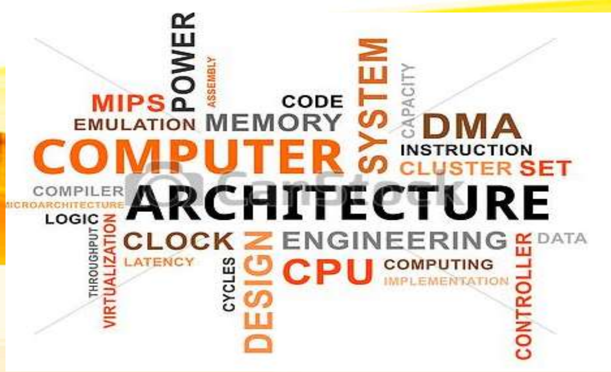
UNIT II
ARITHMETIC OPERATIONS

Addition and subtraction of signed numbers – Design of fast adders – Multiplication of positive numbers - Signed operand multiplication- fast multiplication – Integer division – **Floating point numbers and operations**



Recap the previous Class

Floating point numbers

- Representation for non - integral numbers
  - Including very small and very large numbers
- Like scientific notation
  - -2.34×10^{56} - Normalized
  - $+0.002 \times 10^{-4}$ – Not Normalized
  - $+987.02 \times 10^{9}$ – Not Normalized
- In binary
  - $\pm 1.xxxxxxx_2 \times 2^{yyyy}$

Floating point Standard

- Defined by IEEE Std754 - 1985
- Two representations
  - Single precision (32 - bit)
  - Double precision (64 - bit)

	Sign	Exponent	Fraction	Bias
Single precision	1[31]	8[30-23]	23[22-00]	127
Double precision	1[63]	11[62-52]	52[51-00]	1023

IEEE Floating point Representation

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

Exponent : excess representation: actual exponent + Bias

Ensures exponent is unsigned

	single: 8 bits double: 11 bits	single: 23 bits double: 52 bits
S	Exponent	Fraction

S: sign bit

0 –non - negative

1 - negative

Single: Bias = 127
Double: Bias = 1023

Floating Point Example

Represent 0.75 in Single and Double precession

Step 1: Convert decimal to binary Number 0.75 ------ 0.11

Step 2: Scientific Notation 0.11--------0.11 x 2^{0}

Step 3: Normalize the Scientific Notation 0.11 x 2^{0} -------- 1.1 x 2^{-1}

0	01111110	10000000000000000000000

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

Exponent = Actual Exponent +Bias
= -1 +127 = 126

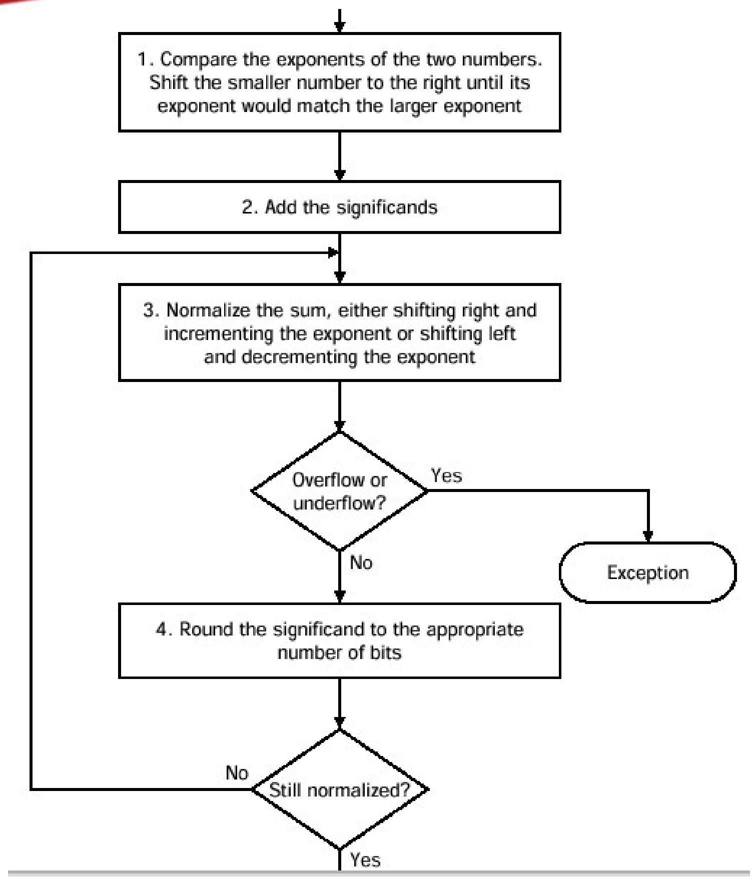
$$X = 1 \times (1.1) \times 2^{126-127}$$

Example for Floating Point Representation

- Represent –0.75
  - $-0.75 = (-1)^1 \times 1.1_2 \times 2^{-1}$
    $= -1 \times 1.\ ½ \times ½$
    $= -1.5 * .5 = -0.75$
  - $S = 1$
  - Fraction = $1000\ldots00_2$
  - Exponent = –1 + Bias
    - Single: $-1 + 127 = 126 = 01111110_2$
    - Double: $-1 + 1023 = 1022 = 01111111110_2$

- Single: 1011111101000…00
- Double: 1011111111101000…00

Floating Point Operation - Addition



OVERFLOW – Positive Exponent too larger to fit in the exponent Field

UNDERFLOW – negative Exponent too larger to fit in the exponent Field

Example -Floating Point Operation - Addition

Consider a 4-digit decimal example $9.999 \times 10^{1} + 1.610 \times 10^{-1}$

1. Align decimal points

- Shift number with smaller exponent $9.999 \times 10^{1} + 0.016 \times 10^{1}$

2. Add significands

- $9.999 \times 10^{1} + 0.016 \times 10^{1} = 10.015 \times 10^{1}$

3. Normalize result & check for over/underflow

- 1.0015×10^{2}

4. Round and renormalize if necessary

- 1.002×10^{2}

Example -Floating Point Operation - Addition

Now consider a 4-digit binary example (0.5 + –0.4375)

$1.000_2 \times 2^{-1} + -1.110_2 \times 2^{-2}$

1. Align binary points

- Shift number with smaller exponent $1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-1}$

2. Add significands

- $1.000_2 \times 2^{-1} + -0.111_2 \times 2^{-}1 = 0.001_2 \times 2^{-1}$

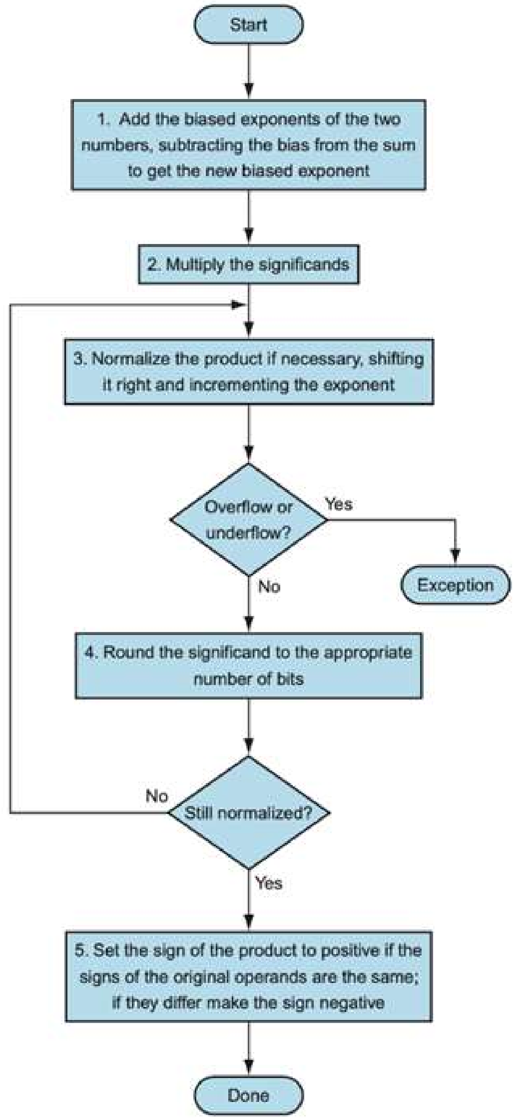
3. Normalize result & check for over/underflow

- $1.000_2 \times 2^{-4}$, with no over/underflow

4. Round and renormalize if necessary

$1.000_2 \times 2^{-4}$ (no change) = 0.0625

Floating Point Operation - Multiplication



Floating Point Multiplication

Consider a 4-digit decimal $1.110 \times 10^{10} \times 9.200 \times 10^{-5}$

Add exponents - For biased exponents, subtract bias from sum

New exponent = 10 + –5 = 5 Unbiased = 5+127 = 132

Multiply significands

$1.110 \times 9.200 = 10.212$ = 10.212×10^{5}

Normalize result & check for over/underflow

1.0212×10^{6}

Round and renormalize if necessary

1.021×10^{6}

Determine sign of result from signs of operands

$+1.021 \times 10^{6}$

Floating Point Multiplication

Consider a 4-digit binary $1.000_2 \times 2^{-1} \times -1.110_2 \times 2^{-2}$ (0.5×-0.4375)

Add exponents

Unbiased: $-1 + -2 = -3$; Biased: $= -3 + 127 = 124$

Multiply significands

$1.000_2 \times 1.110_2 = 1.1102$ => $1.110_2 \times 2^{-3}$

Normalize result & check for over/underflow

$1.110_2 \times 2^{-3}$ (no change) with no over/underflow

Round and renormalize if necessary

$1.110_2 \times 2^{-3}$ (no change)

Determine sign: +ve × –ve => –ve

$-1.1102 \times 2^{-3} = -0.21875$

Guard bit and Truncation

- Guard bits
  - Extra bits during intermediate steps to yield maximum accuracy in the final result
- They need to be removed when generating the final result
  - Chopping
    - simply remove guard bits
  - Von Neumann rounding
    - if all guard bits 0, chop, else 1
  - Rounding
    - Add 1 to LSB if guard MSB = 1

TEXT BOOK

Carl Hamacher, Zvonko Vranesic and Safwat Zaky, “Computer Organization”, McGraw-Hill, 6th Edition 2012.

REFERENCES

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4. M. Morris R. Mano “Computer System Architecture” 3rd Edition 2007

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THANK YOU