# SNS COLLEGE OF TECHNOLOGY

**Coimbatore-35**
**An Autonomous Institution**

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

## DEPARTMENT OF BIOMEDICAL ENGINEERING

## 19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

# UNIT III INFINITE IMPULSE RESPONSE FILTERS

# UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

**Example 5.30** Design a digital Butterworth filter satisfying the constraints .

$$0.707 \leq |H(e^{j\omega})| \leq 1 \qquad \text{for } 0 \leq \omega \leq \frac{\pi}{2}$$

$$|H(e^{j\omega})| \leq 0.2 \qquad \text{for } \frac{3\pi}{4} \leq \omega \leq \pi$$

with $T = 1$ sec using (a) The bilinear transformation (b) Impulse invariance. Realize the filter in each case using the most convenient realization form.

**Solution**

**(a) Bilinear transformation**

Given data $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.707; \quad \frac{1}{\sqrt{1+\lambda^2}} = 0.2; \omega_p = \frac{\pi}{2}; \omega_s = \frac{3\pi}{4}.$

The analog frequency ratio is

$$\frac{\Omega_s}{\Omega_p} = \frac{\frac{2}{T}\tan\frac{\omega_s}{2}}{\frac{2}{T}\tan\frac{\omega_p}{2}} = \frac{\tan\frac{3\pi}{8}}{\tan\frac{\pi}{4}} = 2.414$$

The order of the filter $N \geq \frac{\log \frac{\lambda}{\varepsilon}}{\log \frac{\Omega_s}{\Omega_p}}$.

From the given data $\lambda = 4.898 \quad \varepsilon = 1$.

So $N \geq \frac{\log 4.898}{\log 2.414} = 1.803$.

Rounding $N$ to nearest higher value we get $N = 2$. We know

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \Omega_p \quad (\because \varepsilon = 1)$$

$$= \frac{2}{T} \tan \frac{\omega_p}{2} = 2 \tan \frac{\pi}{4} = 2 \text{ rad/sec}$$

The transfer function of second order normalized Butterworth filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$H_a(s)$ for $\Omega_c = 2$ rad/sec can be obtained by substituting $s \rightarrow s/2$ in $H(s)$

$$\text{i.e.,} \quad H_a(s) = \frac{1}{(s/2)^2 + \sqrt{2} \cdot (s/2) + 1}$$

$$= \frac{4}{s^2 + 2.828s + 4}$$

By using bilinear transformation $H(z)$ can be obtained as

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\text{Thus } H(z) = \frac{4}{s^2 + 2.828s + 4}\Bigg|_{s=2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} \qquad (\because T = 1\,\text{sec})$$

$$= \frac{4(1+z^{-1})^2}{4(1-z^{-1})^2 + 5.656(1-z^{-2}) + 4(1+z^{-1})^2}$$

$$= \frac{0.2929(1+z^{-1})^2}{1 + 0.1716z^{-2}}$$

The above system function can be realized in direct form II as shown in Fig. 5.62.

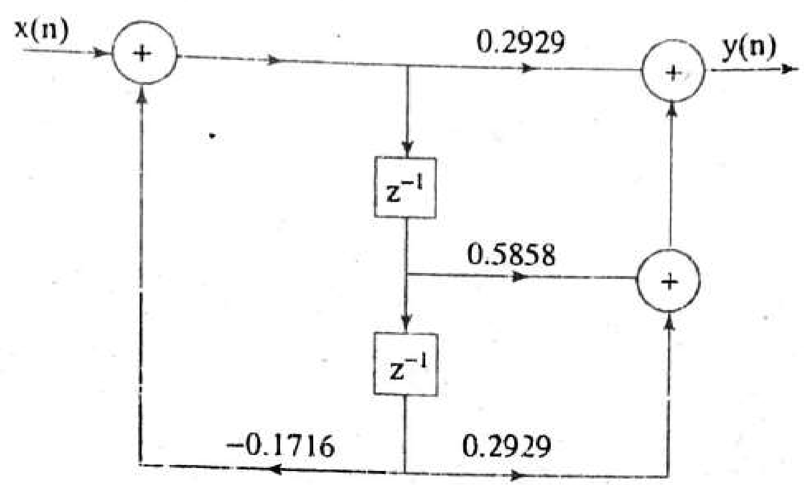


**Fig. 5.62**

## (b) Impulse Invariant Method

### Solution

The relationship between analog & digital frequencies in Impulse invariant method is $\omega = \Omega T$.

From the given data $T = 1$ sec i.e., $\omega = \Omega$

$$\Rightarrow \quad \Omega_p = \omega_p; \quad \Omega_s = \omega_s$$

We know $\lambda = 4.898$; $\varepsilon = 1$.

The order of the filter

$$N \geq \frac{\log \frac{\lambda}{\varepsilon}}{\log \frac{\Omega_s}{\Omega_p}} = \frac{\log 4.989}{\log \frac{3\pi/4}{\pi/2}}$$

$$N \geq 3.924$$

i.e., $N = 4$

The transfer function of a fourth order normalized Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.76537 + 1)(s^2 + 1.8477s + 1)}$$

As $\varepsilon = 1$: $\Omega_p = \Omega_c = 0.5\pi = 1.57$

$$H_a(s) = H(s)\Big|_{s \to \frac{s}{1.57}}$$

$$= \frac{(1.57)^4}{(s^2 + 1.202s + 2.465)(s^2 + 2.902s + 2.465)}$$

$H_a(s)$ in the partial fraction form is given by

$$H_a(s) = \frac{A}{(s+1.45+j0.6)} + \frac{A^*}{(s+1.45-j0.6)} + \frac{B}{(s+0.6+j1.45)} + \frac{B^*}{(s+0.6-j1.45)}$$

$$A = (s+1.45+j0.6)\frac{(1.57)^4}{(s+1.45+j0.6)(s+1.45-j0.6)(s^2+1.202s+2.465)}\Bigg|_{s=-1.45-j0.6}$$

$$= \frac{(1.57)^4}{(-j0.6-0.6)\left[(-1.45-j0.6)^2+1.202(-1.45-j0.6)+2.465\right]}$$

$$= \frac{(1.57)^4}{-j(1.2)\left[1.7425+1.74j-1.7429-j0.7212+2.465\right]}$$

$$= \frac{(1.57)^4}{-j(1.2)(2.465+j1.0188)}$$

$$= \frac{5.063}{1.0188-j2.465} = \frac{5.063(1.0188+j2.465)}{7.114}$$

$$= 0.7116(1.0188+j2.465) = 0.7253+j1.754$$

$$B = (s + 0.6 + j1.45) \frac{(1.57)^4}{(s + 0.6 + j1.45)(s + 0.6 - j1.45)(s^2 + 2.902s + 2.465)} \bigg|_{s=-0.6-j1.45}$$

$$= \frac{(1.57)^4}{-j(2.9)\left[(-0.6 - j1.45)^2 + 2.902(-0.6 - j1.45) + 2.465\right]}$$

$$= \frac{(1.57)^4}{-j(2.9)\left[-1.7425 + j1.74 - 1.7412 - j4.208 + 2.465\right]}$$

$$= \frac{2.095}{-j\left[-1.0187 - j2.468\right]}$$

$$= \frac{2.095}{-2.468 + j1.0187} = \frac{2.095\left[-2.468 - j1.0187\right]}{7.1287}$$

$$= 0.29388\left[-2.468 - j1.0187\right] = -0.7253 - 0.3j$$

$$H_a(s) = \frac{0.7253 + j1.754}{s - (-1.45 - j0.6)} + \frac{0.7253 - j1.754}{s - (-1.45 + j0.6)} + \frac{-0.7253 - 0.3j}{s - (-0.6 - j1.45)} + \frac{-0.7253 + 0.3j}{s - (-0.6 + j1.45)}$$

We know for $T = 1$ sec

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k} z^{-1}}$$

Therefore

$$H_a(s) = \frac{0.7253 + j1.754}{1 - e^{-1.45}e^{-j0.6}z^{-1}} + \frac{0.7253 - j1.754}{1 - e^{-1.45}e^{j0.6}z^{-1}} + \frac{-(0.7253 + 0.3j)}{1 - e^{-0.6}e^{-j1.45}z^{-1}} + \frac{-0.7253 + 0.3j}{1 - e^{-0.6}e^{j1.45}z^{-1}}$$

$$= \frac{1.454 + 0.1839z^{-1}}{1 - 0.387z^{-1} + 0.055z^{-2}} + \frac{-1.454 + 0.2307z^{-1}}{1 - 0.1322z^{-1} + 0.301z^{-2}}$$

This can be realized using parallel form as shown in Fig. 5.63.

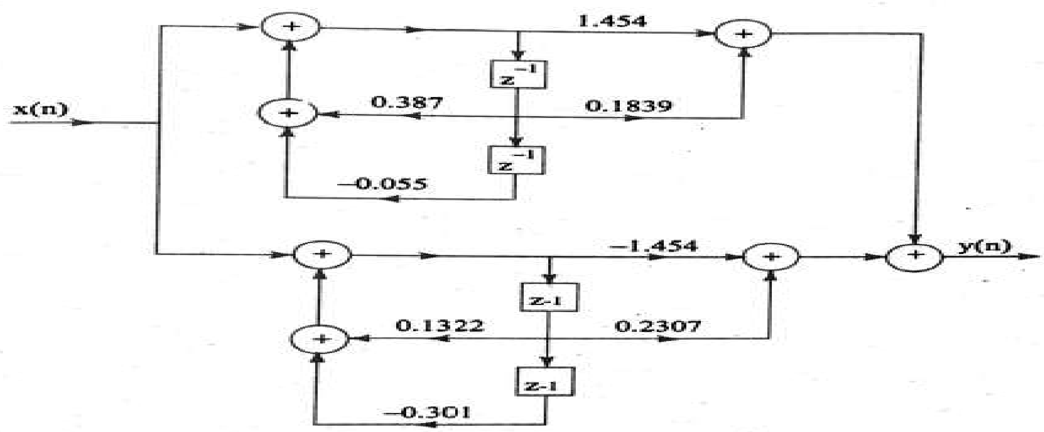


**Fig. 5.63**