If

$RHS = e^{ax+by}\,\phi(x, y)$

Replace D by $D+a$

D' by $D'+b$

II. Find the PI of $(D^2 - 2DD' + D'^2)\,z = x^2 y^2 e^{x+y}$

Soln:

$$PI = \frac{1}{D^2 - 2DD' + D'^2}\, x^2 y^2 e^{x+y}$$

$$= \frac{1}{(D-D')^2}\, e^{x+y}\, x^2 y^2$$

$$= e^{x+y} \frac{1}{(D+1-(D'+1))^2} x^2 y^2 \qquad D \to D+a = D+1$$
$$D' \to D'+b = D'+1$$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2 - 2DD' + D'^2} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2\left[1 - \frac{2D'}{D} + \frac{D'^2}{D^2}\right]} x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 - \left(\frac{2D'}{D} - \frac{D'^2}{D^2}\right)\right]^{-1} x^2 y 2$$

$$= e^{x+y} \frac{1}{D^2} \left[1 + \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2}\right] x^2 y^2$$

$$= e^{x+y} \frac{1}{D^2} \left[x^2 y^2 + \frac{2D'}{D} x^2 y 2 + \frac{3D'^2}{D^2} x^2 y^2\right]$$

$$= e^{x+y} \left[\frac{1}{D^2} x^2 y^2 + \frac{2}{D^3} 2x^2 y + \frac{3}{D^4}(2x^2)\right]$$

$$= e^{x+y} [$$

$$\frac{1}{D^2} x^2 y^2 \to \overset{1st}{\frac{x^3}{3} y^2}$$

$$\to \overset{2nd}{\frac{x^4}{12} y^2}$$

$$\frac{1}{D^3} 4x^2 y \to 4\frac{x^3}{3} y$$

$$\overset{2nd}{\frac{4x^4}{12} y} \to \overset{3rd}{\frac{4x^5}{60} y}$$

$$\frac{1}{D4} 6x^2 \to \frac{6x^3}{3} \to \frac{6x^4}{12}$$

$$\to \frac{6x^5}{60} \to \frac{6x^7}{420}$$

$$PI = e^{x+y}\left[\frac{x^4 y^2}{12} + \frac{x^5 y}{15} + \frac{x^7}{70}\right]$$

Solve $r+s-6t = y\cos x$

Soln:

Given $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$

$$(D^2 + DD' - 6D'^2)z = y\cos x$$

AE $\quad m^2 + m - 6 = 0$

$$(m+3)(m-2) = 0$$

$$m = -3, 2$$

-6 : 3, -2

$$CF = \phi_1(y-3x) + \phi_2(y+2x)$$

$$PI = \frac{1}{(D^2 + DD' - 6D'^2)} y\cos x$$

factor → $D-2D'$

where $y = C-2x$

$D \to C$

$D' \to x$

$$= \frac{1}{(D+3D')(D-2D')} y \cos x$$

$$= \frac{1}{(D+3D')} \int (C-2x) \cos x \, dx$$

$$= \frac{1}{D+3D'} [(C-2x) \sin x - (-2)(-\cos x)]$$

$$= \frac{1}{D+3D'} [y \sin x - 2 \cos x]$$ factor → $D+3D'$, $y \to C+3x$

$$= \int [(C+3x) \sin x - 2 \cos x] \, dx$$

$$= (C+3x)(-\cos x) - 3(-\sin x) - 2 \sin x$$

$$= -y \cos x + 3 \sin x - 2 \sin x$$

$$= -y \cos x + \sin x$$