SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

Example 6.21 Using a rectangular window technique design a lowpass filter with passband gain of unity, cutoff frequency of 1000 Hz and working at a sampling frequency of 5 kHz. The length of the impulse response should be 7.

Solution

Given

$$f_c = 1000\,\text{Hz}$$
$$F = 5000\,\text{Hz}$$

The cutoff frequency $\omega_c = 2\pi fT$

$$= \frac{2\pi(1000)}{5000} = \frac{2\pi}{5}$$

The desired frequency response of the LPF is shown in Fig. 6.66.

The filter coefficients are given by

$$h_d(n) = \frac{1}{2\pi} \int_{-\frac{2\pi}{5}}^{\frac{2\pi}{5}} e^{j\omega n} d\omega$$

$$= \frac{\sin \frac{2\pi}{5} n}{\pi n} \quad -\infty \leq n \leq \infty$$

The rectangular window for $N = 7$ is given by

$$w_R(n) = 1 \quad \text{for } -3 \leq n \leq 3$$
$$= 0 \quad \text{otherwise}$$

For $n = 0$; $h(0) = \lim_{n \to 0} \frac{\sin \frac{2\pi}{5} n}{\pi n} = \frac{2}{5} = 0.4$

$$n = 1; h(1) = h(-1) = \frac{\sin \frac{2\pi}{5}}{\pi} = 0.3027$$

$$n = 1; h(1) = h(-1) = \frac{\sin \frac{2\pi}{5}}{\pi} = 0.3027$$

$$n = 2; h(2) = h(-2) = \frac{\sin \frac{4\pi}{5}}{2\pi} = 0.0935$$

$$n = 3; h(3) = h(-3) = \frac{\sin \frac{6\pi}{5}}{3\pi} = -0.06236$$

The filter coefficients of realizable filter are

$$h(0) = h(6) = -0.06236; h(1) = h(5) = 0.0935; h(2) = h(4) = 0.3027$$
$$h(3) = 0.4$$

$$H_d(e^{j\omega}) = e^{-j3\omega} \qquad \frac{-\pi}{4} \le \omega \le \frac{\pi}{4}$$
$$= 0 \qquad \frac{\pi}{4} < |\omega| \le \pi$$

Using a Hamming window with $N = 7$

Solution

Given $H_d(e^{j\omega}) = e^{-j3\omega}$

The frequency response is having a term $e^{-j\omega(N-1)/2}$ which gives $h(n)$ symmetrical about $n = \frac{N-1}{2} = 3$, i.e., we get a causal sequence.

$$h_d(n) = \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{-j3\omega} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{j(n-3)\omega} d\omega$$

$$= \frac{\sin \frac{\pi}{4}(n-3)}{\pi(n-3)}$$

For $N = 7$ we have

$$h_d(0) = h_d(6) = 0.075$$
$$h_d(1) = h_d(5) = 0.159$$
$$h_d(2) = h_d(4) = 0.22$$
$$h_d(3) = 0.25$$

The non-causal window sequence is

$$
\begin{aligned}
w_{Hn}(n) &= 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \le n \le (N-1)/2 \\
&= \quad 0 \quad \text{otherwise}
\end{aligned}
$$

For $N = 7$

$$
\begin{aligned}
w_{Hn}(n) &= 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -3 \le n \le 3 \\
&= \quad 0 \quad \text{otherwise} \\
w_{Hn}(0) &= 0.5 + 0.5 = 1 \\
w_{Hn}(-1) &= w_{Hn}(1) = 0.5 + 0.5\cos\frac{\pi}{3} = 0.75
\end{aligned}
$$

$$w_{Hn}(-2) = w_{Hn}(2) = 0.5 + 0.5\cos\frac{2\pi}{3} = 0.25$$
$$w_{Hn}(-3) = 0.5 + 0.5\cos\pi = 0$$

The causal window sequence can be obtained by shifting the sequence $w_{Hn}(n)$ to right by 3 samples, i.e.,

$$w_{Hn}(0) = w_{Hn}(6) = 0;\ w_{Hn}(1) = w_{Hn}(5) = 0.25$$
$$w_{Hn}(2) = w_{Hn}(4) = 0.75\ \&\ w_{Hn}(3) = 1$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad 0 \le n \le 6$$
$$h(0) = h(6) = h_d(0)w_{Hn}(0) = (0.075)(0) = 0$$
$$h(1) = h(5) = h_d(1)w_{Hn}(1) = (0.159)(0.25) = 0.03975$$
$$h(2) = h(4) = h_d(2)w_{Hn}(2) = (0.22)(0.75) = 0.165$$
$$h(3) = h_d(3)w_{Hn}(3) = (0.25)(1) = 0.25$$