

Fourier Series: in ' 2π ' period

$$1) f(x) = x^2 \text{ in } (0, 2\pi)$$

$$2) f(x) = x \text{ in } (0, 2\pi)$$

$$3) f(x) = \frac{(\pi-x)^2}{4} \text{ in } (0, 2\pi)$$

$$4) f(x) = x^2 \text{ in } (-\pi, \pi) \text{ \& deduce that}$$

$$(i) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$(ii) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

$$(iii) \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

$$(iv) \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$$

$$5) f(x) = x \text{ in } (-\pi, \pi)$$

$$6) f(x) = x(\pi^2 - x^2) \text{ in } (-\pi, \pi)$$

$$7) f(x) = \begin{cases} 1 + \frac{2}{\pi}x, & -\pi < x < 0 \\ 1 - \frac{2}{\pi}x, & 0 < x < \pi \end{cases}$$

$$8) f(x) = \begin{cases} \frac{\pi}{2} + x, & -\pi < x < 0 \\ \frac{\pi}{2} - x, & 0 < x < \pi \end{cases}$$

$$9) f(x) = |x| \text{ in } (-\pi, \pi)$$

$$10) f(x) = 1 + x + x^2 \text{ in } (-\pi, \pi)$$

Fourier Series in $2l$ period

1) $f(x) = (l-x)^2$ in $(0, 2l)$

2) $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1-x, & 1 < x < 2 \end{cases}$

3) $f(x) = x^2$ in $(-l, l)$

4) $f(x) = \begin{cases} l+x, & -l \leq x \leq 0 \\ l-x, & 0 \leq x \leq l. \end{cases}$

Half-Range: (cosine / sine)

$(0, \pi)$

1) $f(x) = x(\pi-x), 0 < x < \pi$

2) $f(x) = x, (0, \pi)$

3) $f(x) = \begin{cases} \frac{\pi}{4}x, & 0 < x < \pi/2 \\ \frac{\pi}{4}(\pi-x), & \pi/2 < x < \pi \end{cases}$

4) $f(x) = x^2, (0, \pi)$

$(0, l)$

1) $f(x) = x^2, (0, l)$

2) $f(x) = \begin{cases} x & \text{in } (0, l/2) \\ l-x & \text{in } (l/2, l) \end{cases}$

3) $f(x) = l-x$ in $(0, l)$

Harmonic Analysis:

$$1) \begin{array}{l} x: 0 \quad \pi/2 \quad 2\pi/3 \quad \pi \quad 4\pi/3 \quad 5\pi/3 \quad 2\pi \\ y: 1.0 \quad 1.4 \quad 1.9 \quad 1.7 \quad 1.5 \quad 1.2 \quad 1.0 \end{array}$$

$$2) \begin{array}{l} x: 0 \quad \pi/6 \quad \pi/3 \quad \pi/2 \quad 2\pi/3 \quad 5\pi/6 \quad \pi \\ y: 1.98 \quad 1.30 \quad 1.05 \quad 1.30 \quad -0.88 \quad 0.25 \quad 1.98 \end{array}$$

$$3) \begin{array}{l} x: 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ y: 9 \quad 18 \quad 24 \quad 28 \quad 26 \quad 20 \end{array}$$

Two Marks:

1) Dirichlet's condn.

2) Root Mean Square

3) Harmonic Analysis defn.

4) Find the Fourier coefficients (either a_0 , a_n or b_n)

5) Parseval's Identity.