SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

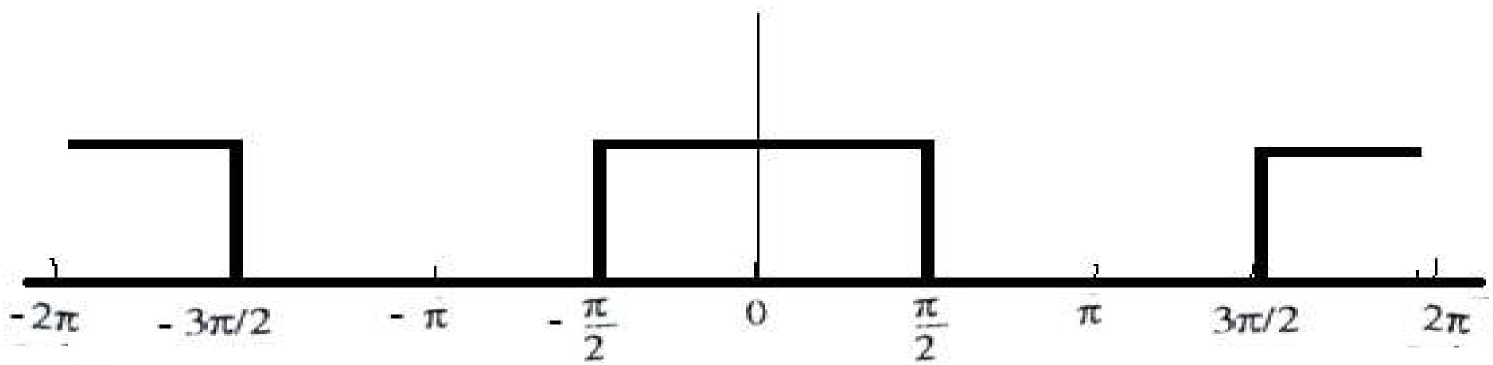
Example 6.15 Determine the filter coefficients $h(n)$ obtained by samping

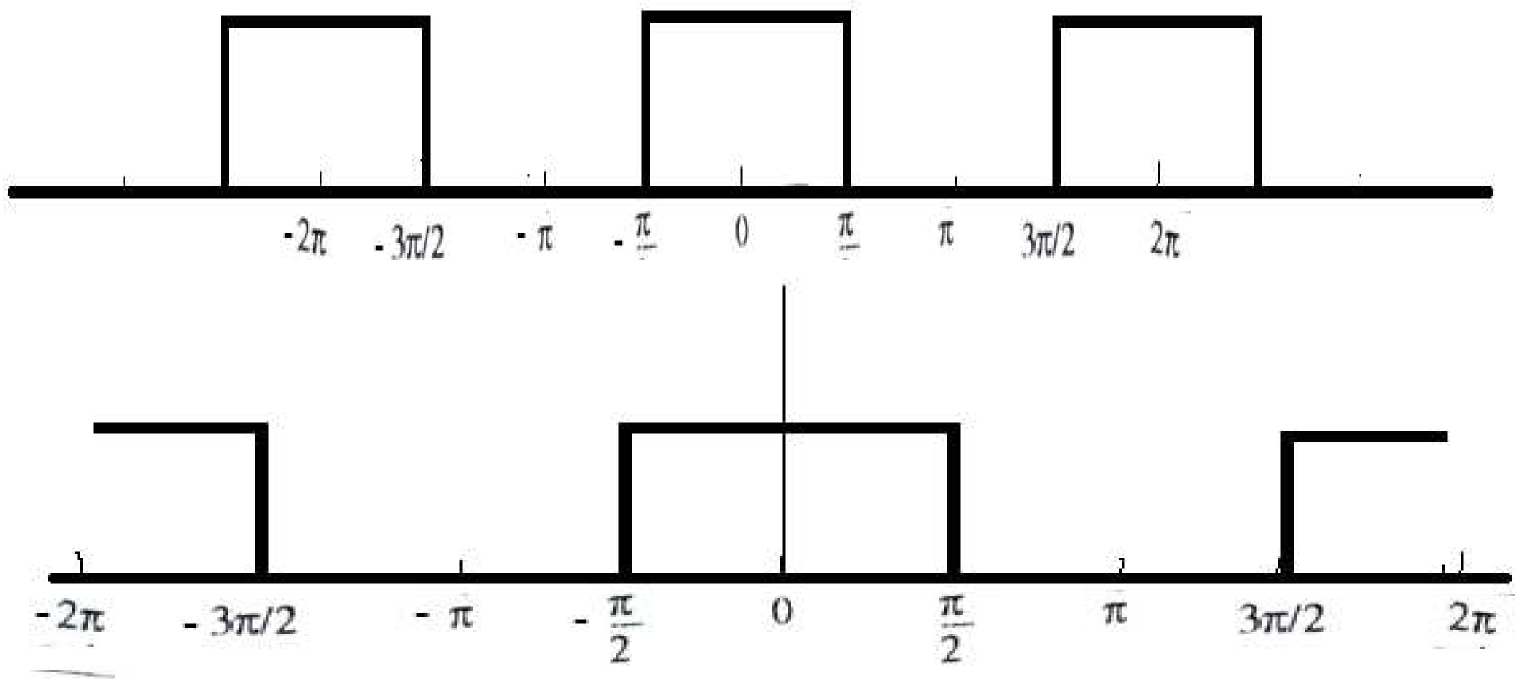
$$H_d(e^{j\omega}) = e^{-j(N-1)\omega/2} \quad 0 \leq |\omega| \leq \frac{\pi}{2}$$
$$= \quad 0 \quad \frac{\pi}{2} \leq |\omega| \leq \pi$$

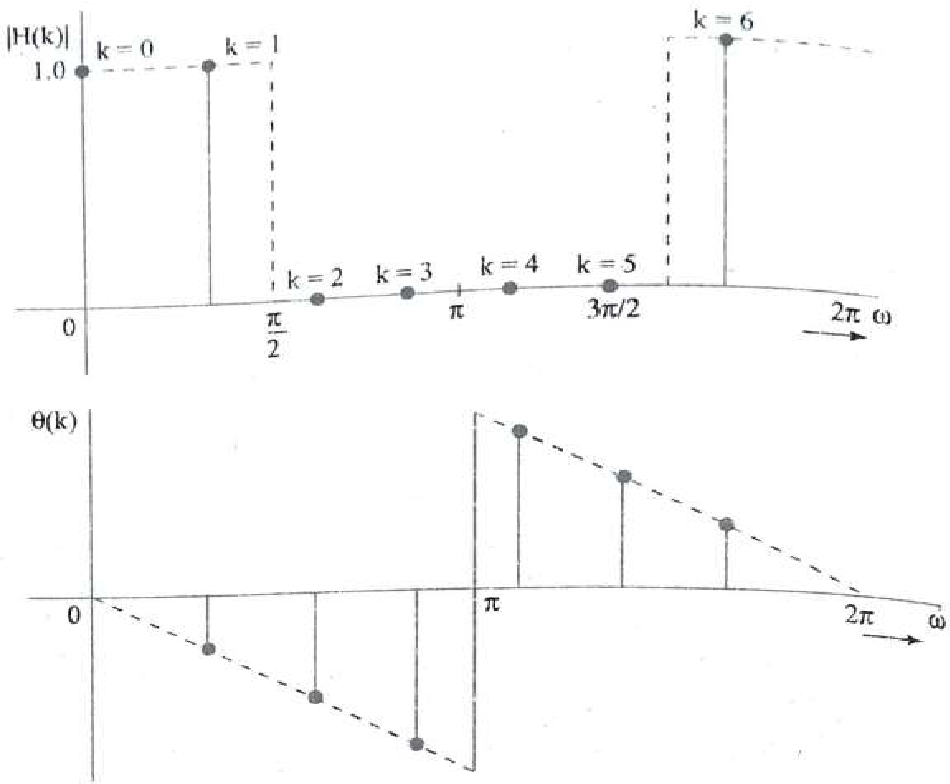
for $N = 7$.

Solution

The ideal magnitude response with samples for the given specification is shown in Fig.




-2π
- 3π/2
- π
- π/2
0
π/2
π
3π/2
2π
-2π
- 3π/2
- π
- π/2
0
π/2
π
3π/2
2π


|H(k)|
1.0
k = 0
k = 1
k = 2
k = 3
k = 4
k = 5
k = 6
0
π/2
π
3π/2
2π
ω
θ(k)
0
π
2π
ω

Given $N = 7$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{7}} \quad k = 0, 1, 2, \ldots 6$$

From Fig. 6.59 we have

$$\begin{aligned} |H(k)| &= 1 \quad \text{for} \quad k = 0, 1, 6 \\ &= 0 \quad \text{for} \quad k = 2, 3, 4, 5 \end{aligned} \qquad (6.14$$

Using Eq.(6.126) we have

$$\begin{aligned} \theta(k) &= -\left(\frac{N-1}{N}\right)\pi k = -\frac{6}{7}\pi k \quad \text{for} \quad k = 0, 1, 2, 3 \\ &= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k = 6\pi - \frac{6\pi k}{7} = \frac{6\pi}{7}(7-k) \quad \text{for} \quad k = 4, 5 \end{aligned}$$

Now the frequency response of the linear phase filter can be wirtten by substi-
ing Eq.(6.140) and Eq.(6.141) in Eq.(6.120)

$$H(k) = e^{-j6\pi k/7} \quad k = 0, 1$$
$$= \quad 0 \quad \text{for} \quad k = 2, 3, 4, 5$$
$$= e^{-j6\pi(k-7)/7} \quad \text{for} \quad k = 6$$

The filter coefficients for N odd are given by

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)e^{j2\pi kn/7}\right]\right\} \quad n = 0, 1, \ldots N-1$$

$$= \frac{1}{7}\left\{1 + 2Re(e^{-j6\pi/7}e^{j2\pi kn/7})\right\}$$

$$= \frac{1}{7}\left\{1 + 2Re(e^{j2\pi(n-3)/7})\right\}$$

$$= \frac{1}{7}\left\{1 + 2\cos\frac{2\pi}{7}(n-3)\right\}$$

$$h(0) = h(6) = \frac{1}{7}\left(1 + 2\cos\frac{6\pi}{7}\right) = -0.11456$$

$$h(1) = h(5) = \frac{1}{7}\left(1 + 2\cos\frac{4\pi}{7}\right) = 0.07928$$

$$h(2) = h(4) = \frac{1}{7}\left(1 + 2\cos\frac{2\pi}{7}\right) = 0.321$$

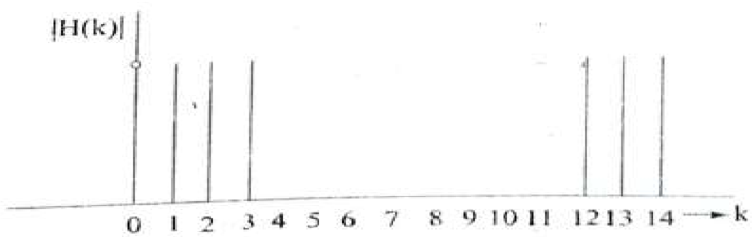
$$h(3) = \frac{1}{7}(1 + 2) = 0.42857$$

Example 6.16 Determine the coefficients of a linear phase FIR filter of length $M = 15$ has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H\left(\frac{2\pi k}{15}\right) = 1 \quad k = 0, 1, 2, 3$$
$$= 0 \quad k = 4, 5, 6, 7$$

Solution

$$|H(k)| = 1 \quad \text{for} \quad 0 \le k \le 3 \quad \text{and} \quad 12 \le k \le 14$$
$$= 0 \quad \text{for} \quad 4 \le k \le 11$$



$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$$
$$= \frac{-14}{15}\pi k \quad 0 \le k \le 7$$

and

$$\theta(k) = 14\pi - \frac{14\pi k}{15} \quad \text{for } 8 \le k \le 14$$

$$\begin{aligned} H(k) &= e^{-j14\pi k/15} \quad \text{for} \quad k = 0, 1, 2, 3 \\ &= 0 \quad \text{for} \quad 4 \le k \le 11 \\ &= e^{-j14\pi(k-15)/15} \quad \text{for} \quad 12 \le k \le 14 \end{aligned}$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} Re\left(e^{-j14\pi k/15}e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$; $h(1) = h(3) = 0.041$ $h(4) = h(10) = -0.1078$

$h(2) = h(12) = 0.0666$; $h(3) = h(11) = -0.0365$ $h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$ $h(7) = 0.466$

Example 6.17 Using frequency sampling method, design a bandpass filter with the following specifications.

$$\text{sampling frequency } F = 8000\text{Hz}$$
$$\text{cut off frequencies } f_{c1} = 1000\text{Hz}$$
$$f_{c2} = 3000\text{Hz}$$

Determine the filter coefficients for $N = 7$.

Solution

$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_{c_1}}{F} = \frac{2\pi(1000)}{8000} = \frac{\pi}{4}$$

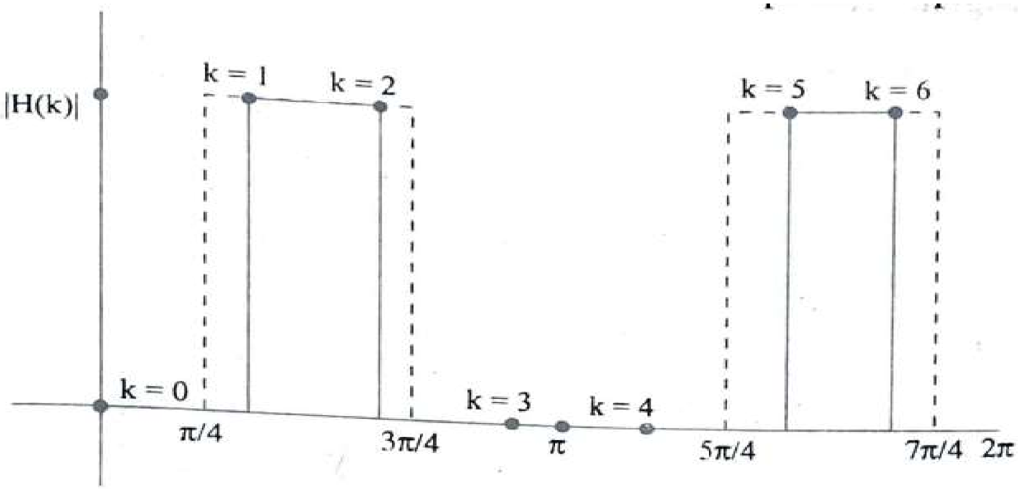


Fig. 6.61 Ideal magnitude response with samples for example 6.17

$$\omega_{c_2} = 2\pi f_{c_2} T = \frac{2\pi f_{c_2}}{F} = \frac{2\pi(3000)}{8000}$$
$$= \frac{3\pi}{4}$$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{7}k} \quad k = 0, 1, \ldots, 6$$

$$|H(k)| = 0 \qquad \text{for } k = 0, 3$$

$$= 1 \qquad \text{for } k = 1, 2$$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi \qquad \text{for } 0 \le k \le \frac{N-1}{2}$$

$$= -\frac{6}{7}\pi k \qquad \text{for } 0 \le k \le 3$$

$$H(k) = 0 \qquad \text{for } k = 0, 3$$

$$= e^{-j6\pi k/7} \qquad \text{for } k = 1, 2$$

The filter coefficients are given by

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} \mathrm{Re}(H(k)e^{j2\pi kn/N})\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{3} \mathrm{Re}(e^{-j6\pi k/7}e^{j2\pi kn/7})\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{2} \cos\frac{2\pi k}{7}(3-n)\right]$$

$$= \frac{2}{7}\left[\cos\frac{2\pi}{7}(3-n) + \cos\frac{4\pi}{7}(3-n)\right]$$

$$h(0) = h(6) = -0.07928$$

$$h(1) = h(5) = -0.321$$

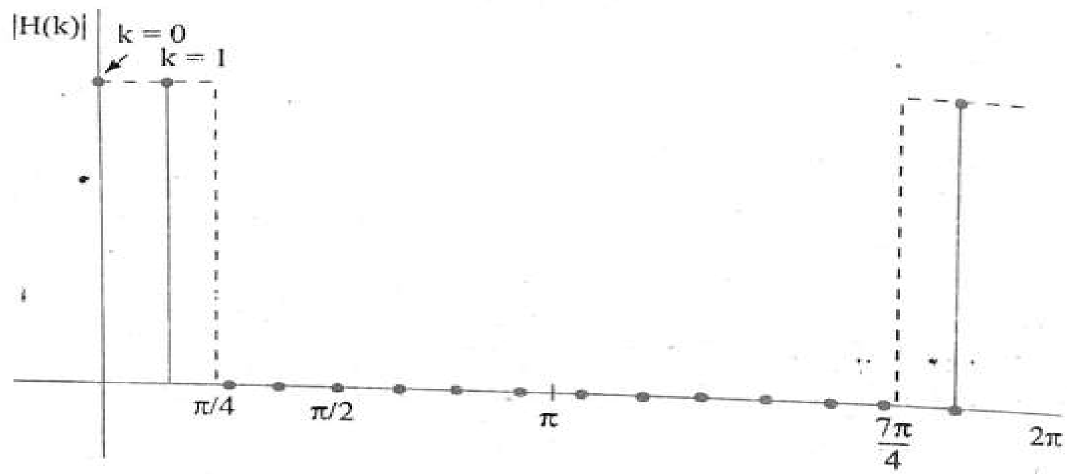
$$h(2) = h(4) = 0.11456$$

$$h(3) = 0.57$$

Example 6.18 (a) Use frequency sampling method to design a FIR lowpass with $\omega_c = \frac{\pi}{4}$, for $N = 15$. Plot the magnitude response. (b) Repeat part (a) selecting an additional sample $|H(k)| = 0.5$ in transition band. Plot the magn response.

Solution

(a) From Fig. 6.62 the frequency samples can be obtained as



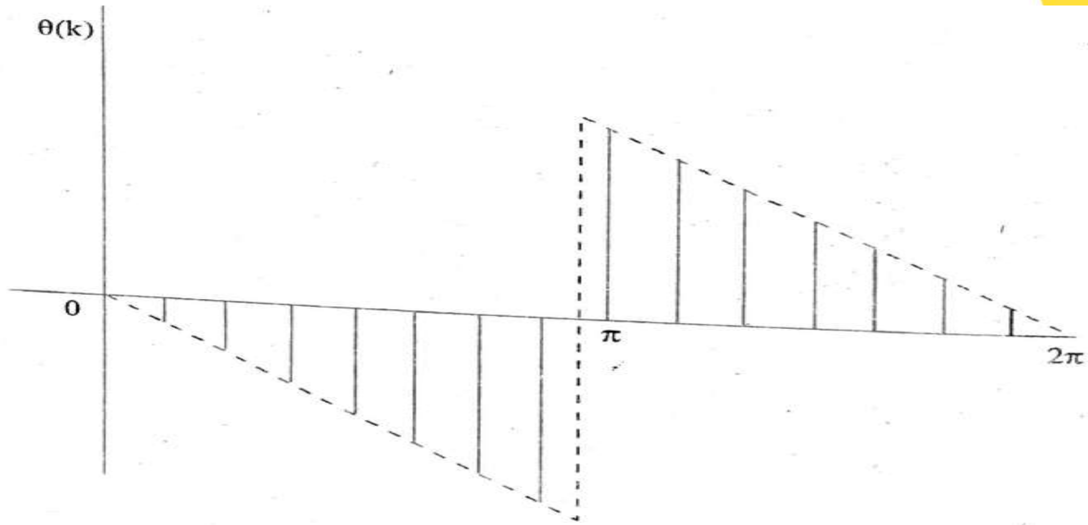


Fig. 6.62 Ideal magnitude and phase response of example 6.18.

$$H(k) = e^{-j14\pi k/15}$$

$$= 0 \quad \text{for } k = 0, 1$$

$$= e^{-j14\pi(k-15)/15} \quad \text{for } 2 \leq k \leq 13$$

for $k = 14$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} \mathrm{Re}\left(H(k)e^{j2\pi kn/N}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} \mathrm{Re}(e^{-j14\pi k/15}e^{j2\pi kn/15})\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(7-n)\right]$$

$$h(0) = h(14) = -0.0637$$
$$h(1) = h(13) = -0.0412$$
$$h(2) = h(12) = 0$$
$$h(3) = h(11) = 0.05273$$
$$h(4) = h(10) = 0.1078$$
$$h(5) = h(9) = 0.156$$
$$h(6) = h(8) = 0.188$$
$$h(7) = 0.2$$

The frequency response is given by

$$\overline{\mathrm{H}}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

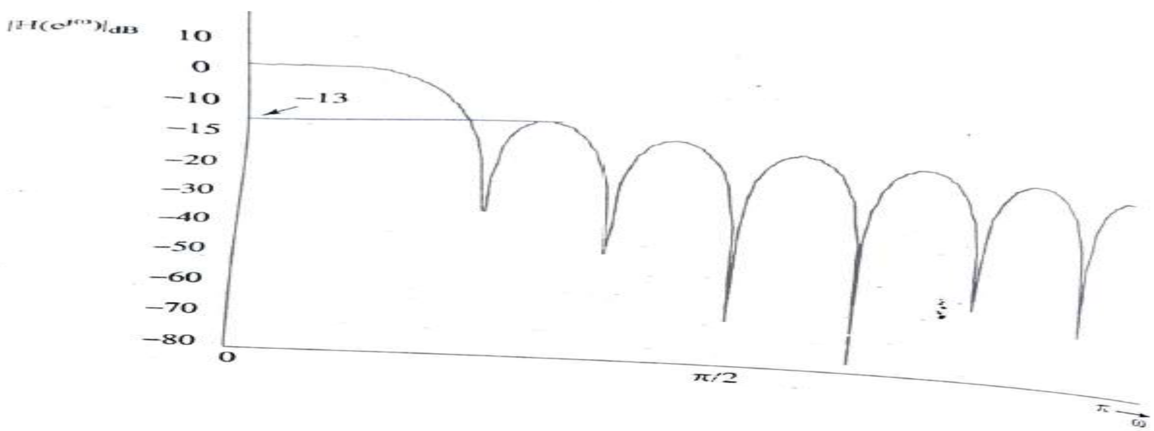
where

$$a(0) = h\left(\frac{N-1}{2}\right)$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$\Rightarrow \overline{\mathrm{H}}(e^{j\omega}) = 0.2 + 0.376\cos\omega + 0.312\cos 2\omega + 0.2156\cos 3\omega + 0.10546\cos 4\omega - 0.0824\cos 6\omega - 0.1274\cos 7\omega$$

ω (in degrees)	0	15	30	60	75	105	135	165	180
$\overline{H}(e^{j\omega})$	0.999	1.083	0.8216	−0.1824	0.0504	−0.0854	0.0712	−0.025	0.07086
$\lvert H(e^{j\omega})\rvert_{dB}$	−0.0064	0.69	−1.7	−14	−26	−21.37	−23	−32.04	−23



(b) From Fig. 6.64

$$|H(k)| = 1 \quad \text{for } k = 0, 1$$
$$= 0.5 \quad \text{for } k = 2$$
$$= 0 \quad \text{for } 3 \leq k \leq 7.$$

We can write

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k = 0, 1$$
$$= 0.5e^{-j14\pi k/15} \quad \text{for } k = 2$$
$$= 0 \quad \text{for } 3 \leq k \leq 7$$

We have

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{N} \mathrm{Re}(H(k)e^{j2\pi kn/N})\right]$$

$$= \frac{1}{15}\left[1 + 2\mathrm{Re}(e^{-j14/15}e^{j2\pi/15}) + \mathrm{Re}(e^{-j28\pi/15}e^{j4\pi/15})\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(7-n) + \cos\frac{4\pi}{15}(7-n)\right]$$

$$h(0) = h(14) = -0.00285$$

$$h(1) = h(13) = -0.0206$$

$$h(2) = h(12) = -0.0333$$

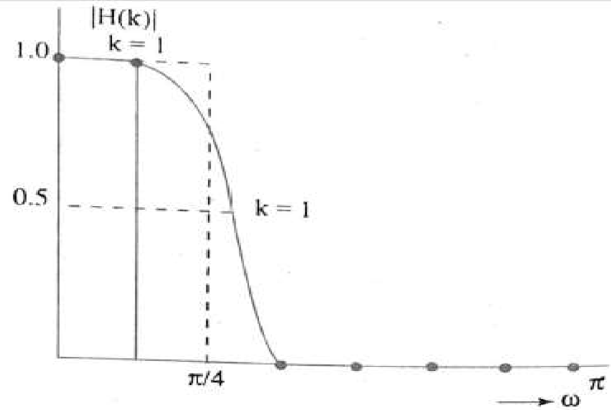


Fig. 6.64 Magnitude response of Example 6.18b with a transition sample

$$h(3) = h(11) = -0.0125$$
$$h(4) = h(10) = 0.054$$
$$h(5) = h(9) = 0.1489$$
$$h(6) = h(8) = 0.233$$
$$h(7) = 0.267$$
$$\overline{H}(e^{j\omega}) = 0.267 + 0.466\cos\omega + 0.2978\cos 2\omega + 0.108\cos 3\omega - 0.025\cos 4\omega - 0.0666\cos 5\omega - 0.0512\cos 6\omega - 0.0057\cos 7\omega$$

ω	0	15	45	70	75	
$\overline{H}(e^{j\omega})$	0.99	1.023	0.588	0.0127	−0.0179	
$\lvert H(e^{j\omega})\rvert_{dB}$	−0.0847	0.1986	−4.61	−36	−34.9	
	90	105	135	150	165	180
	−0.0046	0.0112	−0.0042	0.0134	0.00157	−0.013
	−46.74	−39	−47	−37	−55	−37.6