$$y_n[8(n+2)-8(n+1)]-1[4(n+2)y_{n+1}-2(n+1)y_{n+2}]$$
$$+n[4y_{n+1}-2y_{n+2}]=0$$

$$y_n[8n+16-8n-8]-[(4n+8)y_{n+1}-(2n+2)y_{n+2}]$$
$$+4ny_{n+1}-2ny_{n+2}=0$$

$$8y_n-(4n+8)y_{n+1}+(2n+2)y_{n+2}+4ny_{n+1}-2ny_{n+2}=0$$

$$2y_{n+2}-8y_{n+1}+8y_n=0$$

$$y_{n+2}-4y_{n+1}+4y_n=0$$

Solution of difference equations using Z-transforms:

Formulae:

(1) $Z[y_n]=Y(z)$

(2) $Z[y_{n+1}]=zY(z)-zy(0)$

(3) $Z[y_{n+2}]=z^2Y(z)-z^2y(0)-zy(1)$

(4) $Z[y_{n+3}]=z^3Y(z)-z^3y(0)-z^2y(1)-zy(2)$

(5) $Z[y_{n-1}]=z^{-1}Y(z)$.

Problems:

① Solve $y_{n+1}-2y_n=0$ given $y_0=3$.

Soln: Taking Z-transforms on both sides of the difference eqn, we get

$$Z[y_{n+1}]-2Z[y_n]=Z[0]$$

$$[zY(z)-zy(0)]-2Y(z)=0$$

$$zY(z)-z(3)-2Y(z)=0 \qquad [\because y_0=y(0)=3]$$

$$(z-2)Y(z)-3z=0$$

$$Y(z)=\frac{3z}{z-2}$$

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$$Z[y_n] = \frac{3z}{z-2}$$

$$y_n = Z^{-1}\left[\frac{3z}{z-2}\right] = 3Z^{-1}\left[\frac{z}{z-2}\right] \quad \left[\because Z[a^n] = \frac{z}{z-a}\right]$$

$$\boxed{y_n = 3(2^n)}$$

(2) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given $y_0 = y_1 = 0$.

Soln: Given: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$.

$$Z[y_{n+2}] + 6Z[y_{n+1}] + 9Z[y_n] = Z[2^n]$$

$$z^2 Y(z) - z^2 y(0) - zy(1) + 6[zY(z) - zY(0)] + 9Y(z) = \frac{z}{z-2}$$

$$z^2 Y(z) + 6z Y(z) + 9(Y(z)) = \frac{z}{z-2} \quad [\because y(0) = y(1) = 0]$$

$$(z^2+6z+9)Y(z) = \frac{z}{z-2}$$

$$(z+3)^2 Y(z) = \frac{z}{z-2}$$

$$Y(z) = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{Y(z)}{z} = \frac{1}{(z-2)(z+3)^2} = \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2} \rightarrow (1)$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

Put $z = 2 \Rightarrow 25A = 1 \Rightarrow A = 1/25$

Put $z = -3 \Rightarrow -5C = 1 \Rightarrow C = -1/5$

Equating z^2 coeff on both sides, $0 = A + B \Rightarrow B = -A$

$B = -1/25$

$$(1) \Rightarrow \frac{Y(z)}{z} = \frac{1}{25(z-2)} - \frac{1}{25(z+3)} - \frac{1}{5(z+3)^2}$$

$$Y(z) = \frac{z}{25(z-2)} - \frac{z}{25(z+3)} - \frac{z}{5(z+3)^2}$$

$$y(n) = \frac{1}{25}Z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25}Z^{-1}\left[\frac{z}{z+3}\right] - \frac{1}{5}Z^{-1}\left[\frac{z}{(z+3)^2}\right]$$

$$y(n) = \frac{1}{25} z^{-1}\left[\frac{z}{z-2}\right] - \frac{1}{25} z^{-1}\left[\frac{z}{z-(-3)}\right] + \frac{1}{15} z^{-1}\left[\frac{-3z}{[z-(-3)]^2}\right]$$

$$= \frac{1}{25}(2^n) - \frac{1}{25}(-3)^n + \frac{1}{15}(-3)^n \cdot n \qquad \left[\because z[na^n] = \frac{az}{(z-a)^2}\right]$$

(3) Solve the difference eqn $y(k+2) - 4y(k+1) + 4y(k) = 0$ where $y(0) = 1,\ y(1) = 0$.

Soln: $y(k+2) - 4y(k+1) + 4y(k) = 0$

$$z[y(k+2)] - 4z[y(k+1)] + 4z[y(k)] = 0$$

$$[z^2 Y(z) - z^2 y(0) - z y(1)] - 4[z Y(z) - z y(0)] + 4 Y(z) = 0$$

$$[z^2 Y(z) - z^2 - 0] - 4[z Y(z) - z] + 4(Y(z)) = 0.$$

$$Y(z)(z^2 - 4z + 4) - z^2 + 4z = 0$$

$$Y(z) = \frac{z^2 - 4z}{z^2 - 4z + 4} = \frac{z(z-4)}{z^2 - 4z + 4}$$

$$\frac{Y(z)}{z} = \frac{z-4}{z^2 - 4z + 4} = \frac{z-4}{(z-2)^2} = \frac{A}{z-2} + \frac{B}{(z-2)^2}$$

$$z - 4 = A(z-2) + B$$

Put $z = 2 \Rightarrow B = -2$

Put $z = 0 \Rightarrow -4 = -2A + B \Rightarrow A = 1$

$$\frac{Y(z)}{z} = \frac{1}{z-2} - \frac{2}{(z-2)^2}$$

$$Y(z) = \frac{z}{z-2} - 2 \cdot \frac{z}{(z-2)^2}$$

$$z[y(n)] = \frac{z}{z-2} - 2 \cdot \frac{z}{(z-2)^2}$$

$$y(n) = z^{-1}\left[\frac{z}{z-2}\right] - z^{-1}\left[\frac{2z}{(z-2)^2}\right]$$

$$= 2^n - 2^n \cdot n$$

$$\boxed{y(n) = 2^n(1-n)}$$