \leq \omega \leq 0.2\pi$$
$$|H(e^{j\omega})| \leq 0.2 \qquad 0.6\pi \leq \omega \leq \pi$$

Solution

Given $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8$ from which $\varepsilon = 0.75$, $\frac{1}{\sqrt{1+\lambda^2}} = 0.2$ from which $\lambda = 4.899$

$$\omega_s = 0.6\pi \text{ rad}; \quad \omega_p = 0.2\pi \text{ rad}$$
$$\frac{\omega_s}{\omega_p} = \frac{\Omega_s T}{\Omega_p T} = \frac{\Omega_s}{\Omega_p} = \frac{0.6\pi}{0.2\pi} = 3$$
$$N = \frac{\log \lambda/\varepsilon}{\log 1/k} = \frac{\log \frac{4.899}{0.75}}{\log 3} = 1.71$$

Approximating to nearest higher values we have $N = 2$.

For $N = 2$ the transfer function of normalized Butterworth filter is

For $N = 2$ the transfer function of normalized Butterworth filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \frac{0.2\pi}{(0.75)^{1/2}} = 0.231\pi$$

$$H_a(s) = H(s)\Big|_{s \to s/0.231\pi}$$

$$= \frac{0.5266}{s^2 + 1.03s + 0.5266}$$

$$= \frac{0.516j}{s + 0.51 + j0.51} - \frac{0.516j}{s + 0.51 - j0.51}$$

$$= \frac{0.516j}{s - (-0.51 - j0.51)} - \frac{0.516j}{s - (-0.51 + j0.51)}$$

$$H(z) = \frac{0.516j}{1 - e^{-0.51T}e^{-j0.51T}z^{-1}} - \frac{0.516j}{1 - e^{-0.51T}e^{j0.51T}z^{-1}} \quad (\because T = 1 \text{ sec})$$

$$= \frac{0.3019z^{-1}}{1 - 1.048z^{-1} + 0.36z^{-2}}$$