# SNS COLLEGE OF TECHNOLOGY

**Coimbatore-35**
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## DEPARTMENT OF BIOMEDICAL ENGINEERING

## 19BMB302 - BIOMEDICAL SIGNAL PROCESSING

III YEAR/ V SEMESTER

# UNIT II FINITE IMPULSE RESPONSE FILTERS

- Introduction to FIR
- Linear phase FIR filter
- FIR filter design using window method
- Low Pass Filter
- Frequency sampling method
- Realization of FIR filter using direct form 1, Direct form 2
- Realization of FIR filter using Cascade structures
- Realization of FIR filter using parallel structures

# FIR Filter

- Finite impulse response (FIR) filters are digital filters that have a finite impulse response. FIR filters operate only on current and past input values and are the simplest filters to design. FIR filters also are known as non recursive filters.
- FIR filters provide a linear phase response. IIR filters provide a nonlinear phase response.
- FIR filters are always  stable because they are implemented using an all zero transfer function.
- FIR filters are used for applications that require linear phase responses like high quality audio systems.

# Linear-phase filters

The ability to have an exactly linear phase response is the one of the most important of FIR filters

$$H(\omega) = |H(\omega)| e^{j\phi(\omega)} \quad \text{where } \phi(\omega) = -\omega n_0$$

A general FIR filter does not have a linear phase response but this property is satisfied when

$$h(n) = \pm h(M-1-n), \quad n = 0, 1, \ldots, M-1.$$

four linear phase filter types

| Impulse response | # coefs | $H(\omega)$ | Type |
|---|---|---|---|
| $h(n) = h(M-1-n)$ | Odd | $e^{-j\omega(M-1)/2}\left(h\left(\frac{M-1}{2}\right) + 2\sum_{k=1}^{(M-3)/2} h\left(\frac{M-1}{2} - k\right)\cos(\omega k)\right)$ | 1 |
| $h(n) = h(M-1-n)$ | Even | $e^{-j\omega(M-1)/2} 2\sum_{k=1}^{(M-3)/2} h\left(\frac{M}{2} - k\right)\cos\left(\omega\left(k - \frac{1}{2}\right)\right)$ | 2 |
| $h(n) = -h(M-1-n)$ | Odd | $e^{-j[\omega(M-1)/2-\pi/2]}\left(2\sum_{k=1}^{(M-1)/2} h\left(\frac{M-1}{2} - k\right)\sin(\omega k)\right)$ | 3 |
| $h(n) = -h(M-1-n)$ | Even | $e^{-j[\omega(M-1)/2-\pi/2]} 2\sum_{k=1}^{(M-1)/2} h\left(\frac{M}{2} - k\right)\sin\left(\omega\left(k - \frac{1}{2}\right)\right)$ | 4 |

**Example 6.5** Design an ideal lowpass filter with a frequency response

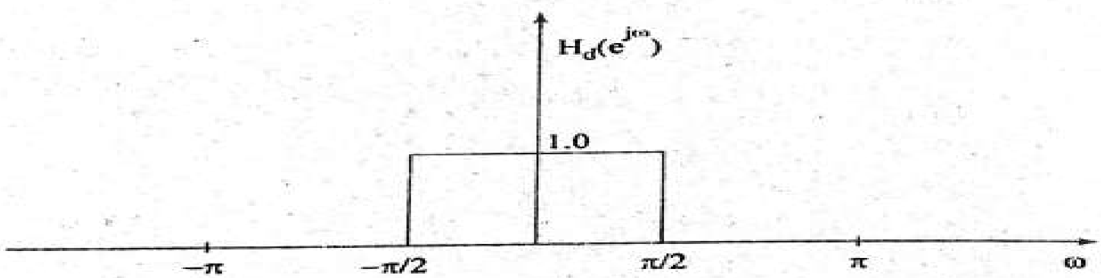
$$H_d(e^{j\omega}) = 1 \text{ for } -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}$$
$$= 0 \text{ for } \frac{\pi}{2} \leq |\omega| \leq \pi$$

Find the values of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

**Solution**

The frequency response of lowpass filter with $\omega_c = \frac{\pi}{2}$ is shown in Fig. 6.8.
Given

$$H_d(e^{j\omega}) = 1 \text{ for } \frac{-\pi}{2} \leq \omega \leq \frac{\pi}{2}$$
$$= 0 \text{ for } \frac{\pi}{2} \leq |\omega| \leq \pi$$



From the frequency response we can find that $\alpha = 0$. Therefore, we get a non-causal filter coefficients symmetrical about $n = 0$, i.e., $h_d(n) = h_d(-n)$. The filter coefficients can be obtained by using the formula given in table 6.2 for zero phase frequency response (or) we can proceed as follows.

We know

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{j\omega n} d\omega \quad (6.54)$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n (2j)} \left[ e^{j\pi n/2} - e^{-j\pi n/2} \right]$$

$$= \frac{\sin \frac{\pi}{2} n}{\pi n} \quad -\infty \le n \le \infty \quad (6.55)$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = \frac{\sin\frac{\pi}{2}n}{\pi n} \quad \text{for} \quad |n| \le 5$$
$$= 0 \quad \text{otherwise}$$

For $n = 0$ Eq. (6.56) becomes indeterminate. So

$$h(0) = \lim_{n\to 0}\frac{\sin\frac{\pi}{2}n}{\pi n} = \frac{1}{2}\lim_{n\to 0}\frac{\sin\frac{\pi}{2}n}{\frac{\pi n}{2}} = \frac{1}{2}$$

$$\therefore \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

(or)

Substitute $n = 0$ in Eq. (6.54) we get

$$h(0) = h_d(0) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} d\omega = \frac{1}{2\pi}\omega\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2\pi} = \frac{1}{2}.$$

For $n = 1$

$$h(1) = h(-1) = \frac{\sin\frac{\pi}{2}}{\pi} = \frac{1}{\pi} = 0.3183.$$

Similarly

$$h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$$

$$h(3) = h(-3) = \frac{\sin \frac{3\pi}{2}}{3\pi} = -\frac{1}{3\pi} = -0.106$$

$$h(4) = h(-4) = \frac{\sin \frac{4\pi}{2}}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin \frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.06366.$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} [h(n)\,(z^n + z^{-n})]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)\,(z^n + z^{-n})$$

$$= 0.5 + 0.3183\,(z^1 + z^{-1}) - 0.106\,(z^3 + z^{-3}) + 0.06366\,(z^5 + z^{-5}).$$

The transfer function of the realizable filter is

$$H'(z) = z^{-(N-1)/2} H(z)$$

$$= z^{-5}\left[0.5 + 0.3183\left(z + z^{-1}\right) - 0.106\left(z^{3} + z^{-3}\right) + 0.06366\left(z^{5} + z^{-5}\right)\right]$$

$$= 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z^{-5} + 0.3183z^{-6} - 0.106z^{-8} + 0.06366z^{-10}$$

From the above Eq. (6.57) the filter coefficients of causal filter are given by

$$h(0) = h(10) = 0.06366; \quad h(1) = h(9) = 0; \quad h(2) = h(8) = -0.106$$

$$h(3) = h(7) = 0; \quad h(4) = h(6) = 0.3183; \quad h(5) = 0.5$$

The frequency response is given by

$$\overline{H}(e^{j\omega}) = \sum_{n=0}^{5} a(n)\cos\omega n \quad \text{where}$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$$a(1) = 2h(5-1) = 2h(4) = 0.6366$$

$$a(2) = 2h(5-2) = 2h(3) = 0$$

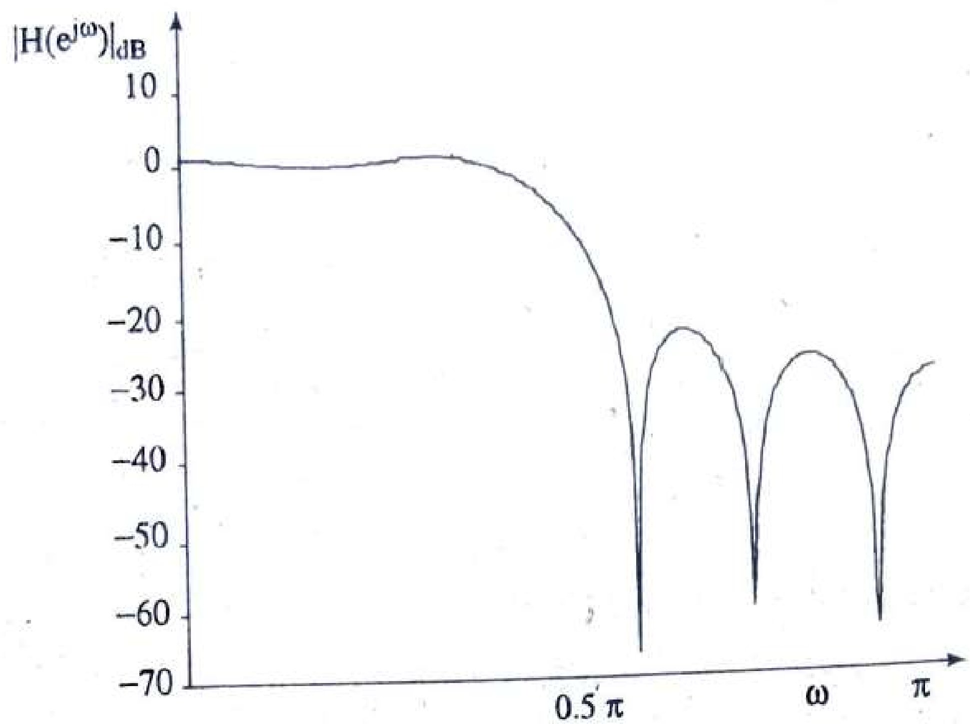
$$a(3) = 2h(5-3) = 2h(2) = -0.212$$
$$a(4) = 2h(5-4) = 2h(1) = 0$$
$$a(5) = 2h(5-5) = 2h(0) = 0.127$$

$$\overline{H}(e^{j\omega}) = 0.5 + 0.6366\cos\omega - 0.212\cos 3\omega + 0.127\cos 5\omega$$

The magnitude in dB is calculated by varying $\omega$ from 0 to $\pi$ and tabulated b
The magnitude $|H(e^{j\omega})|_{dB} = 20\ \log|\overline{H}(e^{j\omega})|$.

| $\omega$(in degrees) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| $\left\|H(e^{j\omega})\right\|_{dB}$ | 0.4 | 0.21 | -0.26 | -0.517 | -0.21 | 0.42 | 0.77 | 0.21 | -1.79 |

| 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 |
|---|---|---|---|---|---|---|---|---|---|
| -6 | -14.56 | -31.89 | -20.6 | -26 | -32 | -24.7 | -30.55 | -32 | -26 |



$|H(e^{j\omega})|_{dB}$
10
0
−10
−20
−30
−40
−50
−60
−70
0.5 π
ω
π