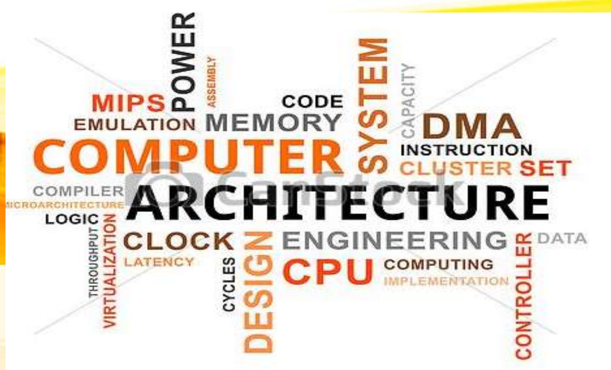
UNIT II
ARITHMETIC OPERATIONS

Addition and subtraction of signed numbers – Design of fast adders – Multiplication of positive numbers - Signed operand multiplication- fast multiplication – **Integer division** – Floating point numbers and operations



Recap the previous Class

Introduction

- Division is more complex than multiplication.
- *Example*: Typical values in Pentium-3 processor.

– Not easy to construct high-speed dividers.

- The ratios have not changed much in later processors.

Instruction	Latency	Cycles / Issue
Load / Store	3	1
Integer Multiply	4	1
Integer Divide	36	36
Floating-point Add	3	1
Floating-point Multiply	5	2
Floating-point Divide	38	38

The Process of Integer Division

- In integer division, a ***divisor*** M and a ***dividend*** D are given.
- The objective is to find a third number Q, called the ***quotient***,

  such that $\mathbf{D = Q \times M + R}$ where R is the ***remainder*** such that $0 \le R < M$.
- The relationship D = Q x M suggests that there is a close correspondence between division and multiplication.
  - Dividend, quotient and divisor correspond to product, multiplicand and multiplier, respectively.

- One of the simplest division methods is the **sequential digit-by-digit algorithm** similar to that used in pencil-and-paper methods.

```
                          0 1 1 0          Quotient Q = Q0Q1Q2Q3
Divisor M        1 1 0  | 1 0 0 1 0 1      Dividend D = R0
                          1 1 0            Q0.M       (Does not go; Q0 = 0)
                        -------------
                          1 0 0 1 0 1      R1
                        -   1 1 0          Q1.2^-1.M  (Does go; Q1 = 1)
                        -------------
                            0 1 1 0 1      R2
                        -     1 1 0        Q2.2^-2.M  (Does go; Q2 = 1)
                        -------------
                              0 0 0 1      R3
                                1 1 0      Q3.2^-3.M  (Does not go; Q3 = 0)
                        -------------
                                0 0 1      R4 = Remainder R
```

$D = 37 = (1\,0\,0\,1\,0\,1)_2$

$M = 6 = (1\,1\,0)_2$

Quotient $Q = 6$

Remainder $R = 1$

- Machine implementation:
  - For hardware implementation, it is more convenient to shift the partial remainder to the left relative to a fixed divisor; thus

$$R_{i+1} = 2R_i - Q_i.M \quad (\text{instead of } R_{i+1} = R_i - Q_i.2^{-i}.M)$$

  - The final partial remainder is the required remainder shifted to the left, so that $R = 2^{-3}.R_4$

Machine implementation

Divisor M			Quotient Q
1 1 0	1 0 0 1 0 1	Dividend = $2R_0$	
Do not subtract	1 1 0	$Q_0.M$	0
	---------------		
	1 0 0 1 0 1	R_1	
	1 0 0 1 0 1 0	$2R_1$	
	1 1 0	$Q_1.M$	0 1
	---------------		
	0 1 1 0 1 0	R_2	
	0 1 1 0 1 0 0	$2R_2$	
	1 1 0	$Q_2.M$	0 1 1
	---------------		
	0 0 0 1 0 0	R_3	
	0 0 0 1 0 0 0	$2R_3$	
	1 1 0	$Q_3.M$	0 1 1 0
	---------------		
	0 0 1 0 0 0	$R_4 = 2^3.R$	

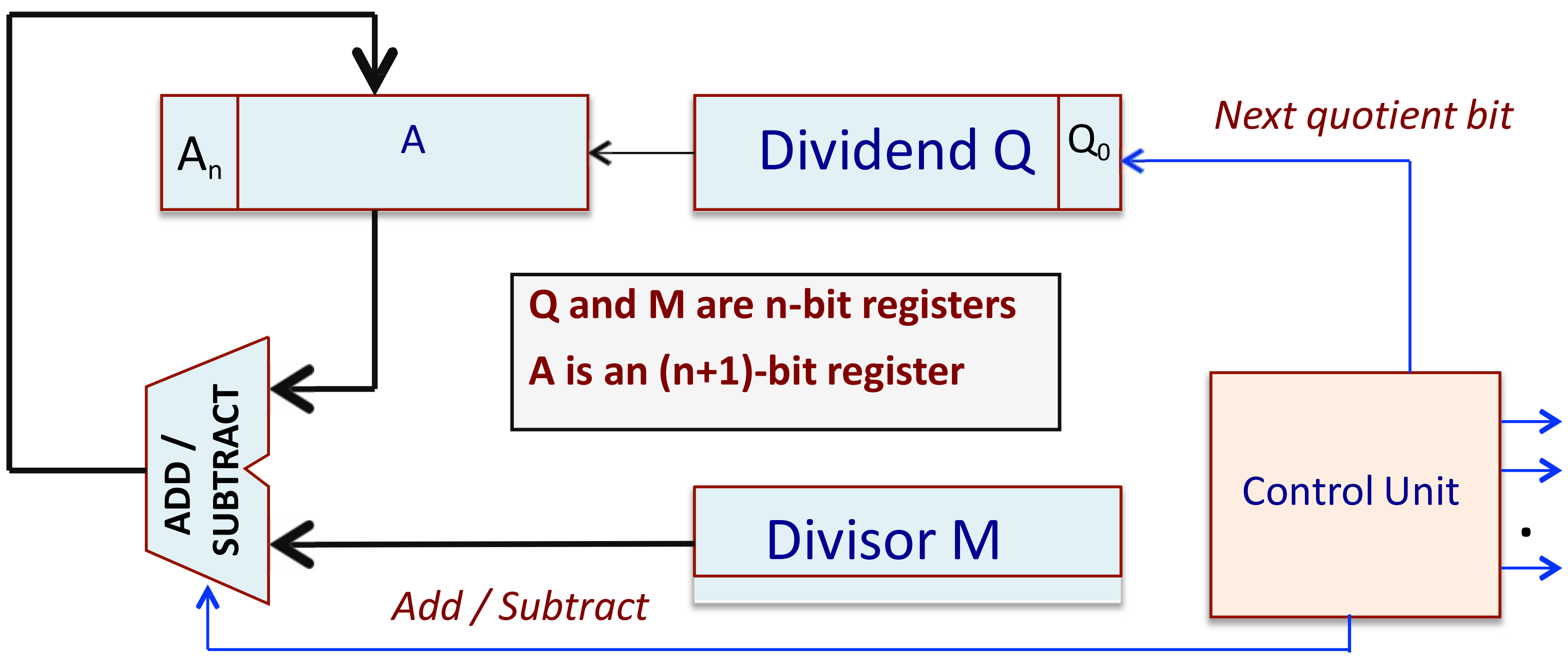
$D = 37 = (1\,0\,0\,1\,0\,1)_2$

$M = 6 = (1\,1\,0)_2$

Quotient $Q = 6$

Remainder $R = 1$

Restoring Division: The Data Path



Basic Steps

Repeat the following steps n times:

a) Shift the dividend one bit at a time starting into register A.

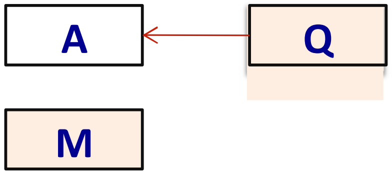
b)Subtract the divisor M from this register A (***trial subtraction***).

c) If the result is negative (***i.e. not going***):

- Add the divisor M back into the register A (***i.e. restoring back***).
- Record 0 as the next quotient bit.

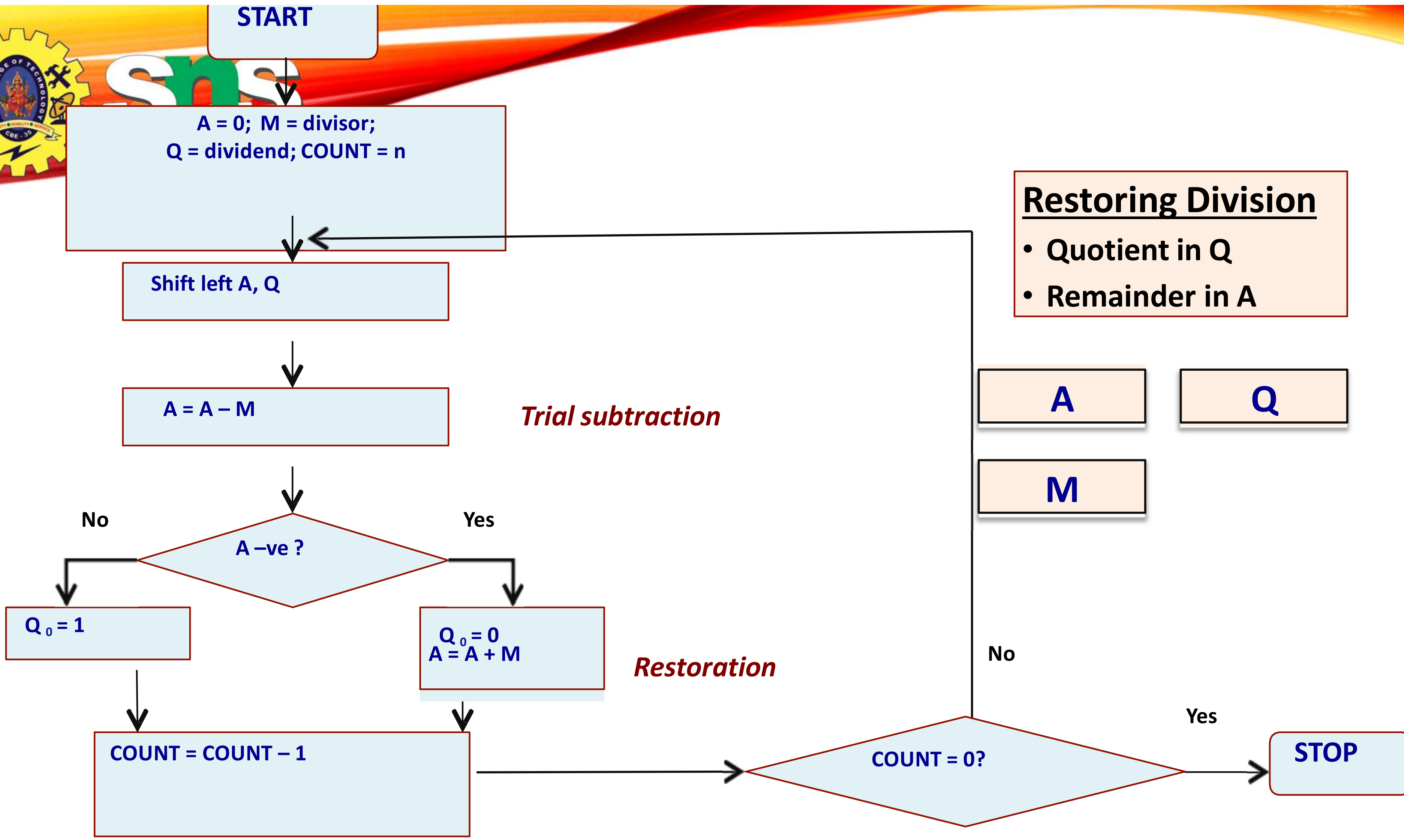
d)If the result is positive:

- Do not restore the intermediate result.
- Record 1 as the next quotient bit.



Restoring Division

- Quotient in Q
- Remainder in A



A Simple Example: 8/3 for 4-bit representation (n=4)

Initially:	0 0 0 0 0	1 0 0 0
	0 0 0 1 1	
Shift:	0 0 0 0 1	0 0 0 –
Subtract:		
Set Q_0:	1 1 1 1 0	
Restore:	0 0 0 1 1	
	0 0 0 0 1	0 0 0 0
Shift:	0 0 0 1 0	0 0 0 –
Subtract:		
Set Q_0:	1 1 1 1 1	
Restore:	0 0 0 1 1	
	0 0 0 1 0	0 0 0 0

Shift:	0 0 1 0 0	0 0 0 –
Subtract:		
Set Q_0:	0 0 0 0 1	
	0 0 0 0 0	0 0 0 1
Shift:	0 0 0 1 0	0 0 1 –
Subtract:		
Set Q_0:	1 1 1 1 1	
Restore:	0 0 0 1 1	
	0 0 0 1 0	0 0 1 0

Remainder	Quotient
00010 = 2	0010 = 2

Perform the restoring division algorithm for the number 11_{10} / 3_{10}

11/3

n = 4

Dividend = 1010 Q

Divisor = 0011   M = 00011

2's M = 11101

	A (n+1 bit)	Q (n bit)	
Initial Value	00000	1010	Cycle 1.
Shift left A&Q	00001	010□	
A = A-M	11101 11110		
Set Q_0 = 0 & A = A+M	00011 00001	0100	
Shift left A&Q	00010	100□	Cycle 2
A = A-M	11101 11111		
Set Q_0 = 0 & A = A+M	00011 00010	1000	
Shift left A&Q	00101	000□	Cycle 3
A = A-M	11101 00010		
Set Q_0=1	00010	0001	
Shift left A&Q	00100	001□	Cycle 4.
A = A-M	11101 00001		
Set Q_0=1	00001	0011	
	Remainder	Quotient	

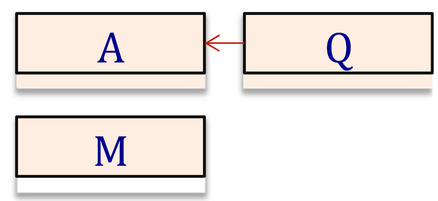
Example : $7_{10} / 3_{10}$

A	Q	M = 0011	
0 0 0 0	0 1 1 1	Initial Value	
0 0 0 0	1 1 1 0	Shift	First cycle
1 1 0 1		Subtract	
0 0 0 0	1 1 1 0	Restore	
0 0 0 1	1 1 0 0	Shift	Secon cycl
1 1 1 0		Subtract	
0 0 0 1	1 1 0 0	Restore	
0 0 1 1	1 0 0 0	Shift	Third cycle
0 0 0 0		Subtract	
0 0 0 0	1 0 0 1	Set $Q_0 = 1$	
0 0 0 1	0 0 1 0	Shift	Fourt cycl
1 1 1 0	Quotient	Subtract	
Remainder 0 0 0 1	0 0 1 0	Restore	

Non-Restoring Division

- The performance of restoring division algorithm can be improved by exploiting the following observation.

- In restoring division, what we do actually is:
  - –If A is positive, we shift it left and subtract M. That is, we compute 2A – M.
  - –If A is negative, we restore is by doing A+M, shift it left, and then subtract M.
    - That is, we compute 2(A + M) – M = 2A + M.
  - We can accordingly modify the basic division algorithm by eliminating the restoring step ⍰ ***NON-RESTORING DIVISION***.



Shift left means multiplying by 2.

Basic steps in non-restoring division:

a)Start by initializing register A to 0, and repeat steps (b)-(d) *n* times.

b)If the value in register A is positive,

- Shift A and Q left by one bit position.
- Subtract M from A.

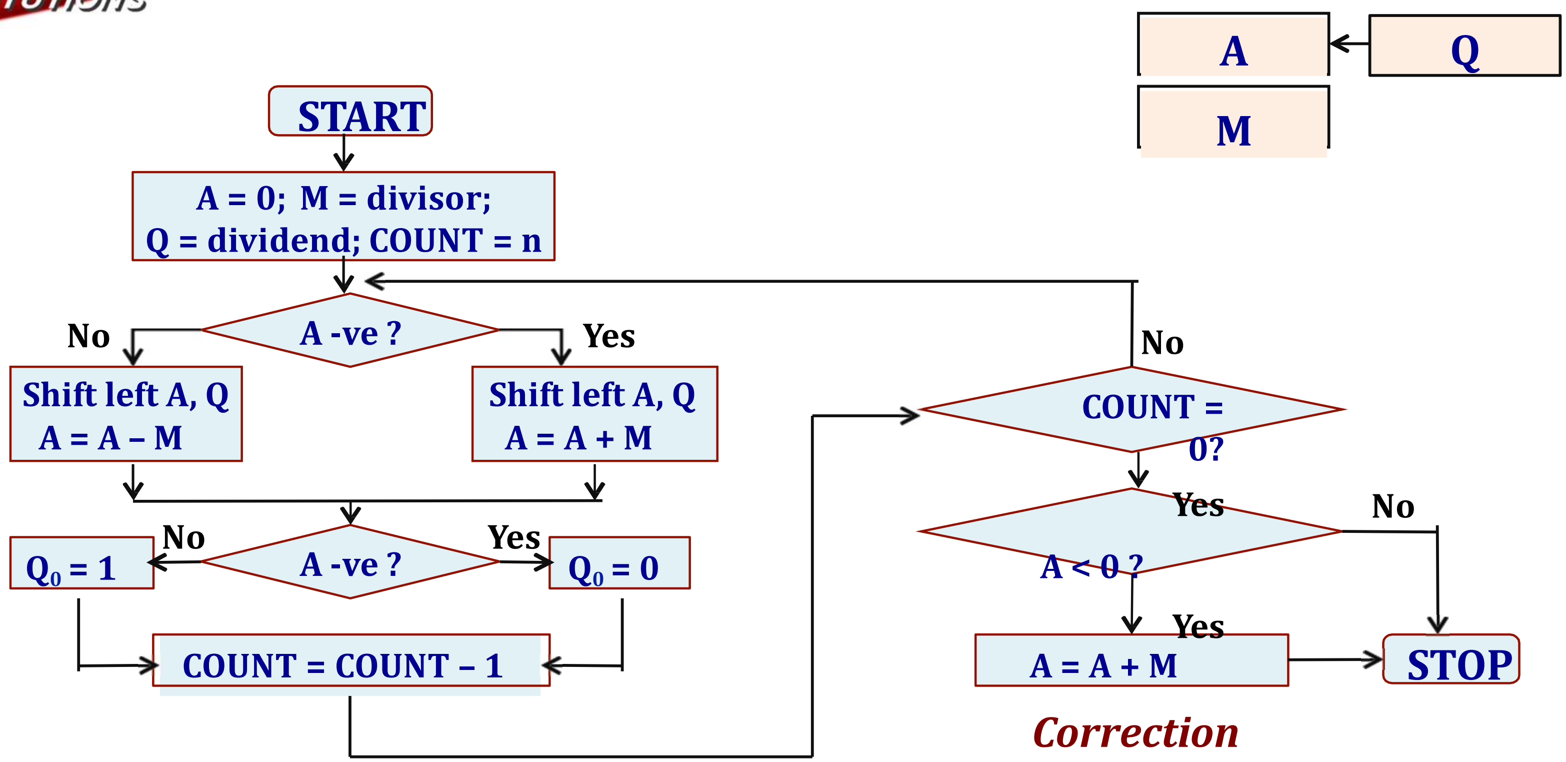
c) If the value in register A is negative,

- Shift A and Q left by one bit position.
- Add M to A.

c)If A is positive, set $Q_0 = 1$; else, set $Q_0 = 0$.

d)If A is negative, add M to A as a final corrective step.

Non-Restoring Division



A Simple Example: 8/3 for n=4

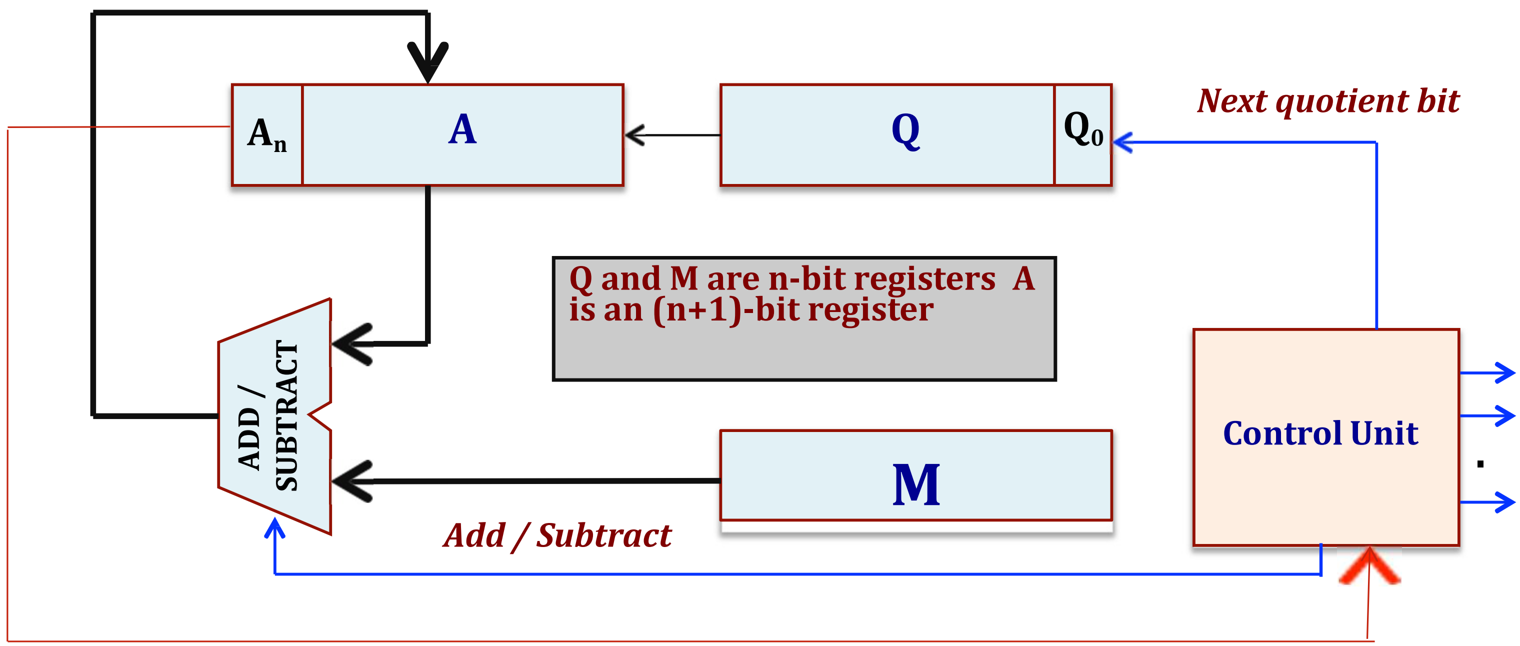
Initially:	0 0 0 0 0	1 0 0 0
Shift:	0 0 0 0 1	0 0 0 –
Subtract:	- 0 0 1 1	
Set Q_0:	1 1 1 1 0	0 0 0 0
Shift:	1 1 1 0 0	0 0 0 –
Add:	0 0 1 1	
Set Q_0:	1 1 1 1 1	0 0 0 0
Shift:	1 1 1 1 0	0 0 0 –
Add:	0 0 1 1	
Set Q_0:	0 0 0 0 1	0 0 0 1

Shift:	0 0 0 1 0	0 0 1 –
Subtract:	- 0 0 1 1	
Set Q_0:	1 1 1 1 1	0 0 1 0
Correction Add:		
	1 1 1 1 1	
	0 0 0 1 1	
	0 0 0 1 0	

Quotient
0010 = 2

Remainder
00010 = 2

Data Path for Non-Restoring Division



TEXT BOOK

Carl Hamacher, Zvonko Vranesic and Safwat Zaky, “Computer Organization”, McGraw-Hill, 6th Edition 2012.

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THANK YOU