## UNIT 2 – ORTHOGONAL TRANSFORMATION OF A REAL SYMMETRIC MATRIX

Quadratic form

### **DEFENITION OF QUADRATIC FORM:**

A homogeneous polynomial of degree 2 with any number of variables are known as quadratic form

$$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 0$$

$$\begin{bmatrix} co\ eff\ of\ x^2 & \frac{1}{2}co\ eff\ of\ xy & \frac{1}{2}co\ eff\ of\ xz \\ \frac{1}{2}co\ eff\ of\ yx & co\ eff\ of\ x^2 & \frac{1}{2}co\ eff\ of\ yz \\ \frac{1}{2}co\ eff\ of\ zx & \frac{1}{2}co\ eff\ of\ zy & co\ eff\ of\ z^2 \end{bmatrix}$$

## **WORKING RULE:**

**STEP 1**:Write the matrix o the quadratic form .then find  $D=N^TAN$ 

By orthogonal transformation.

**STEP 2**: Find  $Q = Y^T DY$ 

## **INDEX OF QUADRATIC FORM**

The no of positive square terms in the canonical form is called the index of the quadratic form.It is denoted by p

### SIGNATURE OF QUADRATIC FORM

The different of positive and negative square terms are called signature of quadratic terms . denoted by  $\boldsymbol{s}$ 

$$s=2p-r$$

#### **NATURE OF QUDARTIC FORM:**

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Positive definite Ex: 1,2,2

Negative definite Ex: -1,-2-,2

Semi Positive Ex: 0,1,2

Semi negative Ex: 0,-1,-2

Indefinite Ex: -1,1,2

-2,-1,1

## **Problems:**

1. Find the nature of the given equation  $2x^2 + 2xy + 3y^2 = 0$ 

**STEP 1**: The matrix form

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

 $c_1$  = Sum Of Diagonal Elements

$$= 2+3 =5$$

$$c_2 = |A| = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$$

The characteristic equation is

$$\lambda^2 - 5\lambda + 5 = 0$$

Here  $c_1$  and  $c_2$  are positive

Hence the nature of the given matrix is positive definite