

## SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)



#### **DEPARTMENT OF MATHEMATICS**

$$\begin{aligned} y_{n} \left[ 8(n+a) - 8(n+1) \right] - 1 \left[ 4(n+a) y_{n+1} - 2(n+1) y_{n+2} \right] \\ &+ n \left[ 4 y_{n+1} - 2 y_{n+2} \right] = 0 \\ y_{n} \left[ 8n + 16 - 8n - 8 \right] - \left[ (4n+8) y_{n+1} - (2n+2) y_{n+2} \right] \\ &+ 4 n y_{n+1} - 2 n y_{n+2} = 0 \\ 8 y_{n} - (4n+8) y_{n+1} + (2n+2) y_{n+2} + 4 n y_{n+1} - 2 n y_{n+2} = 0 \\ 2 y_{n+2} - 8 y_{n+1} + 8 y_{n} = 0 \\ y_{n+2} - 4 y_{n+1} + 4 y_{n} = 0 \end{aligned}$$

Solution of difference equations using Z-transforms:

#### Formulae:

(1) 
$$Z[y_n] = Y(z)$$

(a) 
$$Z[y_{n+1}] = ZY(z) - ZY(0)$$

(3) 
$$Z[y_{n+2}] = z^2 y(z) - z^2 y(0) - zy(1)$$

(4) 
$$Z[y_{n+3}] = z^3 Y(z) - z^3 y(0) - z^2 y(1) - Zy(2)$$

(5) 
$$Z[y_{n-1}] = Z^{-1}y(z)$$
.

### Problems:

1) Solve 
$$y_{n+1} - 2y_n = 0$$
 given  $y_0 = 3$ .

Jaking Z-transforms on both sides of the difference

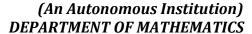
$$z y(z) - z(3) - 2 y(z) = 0$$
 (:  $y_0 = y(0) = 3$ ]

$$(z-a) Y(z) - 3z = 0$$

$$\frac{Y(z) = 3z}{z - 2}$$



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$$Z \left[ y_{n} \right] = \frac{3z}{z - a}$$

$$y_{n} = z^{-1} \left[ \frac{3z}{z - a} \right] = 3z^{-1} \left[ \frac{z}{z - a} \right]$$

$$\left[ y_{n} = 3(a^{n}) \right]$$

$$\left[ y_{n} = 3(a^{n}) \right]$$

② Solve 
$$y_{n+2} + by_{n+1} + 9y_n = a^n$$
 given  $y_0 = y_1 = 0$ .  
Soln: Given:  $y_{n+2} + by_{n+1} + 9y_n = a^n$ .  
 $z [y_{n+2}] + bz [y_{n+1}] + 9z [y_n] = z[a^n]$   
 $z [y_{n+2}] + bz [y_{n+1}] + 9z [y_n] = z[a^n]$   
 $z^2 y(z) - z^2 y(0) - zy(1) + b[zy(z) - zy(0)] + 9y(z) = z^2 z - a$   
 $z^2 y(z) + bz y(z) + 9(y(z)) = z z z = z = 0$ 

$$(z^{2}+6z+9) Y(z) = \frac{z}{z-2}$$
  
 $(z+3)^{2} Y(z) = \frac{z}{z-2}$ 

$$Y(z) = \frac{z}{(z-\lambda)(z+3)^2}$$

$$\frac{Y(z)}{Z} = \frac{1}{(z-a)(z+3)^2} = \frac{A}{Z-a} + \frac{B}{Z+3} + \frac{C}{(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + c(z-2)$$

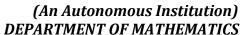
Equating 
$$z^2$$
 coeff on both sides,  $0 = A + B \Rightarrow B = -A$ 

$$Y(z) = \frac{z}{25(z-a)} - \frac{z}{25(z+3)} - \frac{z}{5(z+3)^2}$$

$$y(n) = \frac{1}{25} z^{-1} \left[ \frac{z}{z-a} \right] - \frac{1}{25} z^{-1} \left[ \frac{z}{z+3} \right] - \frac{1}{5} z^{-1} \left[ \frac{z}{(z+3)^2} \right]$$



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$$y(n) = \frac{1}{25} z^{-1} \left[ \frac{z}{z-a} \right] - \frac{1}{25} z^{-1} \left[ \frac{z}{z-(-3)} \right] + \frac{1}{15} z^{-1} \left[ \frac{-3z}{[z-(-3)]^2} \right]$$

$$= \frac{1}{25} (a^n) - \frac{1}{25} (-3)^n + \frac{1}{15} (-3)^n \cdot n \qquad z \left[ na^n \right] = \frac{az}{(z-a)^2}$$

3) Solve the difference even y(k+2) - 4y(k+1) + 4y(k) = 0where y(0) = 1, y(1) = 0.

$$\frac{50\ln x}{y(k+2)-4y(k+1)+4y(k)=0}$$

$$z[y(k+2)]-4z[y(k+1)]+4z[y(k)]=0$$

$$[z^{2}y(z)-z^{2}y(0)-zy(1)]-4[zy(z)-zy(0)]+4y(z)=0$$

$$[z^{2}y(z)-z^{2}-0]-4[zy(z)-z]+4(y(z))=0.$$

$$y(z)(z^{2}-4z+4)-z^{2}+4z=0$$

$$Y(z) = \frac{z^{2} + z}{z^{2} + z + 4} = \frac{z(z - 4)}{z^{2} + z + 4}$$

$$z^{2} = z - 4 = z - 4 + B$$

$$\frac{Y(z)}{z} = \frac{z-4}{z^2-4z+4} = \frac{z-4}{(z-2)^2} = \frac{A}{z-4} + \frac{B}{(z-2)^2}$$

$$z-4 = A(z-2)+B$$

$$Z-4 = A(Z-2) +$$

Put 
$$Z=2 \Rightarrow B=-2$$

Put 
$$z=0 \Rightarrow -4=-2A+B \Rightarrow A=1$$

$$\frac{Y(z)}{z} = \frac{1}{z-2} - \frac{2}{(z-2)^2}$$

$$Y(z) = \frac{Z}{Z-2} - 2 \cdot \frac{Z}{(z-2)^2}$$

$$Z[y(n)] = \frac{Z}{Z-2} - \partial \cdot \frac{Z}{(Z-2)^2}$$

$$y(n) = z^{-1} \left[ \frac{z}{z-2} \right] - 2 z^{-1} \left[ \frac{2z}{(z-2)^2} \right]$$

$$= 2^n - 2^n \cdot n$$