SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT III INFINITE IMPULSE RESPONSE FILTERS

UNIT II INFINITE IMPULSE RESPONSE FILTERS

Characteristics of practical frequency selective filters.
Characteristics of commonly used analog filters
Butterworth filters, Chebyshev filters.
Design of IIR filters from analog filters (LPF, HPF, BPF, BRF)
Approximation of derivatives
Impulse invariance method
Bilinear transformation
Frequency transformation in the analog domain
Structure of IIR filter - direct form I, direct form II
Cascade, parallel realizations

Steps to design an analog Butterworth lowpass filter

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Find the transfer function $H(s)$ for $\Omega_c = 1$ rad/sec for the value of N.
4. Calculate the value of cutoff frequency Ω_c.
5. Find the transfer function $H_a(s)$ for the above value of Ω_c by substituting s → $\frac{s}{\Omega_c}$ in $H(s)$.

Example 5.4 Design an analog Butterworth filter that has a -2 dB passband attenuation at a frequency of 20 rad/sec and atleast -10 dB stopband attenuation at 30 rad/sec.

Solution

Given $\alpha_p = 2\,\text{dB}; \quad \Omega_p = 20\,\text{rad/sec}$

$\alpha_s = 10\,\text{dB}; \quad \Omega_s = 30\,\text{rad/sec}$

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\frac{10-1}{10^{0.2}-1}}}{\log \frac{30}{20}}$$

$$\geq 3.37$$

Rounding off N to the next highest integer we get

$$N = 4$$

The normalized lowpass Butterworth filter for $N = 4$ can be found from table 5.1 as

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

From Eq. (5.31) we have

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{20}{(10^{0.2} - 1)^{1/8}} = 21.3868$$

The transfer function for $\Omega_c = 21.3868$ can be obtained by substituting

$$s \rightarrow \frac{s}{21.3868} \text{ in } H(s)$$

$$\text{i.e.,} \quad H(s) = \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 0.76537\left(\frac{s}{21.3868}\right) + 1} \times \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 1.8477\left(\frac{s}{21.3868}\right) + 1}$$

$$= \frac{0.20921 \times 10^6}{(s^2 + 16.3686s + 457.394)(s^2 + 39.5176s + 457.394)}$$

For the given specifications design an analog Butterworth filter.
$0.9 \le |H(j\Omega)| \le 1$ for $0 \le \Omega \le 0.2\pi$. $|H(j\Omega)| \le 0.2$ for $0.4\pi \le \Omega \le \pi$.

Solution

From the data we find $\Omega_p = 0.2\pi$; $\Omega_s = 0.4\pi$; $\frac{1}{\sqrt{1+\varepsilon^2}} = 0.9$ and $\frac{1}{\sqrt{1+\lambda^2}} = 0.2$ from which we obtain

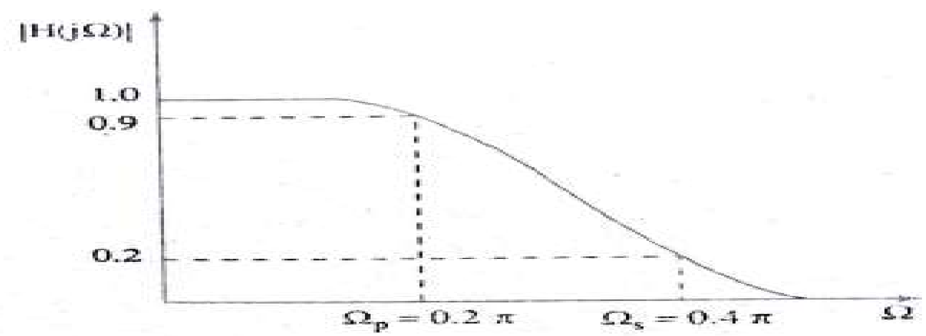

Fig. 5.8 Magnitude response of example 5.5

$$\varepsilon = 0.484 \text{ and } \lambda = 4.898$$

$$N \ge \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\frac{4.898}{0.484}}{\log\left(\frac{0.4\pi}{0.2\pi}\right)} = 3.34$$

i.e., $N = 4$

From the table 5.1, for $N = 4$, the transfer function of normalised Butterworth filter is

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

we know $\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{\Omega_p}{\varepsilon^{1/N}} = \frac{0.2\pi}{(0.484)^{1/4}} = 0.24\pi.$

$H(s)$ for $\Omega_c = 0.24\pi$ can be obtained by substituting $s \to \frac{s}{0.24\pi}$ in $H(s)$ i.e.,

$$H(s) = \frac{1}{\left\{\left(\frac{s}{0.24\pi}\right)^2 + 0.76537\left(\frac{s}{0.24\pi}\right) + 1\right\}} \times \frac{1}{\left(\frac{s}{0.24\pi}\right)^2 + 1.8477\left(\frac{s}{0.24\pi}\right) + 1}$$

$$= \frac{0.323}{(s^2 + 0.577s + 0.0576\pi^2)(s^2 + 1.393s + 0.0576\pi^2)}$$

Comparison between Butterworth Filter and Chebyshev Filter

1. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞, whereas the magnitude response of the Chebyshev filter exhibits ripples in the passband or stopband according to the type.
2. The transition band is more in Butterworth filter when compared to Chebyshev filter.
3. The poles of the Butterworth filter lie on a circle, whereas the poles of the Chebyshev filter lie on an ellipse.
4. For the same specifications, the number of poles in Butterworth are more when compared to the Chebyshev filter i.e., the order of the Chebyshev filter is less than that of Butterworth. This is a great advantage because less number of discrete components will be necessary to construct the filter.

5.9 Steps to design an analog Chebyshev lowpass filter

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Using the following formulas find the values of a and b, which are minor and major axis of the ellipse respectively.

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2}; \quad b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

Ω_p = Passband frequency

α_p = Maximum allowable attenuation in the passband

($\because$ For normalized Chebyshev filter $\Omega p = 1$ rad/sec)

4. Calculate the poles of Chebyshev filter which lie on an ellipse by using the formula.

$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, \ldots, N$$

$$\text{where } \phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \quad k = 1, 2, \ldots, N$$

5. Find the denominator polynomial of the transfer function using the above poles.

6. The numerator of the transfer function depends on the value of N.

   (a) For N odd substitute $s = 0$ in the denominator polynomial and find the value. This value is equal to the numerator of the transfer function.
   ($\because$ For N odd the magnitude response $|H(j\Omega)|$ starts at 1.)

   (b) For N even substitute $s = 0$ in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$. This value is equal to the numerator.

Example 5.6 Given the specifications $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$; $f_p = 1\,\text{KHz}$ and $f_s = 2\,\text{KHz}$. Determine the order of the filter using Chebyshev approximation. Find $H(s)$.

Solution

From the given data we can find

$$\Omega_p = 2\pi \times 1000\,\text{Hz} = 2000\,\pi\ \text{rad/sec}$$
$$\Omega_s = 2\pi \times 2000\,\text{Hz} = 4000\,\pi\ \text{rad/sec}$$

and $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$.

Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}} = \cosh^{-1}\frac{\sqrt{\dfrac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}\dfrac{4000\pi}{2000\pi}}$$
$$= 1.91$$

Step 2: Rounding N to next higher value we get $N = 2$.

For N even, the oscillatory curve starts from $\frac{1}{\sqrt{1+\varepsilon^2}}$.

Step 3: The values of minor axis and major axis can be found as below

$$\varepsilon = (10^{0.1\alpha_p} - 1)^{0.5} = (10^{0.3} - 1)^{0.5} = 1$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.414$$

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} - (2.414)^{-1/2}]}{2} = 910\pi$$

$$b = \Omega_p \frac{[\mu^{1/N} + \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} + (2.414)^{-1/2}]}{2} = 2197\pi$$

Step 4: The poles are given by

$$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1, 2$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$

$$s_1 = a\cos\phi_1 + j\,b\sin\phi_1 = -643.46\pi + j1554\pi$$

$$s_2 = a\cos\phi_2 + j\,b\sin\phi_2 = -643.46\pi - j1554\pi$$

Step 5: The denominator of $H(s) = (s + 643.46\pi)^2 + (1554\pi)^2$

Step 6: The numerator of $H(s) = \frac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}} = (1414.38)^2\pi^2$

The transfer function $H(s) = \frac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$.