SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A+' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

Example 6.6 Design an ideal highpass filter with a frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } \frac{\pi}{4} \leq |\omega| \leq \pi$$
$$= 0 \text{ for } |\omega| \leq \frac{\pi}{4}$$

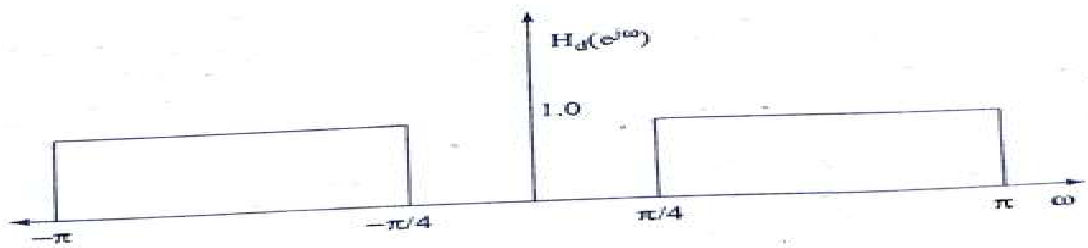
Find the values of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

Solution

The desired frequency response is shown in Fig. 6.10.

We know

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$
$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\pi} e^{j\omega n}d\omega\right]$$
$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\Big|_{-\pi}^{-\pi/4} + e^{j\omega n}\Big|_{\pi/4}^{\pi}\right]$$



$$= \frac{1}{\pi n(2j)}\left[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}\right]$$

$$= \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4} n\right] \qquad -\infty \leq n \leq \infty \qquad (6.$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = h_d(n) \quad \text{for} \quad |n| \leq 5$$
$$= 0 \quad \text{otherwise}$$

For $n = 0$

$$h(0) = \lim_{n \to 0} \frac{\sin \pi n}{\pi n} - \lim_{n \to 0} \frac{\sin \frac{\pi}{4} n}{\pi n}$$
$$= \left((1 - \frac{1}{4}\right)$$

$$\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$
$$\lim_{\theta \to 0} \frac{\sin n\theta}{\theta} = n$$

From the given frequency response we can find that $\alpha = 0$. Therefore, The filt coefficients are symmetrical about $n = 0$ satisfying the condition $h(n) = h(-n)$

For $n = 1$

$$h(1) = h(-1) = \frac{\sin \pi - \sin \frac{\pi}{4}}{\pi} = -0.225$$

Similarly

$$h(2) = h(-2) = \frac{\sin 2\pi - \sin \frac{\pi}{2}}{2\pi} = -0.159$$

$$h(3) = h(-3) = \frac{\sin 3\pi - \sin\frac{3\pi}{4}}{3\pi} = -0.075$$
$$h(4) = h(-4) = \frac{\sin 4\pi - \sin\pi}{4\pi} = 0$$
$$h(5) = h(-5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$$

The transfer function of the filter is given by

$$\begin{aligned} H(z) &= h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)(z^n + z^{-n})] \\ &= 0.75 + \sum_{n=1}^{5} [h(n)(z^n + z^{-n})] \\ &= 0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3}) \\ &\quad + 0.045(z^5 + z^{-5}) \end{aligned} \quad (6.$$

The transfer function of the realizable filter is

$$\begin{aligned} H'(z) &= z^{-5}H(z) \\ &= z^{-5}[0.75 - 0.225(z + z^{-1}) - 0.159(z^2 + z^{-2}) - 0.075(z^3 + z^{-3}) \\ &\quad + 0.045(z^5 + z^{-5})] \\ &= 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.225z^{-4} + 0.75z^{-5} - 0.225z^{-6} \\ &\quad - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10} \end{aligned} \quad (6.$$

From Eq. (6.61) the filter coefficients of causal filter are

$h(0) = h(10) = 0.045; \quad h(1) = h(9) = 0; \quad h(2) = h(8) = -0.075$

$h(3) = h(7) = -0.159; \quad h(4) = h(6) = -0.225; \quad h(5) = 0.75$

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n \quad \text{where}$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = -0.45$$

$$a(2) = 2h(5-2) = 2h(3) = -0.318$$

$$a(3) = 2h(5-3) = 2h(2) = -0.15$$

$$a(4) = 2h(5-4) = 2h(1) = 0$$

$$a(5) = 2h(5-5) = 2h(0) = 0.09$$

$$\begin{aligned}\overline{H}(e^{j\omega}) &= a(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega \\ &\quad + a(4)\cos 4\omega + a(5)\cos 5\omega \\ &= 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 5\omega \quad \textbf{(6.62)}\end{aligned}$$

ω (in degrees)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
$\overline{H}(e^{j\omega})$	−0.08	−0.066	−0.0086	0.122	0.34	0.61	0.88	1.05	1.11	1.07	0.98	0.93	0.94	0.995	1.26	1.05	1.01	0.96	0.94
$\lvert H(e^{j\omega})\rvert_{dB}$	−22	−23.62	−41.3	−18.2	−9.36	−4.2	−1.1	0.504	0.95	0.587	−0.132	−0.625	−0.537	−0.037	2	0.48	0.16	−0.31	−0.537

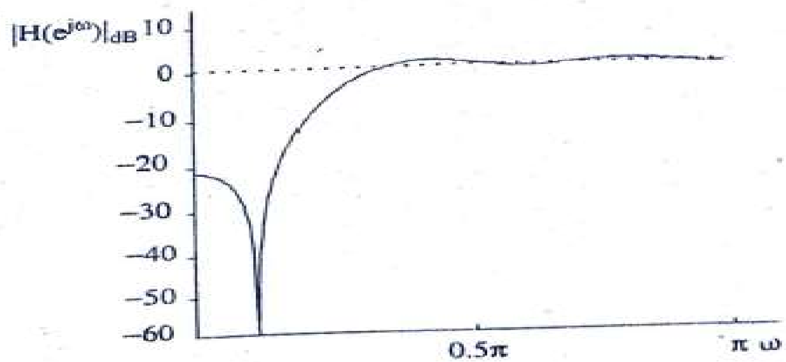


Fig. 6.11 Frequency response of highpass filter of example 6.6