SNS COLLEGE OF TECHNOLOGY
Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Steps to design a digital filter using Impulse Invariance method

1. For the given specifications, find $H_a(s)$, the transfer function of an analog filter.
2. Select the sampling rate of the digital filter, T seconds per sample.
3. Express the analog filter transfer function as the sum of single-pole filters.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}$$

4. Compute the z-transform of the digital filter by using the formula

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}.$$

For high sampling rates use

$$H(z) = \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k T} z^{-1}}$$

Example 5.11 For the analog transfer function $H(s) = \frac{2}{(s+1)(s+2)}$ determine $H(z)$ using impulse invariance method. Assume $T = 1$ sec.

Solution

Given $H(s) = \dfrac{2}{(s+1)(s+2)}$

Using partial fraction we can write

$$H(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

$$A = (s+1)\frac{2}{(s+1)(s+2)}\Big|_{s=-1} = 2$$

$$B = (s+2)\frac{2}{(s+1)(s+2)}\Big|_{s=-2} = -2$$

Using impulse invariance technique we have, if

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \quad \text{then} \quad H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k} \quad \text{then} \quad H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

i.e., $(s - p_k)$ is transformed to $1 - e^{p_k T} z^{-1}$.

There are two poles $p_1 = -1$ and $p_2 = -2$. So

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} - \frac{2}{1 - e^{-2T} z^{-1}}$$

For $T = 1$ sec

$$H(z) = \frac{2}{1 - e^{-1} z^{-1}} - \frac{2}{1 - e^{-2} z^{-1}}$$

$$= \frac{2}{1 - 0.3678 z^{-1}} - \frac{2}{1 - 0.1353 z^{-1}}$$

$$H(z) = \frac{0.465 z^{-1}}{1 - 0.503 z^{-1} + 0.04976 z^{-2}}$$

Example 5.13 Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period $T = 1$ sec.

Solution

From the table 5.1, for $N = 3$, the transfer function of a normalised Butterworth filter is given by

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$
$$= \frac{A}{s+1} + \frac{B}{s+0.5+j0.866} + \frac{C}{s+0.5-j0.866}$$

$$A = (s+1)\frac{1}{(s+1)(s^2+s+1)}\bigg|_{s=-1} = \frac{1}{(-1)^2-1+1} = 1$$

$$B = (s+0.5+j0.866)\frac{1}{(s+1)(s+0.5+j0.866)(s+0.5-j0.866)}\bigg|_{s=-0.5-j0.866}$$

$$= \frac{1}{(-0.5-j0.866+1)(-j0.866-j0.866)}$$

$$= \frac{1}{-j1.732(0.5-j0.866)} = \frac{1}{-j0.866-1.5}$$

$$= \frac{-1.5+j0.866}{3} = -0.5+j0.288$$

$$C = B^* = -0.5-j0.288$$

Hence

$$H(s) = \frac{1}{s+1} + \frac{-0.5+0.288j}{s+0.5+j0.866} + \frac{-0.5-0.288j}{s+0.5-j0.866}$$
$$= \frac{1}{s-(-1)} + \frac{-0.5+0.288j}{s-(-0.5-j0.866)} + \frac{-0.5-0.288j}{s-(-0.5+j0.866)}$$

In impulse invariant technique

$$\text{if} \quad H(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k}, \quad \text{then} \quad H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k T} z^{-1}}$$

Therefore,

$$H(z) = \frac{1}{1-e^{-1}z^{-1}} + \frac{-0.5+j0.288}{1-e^{-0.5}e^{-j0.866}z^{-1}} + \frac{-0.5-j0.288}{1-e^{-0.5}e^{j0.866}z^{-1}}$$
$$= \frac{1}{1-0.368z^{-1}} + \frac{-1+0.66z^{-1}}{1-0.786z^{-1}+0.368z^{-2}}$$

Example 5.15 An analog filter has a transfer function $H(s) = \frac{10}{s^2 + 7s + 10}$. Design a digital filter equivalent to this using impulse invariant method for $T = 0.2$ sec.

Soiution

Given

$$H(s) = \frac{10}{s^2 + 7s + 10}$$
$$= \frac{-3.33}{s+5} + \frac{3.33}{s+2} = \frac{-3.33}{s-(-5)} + \frac{3.33}{s-(-2)}$$

Using Eq. (5.81b) we have

$$H(z) = T\left[\frac{-3.33}{1 - e^{-5T}z^{-1}} + \frac{3.33}{1 - e^{-2T}z^{-1}}\right] = 0.2\left[\frac{-3.33}{1 - e^{-1}z^{-1}} + \frac{3.33}{e^{-0.4}z^{-1}}\right]$$
$$= \left[\frac{-0.666}{1 - 0.3678z^{-1}} + \frac{0.666}{1 - 0.67z^{-1}}\right]$$
$$= \frac{0.2012z^{-1}}{1 - 1.0378z^{-1} + 0.247z^{-2}}$$