SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

(a) Hanning window

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad \text{otherwise}$$

For $N = 11$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{5} \quad -5 \le n \le 5$$
$$= 0 \quad \text{otherwise}$$
$$w_{Hn}(0) = 0.5 + 0.5 = 1$$
$$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5\cos\frac{\pi}{5} = 0.9045$$
$$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5\cos\frac{2\pi}{5} = 0.655$$
$$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5\cos\frac{3\pi}{5} = 0.345$$
$$w_{Hn}(4) = w_{Hn}(-4) = 0.5 + 0.5\cos\frac{4\pi}{5} = 0.0945$$
$$w_{Hn}(5) = w_{Hn}(-5) = 0.5 + 0.5\cos\pi = 0$$

The filter coefficients can be obtained from the example 6.6, i.e.,

$$h_d(n) = \frac{\sin \pi n - \sin \frac{\pi}{4} n}{n\pi}$$

$$h_d(0) = \lim_{n \to 0} \left[\frac{\sin \pi n - \sin \frac{\pi}{4} n}{\pi n} \right] = 1 - \frac{1}{4} = 0.75$$

$$h_d(-1) = h_d(1) = \frac{\sin \pi - \sin \frac{\pi}{4}}{\pi} = -0.225$$

$$h_d(-2) = h_d(2) = \frac{\sin 2\pi - \sin \frac{\pi}{2}}{2\pi} = -0.159$$

$$h_d(-3) = h_d(3) = \frac{\sin 3\pi - \sin \frac{3\pi}{4}}{3\pi} = -0.075$$

$$h_d(-4) = h_d(4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$$

$$h_d(-5) = h_d(5) = \frac{\sin 5\pi - \sin \frac{5\pi}{4}}{5\pi} = 0.045$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} d\omega + \int_{\pi/4}^{\pi} d\omega\right]$$

$$= \frac{1}{2\pi}\left[[\omega]_{-\pi}^{-\pi/4} + [\omega]_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left\{\left[-\frac{\pi}{4} + \pi\right] + \left[\pi - \frac{\pi}{4}\right]\right\}$$

$$= \frac{1}{2\pi}\left[\frac{3\pi}{4} + \frac{3\pi}{4}\right]$$

$$= \frac{1}{2\pi}\left[\frac{3\pi}{2}\right]$$

$$= \frac{3}{4} = 0.75$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad -5 \leq n \leq 5$$
$$= \quad 0 \quad \text{otherwise}$$
$$h(0) = h_d(0)w_{Hn}(0) = (0.75)(1) = 0.75$$
$$h(-1) = h(1) = h_d(1)w_{Hn}(1) = (-0.225)(0.905) = -0.204$$
$$h(-2) = h(2) = h_d(2)w_{Hn}(2) = (-0.159)(0.655) = -0.104$$
$$h(-3) = h(3) = h_d(3)w_{Hn}(3) = (-0.075)(0.345) = -0.026$$
$$h(-4) = h(4) = h_d(4)w_{Hn}(4) = (0)(0.8145) = 0$$
$$h(-5) = h(5) = h_d(5)w_{Hn}(5) = (0.045)(0) = 0$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$
$$= 0.75 - 0.204\left(z + z^{-1}\right) - 0.104\left(z^{2} + z^{-2}\right) - 0.026\left(z^{3} + z^{-3}\right)$$

The transfer function of the realizable filter is

$$H'(z) = z^{-5}H(z)$$
$$= -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6}$$
$$- 0.104z^{-7} - 0.026z^{-8}$$

The causal filter coefficients are

$$h(0) = h(1) = h(9) = h(10) = 0$$
$$h(2) = h(8) = -0.026$$
$$h(3) = h(7) = -0.104$$
$$h(4) = h(6) = -0.204$$
$$h(5) = 0.75$$

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

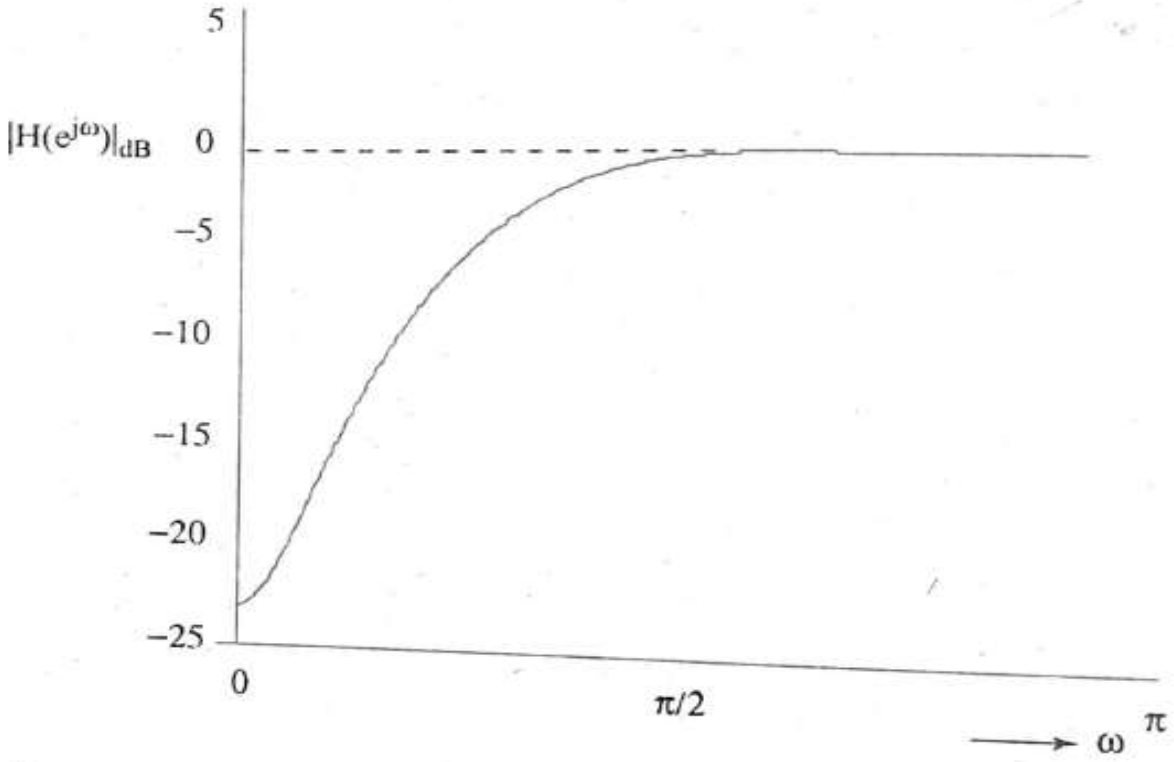
$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = -0.408$$
$$a(2) = 2h(5-2) = 2h(3) = -0.208$$
$$a(3) = 2h(5-3) = 2h(2) = -0.052$$
$$a(4) = 2h(5-4) = 2h(1) = 0$$
$$a(5) = 2h(5-5) = 2h(0) = 0$$

$$\overline{H}(e^{j\omega}) = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$$

ω (in degrees)	0	15	30	45	60	75
$\overline{H}(e^{j\omega})$	0.082	0.139	0.292	0.498	0.702	0.86
$\lvert H(e^{j\omega})\rvert_{dB}$	−21.72	−17.14	−10.67	−6.05	−3.07	−1.297

90	105	120	135	150	165	180
0.96	0.999	1.006	1.0017	1	1	1.002
−0.3726	−0.0087	0.052	0.015	0	0	0.017


5
$|H(e^{j\omega})|_{dB}$
0
−5
−10
−15
−20
−25
0
π/2
π
ω