



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

COIMBATORE-35



$$e^{ax+by} + \cos(ax+by)$$

J. Solve $(D^2 - DD' - 20D'^2)z = e^{5x+y} + \sin(4x-y)$

Soln.

AE

$$m^2 - m - 20 = 0$$

$$D \rightarrow m$$

$$(m+5)(m-4) = 0$$

$$D' \rightarrow 1$$

$$m = 5, -4$$

$$CF = \delta_1 (y - 4x) + \delta_2 (y + 5x)$$

$$PI = \frac{1}{D^2 - DD' - 20D'^2} [e^{5x+y} + \sin(4x-y)]$$

$$= \frac{1}{D^2 - DD' - 20D'^2} e^{5x+y} + \frac{1}{D^2 - DD' - 20D'^2} \sin(4x-y)$$

$$PI = PI_1 + PI_2$$

$$PI_1 = \frac{1}{D^2 - DD' - 20D'^2} e^{5x+y}$$

$$= \frac{1}{25 - 5(1) - 20(1)^2} e^{5x+y} \quad \begin{array}{l} D \rightarrow a = 5 \\ D' \rightarrow b = 1 \end{array}$$

$$= \frac{1}{0} e^{5x+y}$$

$$= x \frac{1}{2D - D'} e^{5x+y}$$

$$= x \frac{1}{2(5) - 1} e^{5x+y}$$

$$PI_1 = \frac{x}{9} e^{5x+y}$$

$$PI_2 = \frac{1}{D^2 - DD' - 20D'^2} \sin(4x-y)$$

$$= \frac{1}{-16 - 4 - 20(-1)} \sin(4x-y) \quad \begin{array}{l} D^2 \rightarrow a^2 = -16 \\ DD' \rightarrow -ab = -4(-1) = 4 \\ D'^2 \rightarrow -b^2 = -(-1)^2 = -1 \end{array}$$

$$= x \frac{1}{2D - D'} \sin(4x-y)$$

$$= x \frac{(2D + D') \sin(4x-y)}{(2D - D')(2D + D')}$$

$$= x \frac{(2D + D') \sin(4x-y)}{4D^2 - D'^2}$$

$$= \frac{x}{-64 + 1} (2D + D') \sin(4x-y)$$

$$= -\frac{x}{63} [2D \sin(4x-y) + D' \sin(4x-y)]$$

$$= -\frac{x}{63} [8 \cos(4x-y) - \cos(4x-y)]$$

$$= \frac{-7x \cos(4x-y)}{63}$$

$$= -\frac{x}{9} \cos(4x-y)$$

The soln is, $x = CF + PI$

$$x = f_1(y-4x) + f_2(y+5x) + \frac{x}{9} e^{5x+y} - \frac{x}{9} \cos(4x-y)$$

Hw
1]. $(D^2 + 4DD' - 5D'^2) x = e^{2x-y} + \sin(x-2y)$

2]. $(D^2 - DD' - 20D'^2) x = xy + e^{6x+y}$

Solve $x + y - 6t = y \cos x$

Soln:

$$\text{Given } \frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 x}{\partial x \partial y} - 6 \frac{\partial^2 x}{\partial y^2} = y \cos x$$

$$(D^2 + DD' - 6D'^2) x = y \cos x$$

AE

$$m^2 + m - 6 = 0$$

$$(m+3)(m-2) = 0$$

$$m = -3, 2$$

$$CF = f_1(y-3x) + f_2(y+2x)$$

$$PI = \frac{1}{(D^2 + DD' - 6D'^2)} y \cos x$$

factor $\rightarrow D - 2D'$

where $y = C - 2x$

$D \rightarrow C$

$D' \rightarrow x$

$$= \frac{1}{(D+3D')(D-2D')} y \cos x$$

$$= \frac{1}{(D+3D')} \int (C-2x) \cos x dx$$

$$= \frac{1}{D+3D'} [(C-2x) \sin x - (-2)(-\cos x)]$$

$$= \frac{1}{D+3D'} [y \sin x - 2 \cos x] \quad \text{factor} \rightarrow D+3D'$$

$y \rightarrow C+3x$

$$= \int [(C+3x) \sin x - 2 \cos x] dx$$

$$= (C+3x)(-\cos x) - 3(-\sin x) - 2 \sin x$$

$$= -y \cos x + 3 \sin x - 2 \sin x$$

$$= -y \cos x + \sin x$$

Type

$$\text{RHS} = e^{ax+by} \phi(x, y)$$

Replace D by $D+a$

D' by $D'+b$

II. So find the PI of $(D^2 - 2DD' + D'^2) z = x^2 y^2 e^{x+y}$

Soln.:

$$\text{PI} = \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2 e^{x+y}$$

$$= \frac{1}{(D-D')^2} e^{x+y} x^2 y^2$$

$$= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2$$

$$D \rightarrow D+a = D+1$$

$$D' \rightarrow D'+b = D'+1$$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 \left[1 - \frac{2D'}{D} + \frac{D'^2}{D^2} \right]} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2} \right) \right]^{-1} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2} \right] x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[x^2 y^2 + \frac{2D'}{D} x^2 y^2 + \frac{3D'^2}{D^2} x^2 y^2 \right]$$

$$= e^{x+y} \left[\frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4} (2x^2) \right]$$

$$= e^{x+y} \left[\right.$$

$$\frac{1}{D^2} x^2 y^2 \xrightarrow{1^{st}} \frac{x^3}{3} y^2$$

$$2^{nd} \rightarrow \frac{x^4}{12} y^2$$

$$\frac{1}{D^3} 4x^2 y \rightarrow 4 \frac{x^3}{3} y$$

$$\frac{4x^4}{12} y \rightarrow \frac{4x^5}{60} y$$

$$\frac{1}{D^4} 6x^2 \rightarrow \frac{6x^3}{3} \rightarrow \frac{6x^4}{12}$$

$$\rightarrow \frac{6x^5}{60} \rightarrow \frac{6x^7}{420}$$

$$PI = e^{x+y} \left[\frac{x^4 y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70} \right]$$