SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

Table 6.2 Filter coefficients of FIR filters.

Type	Coefficients of zero phase filter	Coefficients of linear phase filter with delay $\alpha = \frac{N-1}{2}$
1. Lowpass filter with cutoff frequency ω_c	$h_d(0) = \frac{\omega_c}{\pi}$ $h_d(n) = \frac{\sin \omega_c n}{n\pi} \|n\| > 0$	$h_d(n) = \frac{\omega_c}{\pi}$ for $n = \alpha$ $= \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}$ for $n \neq \alpha$
2. Highpass filter with cutoff frequency ω_c	$h_d(0) = 1 - \frac{\omega_c}{\pi}$ $h_d(n) = \frac{-\sin \omega_c n}{n\pi} \|n\| > 0$	$h_d(n) = 1 - \frac{\omega_c}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin(n-\alpha)\pi - \sin(n-\alpha)\omega_c] n \neq \alpha$
3. Bandpass filter with cutoff frequencies ω_{c_1} and ω_{c_2}	$h_d(0) = \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ $h_d(n) = \frac{1}{n\pi}[\sin(\omega_{c_2} n) - \sin(\omega_{c_1} n)] \|n\| > 0$	$h_d(n) = \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin \omega_{c_2}(n-\alpha) - \sin \omega_{c_1}(n-\alpha)]$
4. Bandreject filter with cutoff frequencies ω_{c_1} and ω_{c_2}	$h_d(0) = 1 - \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ $= \frac{1}{n\pi}[\sin(\omega_{c_1} n) - \sin(\omega_{c_2} n)]$	$h_d(n) = 1 - \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$ for $n = \alpha$ $= \frac{1}{\pi(n-\alpha)}[\sin \omega_{c_1}(n-\alpha) - \sin \omega_{c_2}(n-\alpha) + \sin(n-\alpha)\pi]$

6.6 Design of FIR filters using windows

The desired frequency response $H_d(e^{j\omega})$ of a filter is periodic in frequency and can be expanded in a Fourier series. The resultant series is given by

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n} \tag{6.70}$$

where

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \tag{6.71}$$

and known as Fourier coefficents having infinite length. One possible way of obtaining FIR filter is to truncate the infinite Fourier series at $n = \pm\left(\frac{N-1}{2}\right)$, where N is the length of the desired sequence. But abrupt truncation of the Fourier series results in oscillation in the passband and stopband. These oscillations are due to slow convergence of the Fourier series and this effect is known as the Gibbs phenomenon. To reduce these oscillations, the Fourier coefficients of the filter are modified by multiplying the infinite impulse response with a finite weighing sequence $w(n)$ called a window where

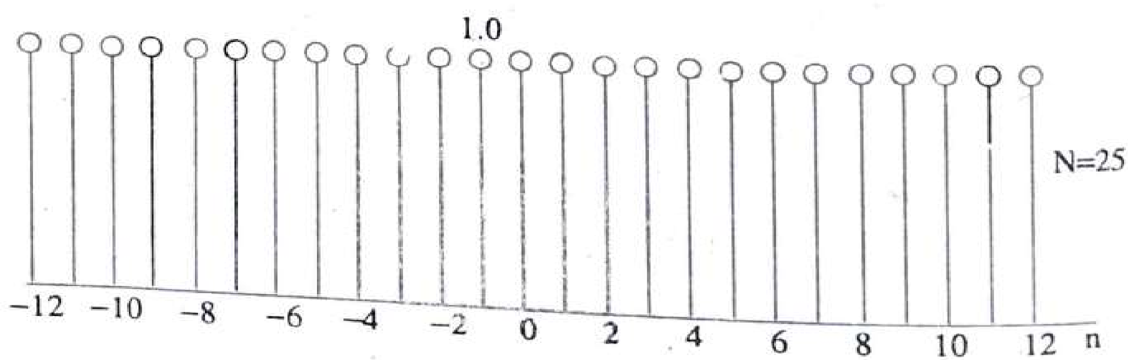
$$\begin{aligned} w(n) = w(-n) &\neq 0 \quad \text{for} \quad |n| \leq \left(\frac{N-1}{2}\right) \\ &= 0 \quad \text{for} \quad |n| > \left(\frac{N-1}{2}\right) \end{aligned} \tag{6.72}$$

6.6.1 *Rectangular window*

The rectangular window sequence is given by

$$w_R(n) = 1 \quad \text{for} -(N-1)/2 \leq n \leq (N-1)/2$$
$$= 0 \quad \text{otherwise}$$

An example is shown in Fig. 6.17 for $N = 25$.



6.6.4 *Hanning window*

The Hanning window sequence can be obtained by substituting $\alpha = 0.5$ in Eq.(6.82)

$$w_{Hn}(n) = 0.5 + 0.5\cos 2\pi n/(N-1) \quad \text{for} -(N-1)/2 \le n \le (N-1)/2 \tag{6.84}$$

$$= \quad 0 \quad \text{otherwise}$$

The frequency response of Hanning window is

$$W_{Hn}(e^{j\omega}) = 0.5\frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} + 0.25\frac{\sin(\omega N/2 - \pi N/(N-1))}{\sin(\omega/2 - \pi/(N-1))} + 0.25\frac{\sin(\omega N/2 + \pi N/(N-1))}{\sin(\omega/2 + \pi/(N-1))} \tag{6.85}$$

6.6.5 *Hamming window*

The equation for Hamming window can be obtained by substituting $\alpha = 0$ Eq.(6.82)

$$\omega_H(n) = 0.54 + 0.46\cos(2\pi n/N - 1) \quad \text{for} \quad -(N-1)/2 \le n \le (N-$$
$$= \qquad 0 \qquad \text{otherwise}$$

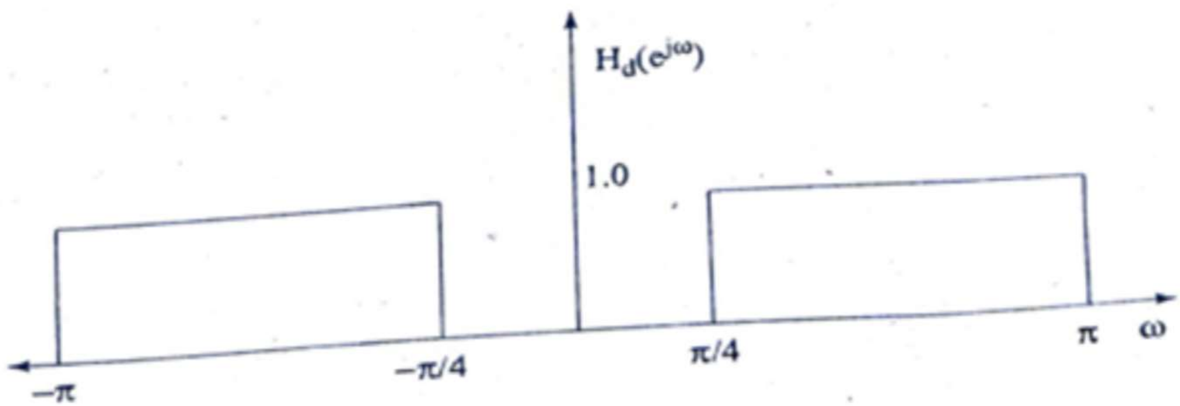
The frequency response of Hamming window is

$$W_H(e^{j\omega}) = 0.54\frac{\sin \omega N/2}{\sin \omega/2} + 0.23\frac{\sin(\omega N/2 - \pi N/(N-1))}{\sin(\omega/2 - \pi/(N-1))} + 0.23\frac{\sin(\omega N/2 + \pi N/(N-1))}{\sin(\omega/2 + \pi/(N-1))}$$

Design an ideal highpass filter with a frequency response Using Hanning and Hamming Window for

$$H_d(e^{j\omega}) = 1 \text{ for } \frac{\pi}{4} \leq |\omega| \leq \pi$$
$$= 0 \text{ for } |\omega| \leq \frac{\pi}{4}$$

Find the values of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\pi} e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4} n\right] \qquad -\infty \le n \le \infty$$