SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 35

DEPARTMENT OF MATHEMATICS

UNIT - II ORTHOGONAL TRANSFORMATION OF REAL SYMMETRIC MATRIX

Defn: of quadratic form:-

A homogeneous poly. of degree 2 in any no. of variables is called as quadratic form.

general form is $q = \sum_{j=1}^{n} \sum_{i=1}^{n} a_{ij} x_i x_j = x^T A x$

where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ with $a_{ij} = a_{ji}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ n \end{bmatrix}$

Canonical form:-

If a quadratic form $q = x^T A x$ can be reduced by a non-singular linear transformation $x = Ny$ to $q = y^T D y$ where $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ then $q = y^T D y = \lambda y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 + \cdots + \lambda_n y_n^2$ is known as canonical form.

Matrix of quadratic form:-

The symmetric matrix A of quadratic form obtained by placing the coeff. of x_i^2 in a_{ii} & placing ½ (coeff. of $x_i x_j$) in remaining a_{ij} & a_{ji} position.

Write the matrix of quadratic form.

1) $x_1^2+2x_2^2+2x_1x_2$ is $\begin{array}{c c} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 1 & \frac{1}{2}(2) \\ \frac{1}{2}(2) & 2 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

2) $x_1^2+x_3^2$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3] $x_1^2+4x_2^2-3x_1x_2$ is $\begin{array}{c c} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 1 & \frac{1}{2}(-3) \\ \frac{1}{2}(-3) & 4 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & -3/2 \\ -3/2 & 4 \end{bmatrix}$

Write the following matrix in quadratic form

1) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is $0x_1^2+x_1x_2+x_1+x_2+0x_2^2 = 2x_1x_2$

2) $\begin{bmatrix} a & -b \\ -b & a \end{bmatrix}$ is $ax_1^2-bx_1x_2-bx_2x_1+ax_2^2 = ax_1^2-2bx_1x_2+a$

3) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ is $x_1^2+x_2^2+0x_3^2+0x_1x_2+x_1x_3-x_2x_3+x_3x_1-x_3$

$= x_1^2+x_2^2+2x_1x_3-2x_2x_3$