

Approaches to Line Balancing COMSOAL & RPW

Active Learning Module 2



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Background Material

- ◆ **Modeling and Analysis of Manufacturing Systems** by Ronald G. Askin , Charles R. Standridge, John Wiley & Sons, 1993, Chapter 2.
- ◆ **Manufacturing Systems Engineering** by Stanley B. Gershwin, Prentice – Hall, 1994, Chapter 2.
- ◆ Any good manufacturing systems textbook which has detailed explanation on reliable serial systems.

Lecture Objectives

- ◆ At the end of this module, the students should be able to
 - Explain the approaches to line balancing
 - ◆ COMSOAL Random Sequence Generation
 - ◆ Ranked Positional Weight Heuristics
 - Solve and find the optimal solutions to line balancing problems using the above techniques

Time Management

| | |
|---------------------------------|----------------|
| Introduction | 5 |
| Readiness Assessment Test (RAT) | 5 |
| COMSOAL Procedure | 12 |
| Spot Exercise | 5 |
| RPW Procedure | 15 |
| Team Exercise | 5 |
| Assignment | 3 |
| Total Time | 50 Mins |

RAT – Solution

1. In a **Process** layout, work stations are arranged according to the general function they perform without regard to any particular product.
2. **True.** A product layout is more suited to situations where product demand is stable than when it is fluctuating.
3. **True.** Fixed position layouts are used in projects where the product cannot be moved, and therefore equipment, workers, and materials are brought to it.
4. **False.** In general, work-in-process inventory is large for a process layout and small for a product layout.
5. **Low Volume** is associated with process layout.

Approaches to Line Balancing

Three Basic Approaches for finding a solution

- **COMSOAL** – Basic random solution generation method
- **Ranked Positional Weight Heuristic** – Good solutions found quickly
- **Implicit Enumeration Scheme**

Assumptions

Required cycle time, sequencing restrictions and task times are known

COMSOAL Random Sequence Generation

- ◆ A simple record-keeping approach that allows a large number of possible sequences to be examined quickly
- ◆ Only tasks that satisfy all the constraints are considered at each step.
- ◆ Sequence discarded as soon as it exceeds the upper bound.
- ◆ Sequence saved if it is better than the previous upper bound and the bound is updated.
- ◆ Efficiency depends on the data storage and processing structure

COMSOAL – Cont...

- ◆ COMSOAL uses several list for speed computation.
 - $NIP(i)$ → Number of immediate predecessors for each task i .
 - $WIP(i)$ → Indicates for which other tasks i is an immediate predecessor.
 - TK → Consists of N tasks.
- ◆ During each sequence generation,
 - List of unassigned tasks (A)
 - Tasks from A with all immediate predecessors (B)
 - Tasks from B with task times not exceeding remaining cycle time in the workstation ($F - Fit List$)are updated.

COMSOAL Procedure

1. Set $x = 0$, $UB = \infty$, $C = \text{Cycle Time}$, $c = C$.
2. Start the new sequence : Set $x = x+1$, $A = TK$, $NIPW(i) = NIP(i)$.
3. Precedence Feasibility : For all, if $NIPW(i) = 0$, add i to B .
4. Time Feasibility : For all $i \in B$, if $t_i \leq c$, add i to F . If F empty, Step 5; otherwise Step 6.
5. Open new station : $IDLE = IDLE + c$. $c = C$. If $IDLE > UB$ go to Step 2; Otherwise Step 3.

COMSOAL Procedure – Cont...

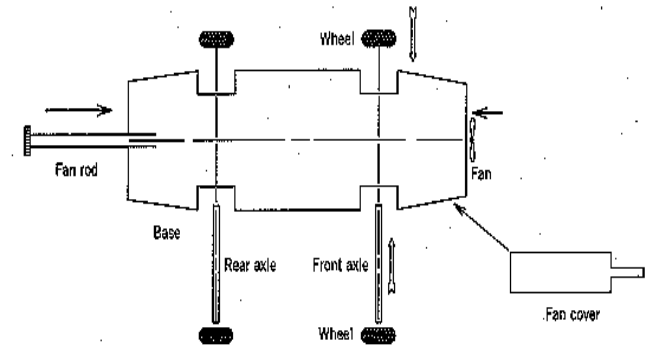
6. Select Task : Set $m = \text{card } \{F\}$. Randomly generate $RN \in U(0,1)$. Let $i^* = [m.RN]_{th}$ task from F . Remove i^* from A, B, F . $c = c - t_i^*$. For all $i \in WIP(i^*)$, $NIPW(i) = NIPW(i) - 1$. If A empty, go to Step 7; otherwise go to Step 3.
7. Schedule completion : $IDLE = IDLE + c$. If $IDLE \leq UB$, $UB = IDLE$ and store schedule. If $x = X$, stop; otherwise go to Step 2.

COMSOAL – Advantages

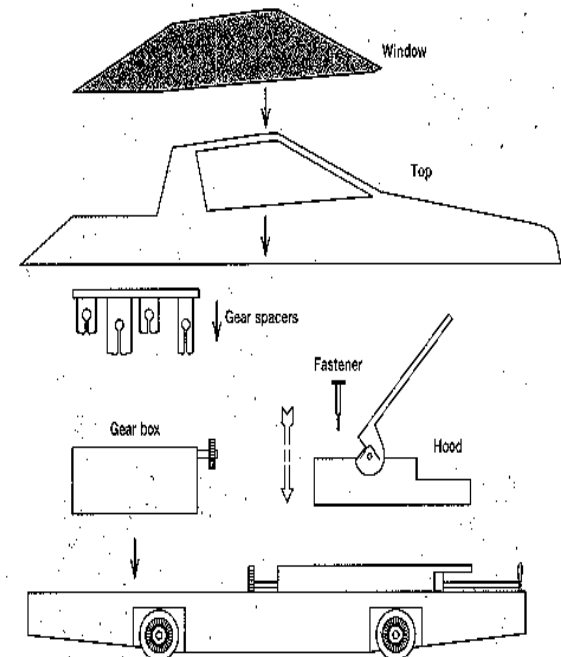
- ◆ The technique is relatively easy to program.
- ◆ Feasible solutions are found quickly.
- ◆ Greater the computational effort expended, the better the expected solution .
- ◆ Basic idea can be applied to many decision problems, the only requirement being that we can build solutions sequentially and a function evaluation can be performed to rank candidate solutions.

COMSOAL – Example

| Task | Activity | Assembly Time | Immediate Predecessor |
|----------|----------------------------|---------------|-----------------------|
| <i>a</i> | Insert Front Axle / Wheels | 20 | - |
| <i>b</i> | Insert Fan Rod | 6 | <i>a</i> |
| <i>c</i> | Insert Fan Rod Cover | 5 | <i>b</i> |
| <i>d</i> | Insert Rear Axle / Wheels | 21 | - |
| <i>e</i> | Insert Hood to Wheel Frame | 8 | - |
| <i>f</i> | Glue Windows to top | 35 | - |
| <i>g</i> | Insert Gear Assembly | 15 | <i>c, d</i> |
| <i>h</i> | Insert Gear Spacers | 10 | <i>g</i> |
| <i>i</i> | Secure Front Wheel Frame | 15 | <i>e, h</i> |
| <i>j</i> | Insert Engine | 5 | <i>c</i> |
| <i>k</i> | Attach Top | 46 | <i>f, i, j</i> |
| <i>l</i> | Add Decals | 16 | <i>k</i> |



(a) Tasks *a* through *d*



(b) Tasks *e* through *k*

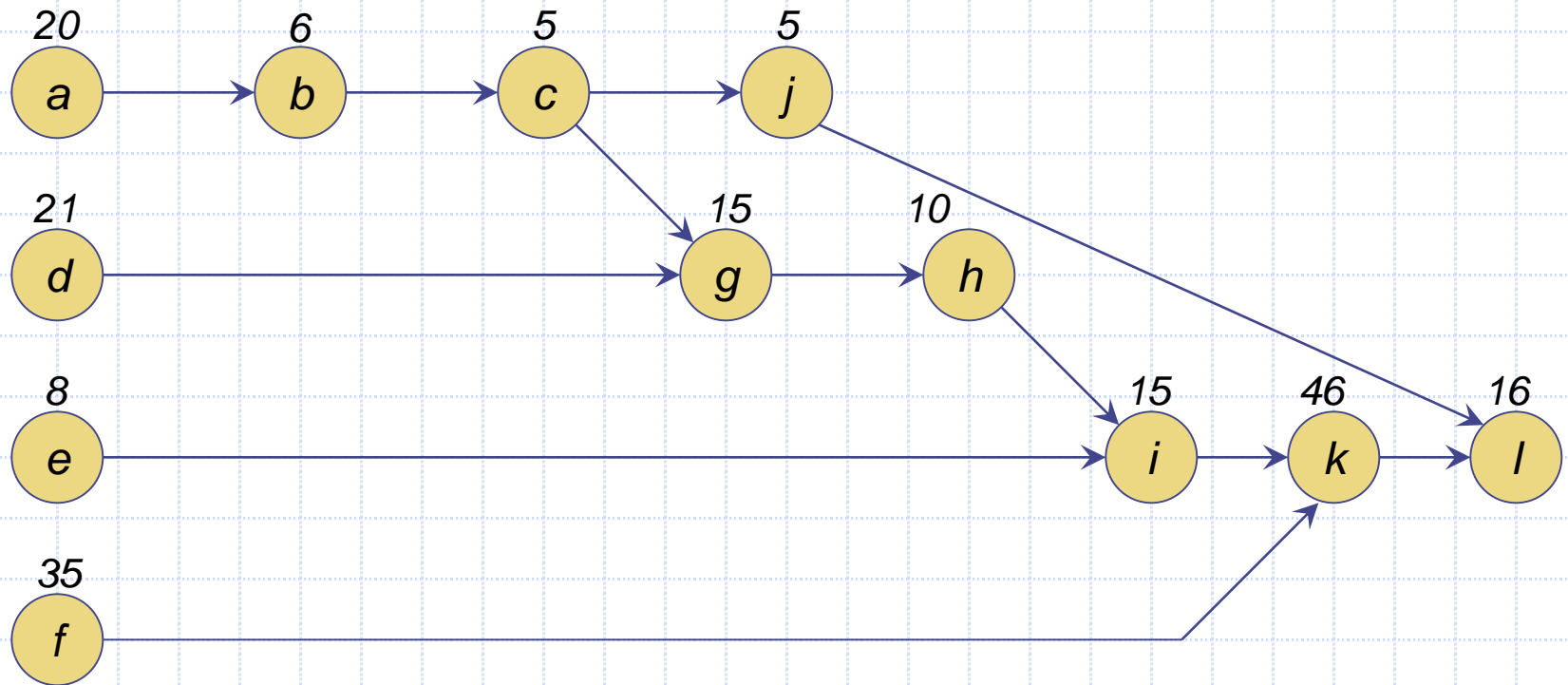
COMSOAL – Example

Data Known :

- Two 4 hour-shifts, 4 days a week will be used for assembly.
- Each shift receives two 10 minute breaks.
- Planned production rate of 1500 units/week.
- No Zoning constraints exist.

Example Solution

Model Car Precedence Structure



Example Solution – Cont...

$$C = \frac{1 \text{ Week}}{1500 \text{ Units}} \times 4 \frac{\text{days}}{\text{week}} \times 2 \frac{\text{shifts}}{\text{day}} \times 220 \frac{\text{minutes}}{\text{shift}} = 1.17 \frac{\text{minutes}}{\text{unit}}$$

- ◆ To meet demand $C = 70$ Seconds.
- ◆ Initially four potential tasks $a, d, e, \text{ or } f$
- ◆ Generate random number between 0 and 1. Say outcome in our case is 0.34
- ◆ R is in second quadrant so keep d as first task.
- ◆ Continue the random generation.
- ◆ Quick check of lower bound

$$K^0 = \left\lceil \sum_{r=a}^l t_r / C \right\rceil = \lceil 202/70 \rceil = 3$$

Thus Better
Solutions may exist

Single COMSOAL Sequence Results

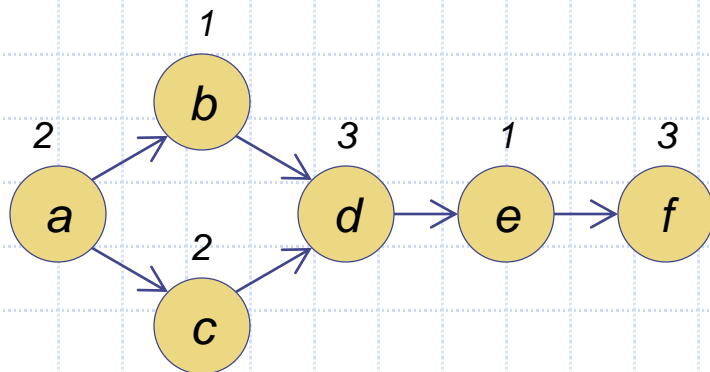
| Step | List A | List B | List F | $U(0,1)$ | Selected Tasks | Station (Idle Time) |
|------|-------------------------------------|-------------------|-------------------|--------------|----------------|---------------------|
| 1 | <i>a through l</i> | <i>a, d, e, f</i> | <i>a, d, e, f</i> | 0.34 | <i>d</i> | 1(49) |
| 2 | <i>a through l, -d</i> | <i>a, e, f</i> | <i>a, e, f</i> | 0.83 | <i>f</i> | 1(14) |
| 3 | <i>a, b, c, e, g, h, i, j, k, l</i> | <i>a, e</i> | <i>e</i> | - | <i>e</i> | 1(6) |
| 4 | <i>a, b, c, g, h, i, j, k, l</i> | <i>a</i> | - | Open Station | | |
| 4 | <i>a, b, c, g, h, i, j, k, l</i> | <i>a</i> | <i>a</i> | - | <i>a</i> | 2(50) |
| 5 | <i>b, c, g, h, i, j, k, l</i> | <i>b</i> | <i>b</i> | - | <i>b</i> | 2(44) |
| 6 | <i>c, g, h, i, j, k, l</i> | <i>c</i> | <i>c</i> | - | <i>c</i> | 2(39) |
| 7 | <i>g, h, i, j, k, l</i> | <i>g, j</i> | <i>g, j</i> | 0.21 | <i>g</i> | 2(24) |
| 8 | <i>h, i, j, k, l</i> | <i>j, h</i> | <i>h, j</i> | 0.42 | <i>h</i> | 2(14) |
| 9 | <i>i, j, k, l</i> | <i>i, j</i> | <i>j</i> | - | <i>j</i> | 2(9) |
| 10 | <i>i, k, l</i> | <i>i</i> | - | Open Station | | |
| 10 | <i>i, k, l</i> | <i>i</i> | <i>i</i> | - | <i>i</i> | 3(55) |
| 11 | <i>k, l</i> | <i>k</i> | <i>k</i> | - | <i>k</i> | 3(9) |
| 12 | <i>l</i> | <i>l</i> | - | Open Station | | |
| 12 | <i>l</i> | <i>l</i> | <i>l</i> | - | <i>l</i> | 4(54) |

Spot Exercise

Solve the following line balancing problem using COMSOAL procedure. Assume demand is 100/day.

| Task | Time | Immediate Predecessor |
|------|------|-----------------------|
| a | 2 | - |
| b | 1 | a |
| c | 2 | a |
| d | 3 | b, c |
| e | 1 | d |
| f | 3 | e |

Exercise Solution



$$C = \frac{\text{Production time available}}{\text{Desired units of output}}$$
$$= \frac{8 \text{ Hours} \times 60 \text{ Minutes/hour}}{100}$$
$$= 4.8 \text{ Minutes}$$

$$K^0 = \left\lceil \sum_{r=a}^l t_r / C \right\rceil = \lceil 12 / 4.8 \rceil = 2.5 \approx 3$$

Exercise Solution – Cont...

| Step | List A | List B | List F | U(0,1) | Selected Task | Station (Idle Time) |
|------|------------|--------|--------|--------|---------------|---------------------|
| 1 | a to f | a | a | - | a | 1(2.8) |
| 2 | b to f | b, c | b, c | 0.68 | c | 1(0.8) |
| 3 | b to f, -c | b | b | - | b | 2(3.8) |
| 4 | d to f | d | d | - | d | 2(0.8) |
| 5 | e, f | e | e | - | e | 3(3.8) |
| 6 | f | f | f | - | f | 3(0.8) |

Ranked Positional Weight Heuristic

- ◆ A task is prioritized based on the cumulative assembly time associated with itself and its successors.
- ◆ Tasks are assigned in this order to the lowest numbered feasible workstation.
- ◆ Cumulative remaining assembly time constrains the number of workstations required.
- ◆ Illustrates the greedy, single pass heuristics.
- ◆ Procedure requires computation of *positional weight* $PW(i)$ of each task.

RPW Procedure

- ◆ Let $S(i) \rightarrow$ Set of successors of tasks i .
- ◆ Example, $j \in S(i)$ means j cannot begin until i is complete.
- ◆ Compute $PW_i = t_i + \sum_{r \in S(i)} t_r$
- ◆ Tasks ordered such that $i < r$ implies $i \text{ not } \in S(r)$.
- ◆ Task r is then a member of $S(i)$ only if there exists an immediate successor relationship from i to r .
- ◆ Immediate successors $IS(i)$ are known from the inverse of the $IP(i)$ relationships.

RPW Procedure – Cont...

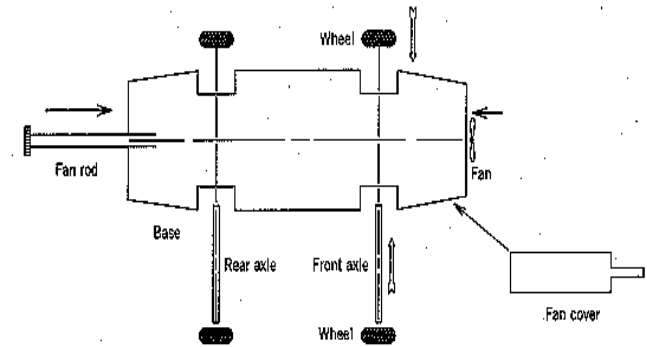
1. **Task Ordering** : For all tasks $i = 1, \dots, N$ compute $PW(i)$.
Order (rank) tasks by nonincreasing $PW(i)$
2. **Task Assignment** : For ranked tasks $i = 1, \dots, N$ assign task i to first feasible workstation.

Precedence Constraints : assignment to any workstation at least as large as that to which its predecessors are assigned

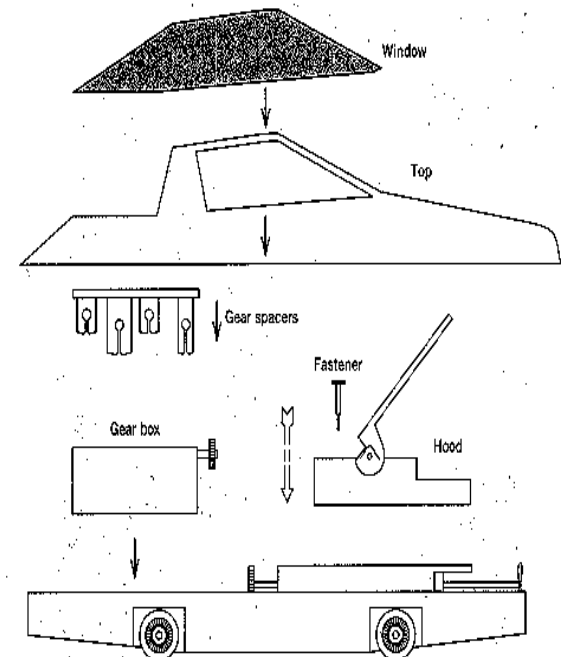
Zoning & Time Restrictions : Checked on placement.

RPW Procedure - Example

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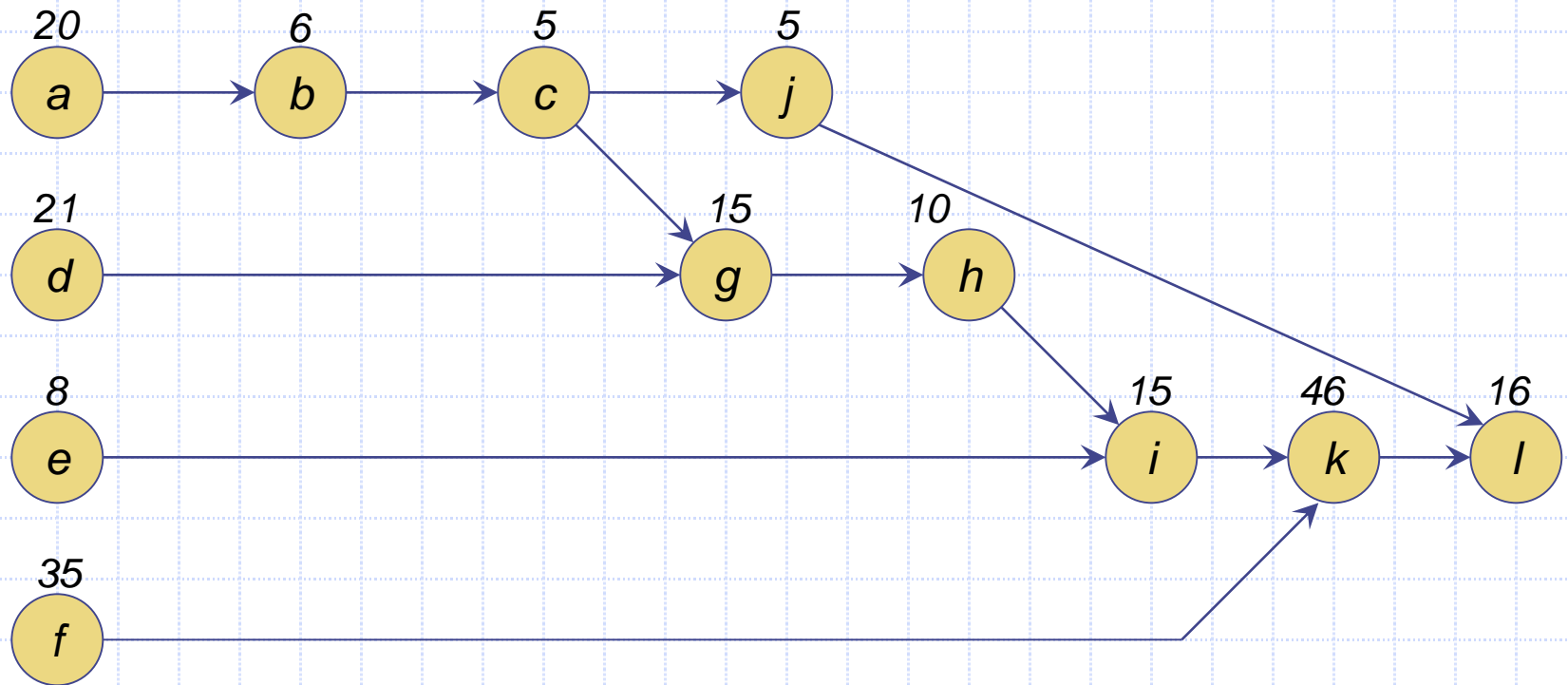
(a) Tasks *a* through *d*



(b) Tasks *e* through *k*

Example Solution

Model Car Precedence Structure



RPW Procedure - Solution

Positional Weight calculated based on the precedence structure (previous slide).

$$PW_l = \text{its task time} = 16$$

$$PW_k = t_k + PW_l = 46 + 16 = 62$$

$$PW_j = t_j + PW_k = 5 + 62 = 67$$

| Task | PW | Ranked PW |
|------|-----|-----------|
| a | 138 | 1 |
| b | 118 | 3 |
| c | 112 | 4 |
| d | 123 | 2 |
| e | 85 | 8 |
| f | 97 | 6 |
| g | 102 | 5 |
| h | 87 | 7 |
| i | 77 | 9 |
| j | 67 | 10 |
| k | 62 | 11 |
| l | 16 | 12 |

RPW Solution Cont...

- ◆ Assignment order is given by the rankings.
- ◆ Task *a* assigned to station 1.
 - $c - t_a = 70 - 20 = 50$ seconds left in Station 1.
- ◆ Next Assign task *d*
 - $50 - 21 = 29$ seconds left in Station 1.

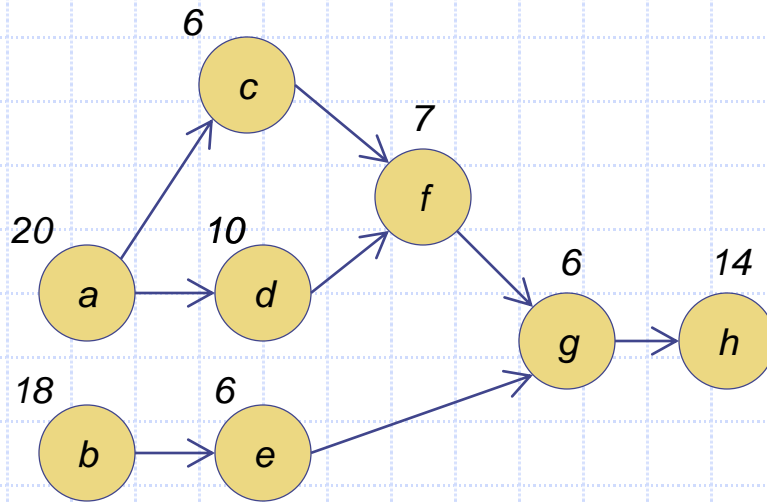
| Station | Time Remaining | Tasks |
|---------|-----------------------|---------------|
| 1 | 70, 50, 29, 23, 18, 3 | a, d, b, c, g |
| 2 | 70, 35, 25, 17, 2 | f, h, e, i |
| 3 | 70, 65, 19, 3 | j, k, l |

Team Exercise

Assembly of a product has been divided into elemental tasks suitable for assignment to unskilled workers. Task times and constraints are given below. Solve by RPW Procedure

| Task | Time | Immediate Predecessors |
|------|------|------------------------|
| a | 20 | - |
| b | 18 | - |
| c | 6 | a |
| d | 10 | a |
| e | 6 | b |
| f | 7 | c, d |
| g | 6 | e, f |
| h | 14 | g |

Exercise Solution



| Task | PW _i | Rank |
|------|-----------------|------|
| a | 63 | 1 |
| b | 44 | 2 |
| c | 33 | 4 |
| d | 37 | 3 |
| e | 26 | 6 |
| f | 27 | 5 |
| g | 20 | 7 |
| h | 14 | 8 |

| Workstation | Assigned Tasks | Remaining Time |
|-------------|----------------|----------------|
| 1 | a, d | 30, 10, 0 |
| 2 | b, c, e | 30, 12, 6, 0 |
| 3 | f, g, h | 30, 23, 17, 3 |

Assignment

Write a flowchart for COMSOAL using the decision rule that feasible tasks are selected with probability proportional to their positional weight.