SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

(b) Hamming window

The Hamming window sequence is given by

$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \leq n \leq (N-1)/2$$

$$= 0 \quad \text{otherwise}$$

The window sequence for $N = 11$ is given by

$$w_H(n) = 0.54 + 0.46 \cos \frac{\pi n}{5} \quad \text{for} \quad -5 \leq n \leq 5$$

$$= 0 \quad \text{otherwise}$$

$$w_H(0) = 0.54 + 0.46 = 1$$

$$w_H(-1) = w_H(1) = 0.54 + 0.46 \cos \frac{\pi}{5} = 0.912$$

$$w_H(-2) = w_H(2) = 0.54 + 0.46 \cos \frac{2\pi}{5} = 0.682$$

$$w_H(-3) = w_H(3) = 0.54 + 0.46 \cos \frac{3\pi}{5} = 0.398$$

$$w_H(-4) = w_H(4) = 0.54 + 0.46 \cos \frac{4\pi}{5} = 0.1678$$

$$w_H(-5) = w_H(5) = 0.54 + 0.46 \cos \pi = 0.08$$

The filter coefficients using Hamming window sequence are

$$
\begin{aligned}
h(n) &= h_d(n)w_H(n) \quad \text{for} \quad -5 \le n \le 5 \\
&= 0 \qquad\qquad \text{otherwise} \\
h(0) &= h_d(0)w_{Hn}(0) = (1)(0.75) = 0.75 \\
h(-1) &= h(1) = h_d(1)w_{Hn}(1) = (-0.225)(0.912) = -0.2052 \\
h(-2) &= h(2) = h_d(2)w_{Hn}(2) = (-0.159)(0.682) = -0.1084 \\
h(-3) &= h(3) = h_d(3)w_{Hn}(3) = (-0.075)(0.398) = -0.03 \\
h(-4) &= h(4) = h_d(4)w_{Hn}(4) = (0)(0.1678) = 0 \\
h(-5) &= h(5) = h_d(5)w_{Hn}(5) = (-0.045)(0.08) = 0.0036
\end{aligned}
$$

The transfer function of the filter is given by

$$
\begin{aligned}
H(z) &= h(0) + \sum_{n=1}^{5} \left[h(n)\left(z^{-n} + z^{n}\right)\right] \\
&= 0.75 - 0.2052\left(z^{-1} + z\right) - 0.1084\left(z^{-2} + z^{2}\right) - 0.03\left(z^{-3} + z^{3}\right) \\
&\quad + 0.0036\left(z^{-5} + z^{5}\right)
\end{aligned}
$$

The transfer function of the realizable filter is

$$\begin{aligned} H'(z) &= z^{-5}H(z) \\ &= 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.2052z^{-4} + 0.75z^{-5} \\ &\quad - 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10} \end{aligned}$$

The filter coefficients of causal filter are

$$h(0) = h(10) = 0.0036; \quad h(1) = h(9) = 0; \quad h(2) = h(8) = -0.03$$
$$h(3) = h(7) = -0.1084; \quad h(4) = h(6) = -0.2052; \quad h(5) = 0.75$$

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = -0.4104$$

$$a(2) = 2h(5-2) = 2h(3) = -0.2168$$

$$a(3) = 2h(5-3) = 2h(2) = -0.06$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.0072$$

$$\overline{H}(e^{j\omega}) = 0.75 - 0.4104\cos\omega - 0.2168\cos 2\omega - 0.06\cos 3\omega + 0.0072\cos 5\omega$$

ω (in degrees)	0	15	30	45	60	75	
$\overline{H(e^{j\omega})}$	0.07	0.125	0.28	0.497	0.7168	0.88	
$\lvert H(e^{j\omega})\rvert_{dB}$	−23.1	−18	−11	−6.07	−2.89	−1.1	

90	105	120	135	150	165	180
0.9668	0.9945	1	1.0026	1.003	1	1.0108
−0.29	−0.0478	0	0.0229	0.028	0	0.093