Formation of difference equations:

Difference equations: A differential equation is a relation between the differences of an unknown function at one or more general values of the argument.

Thus $\Delta y_{(n+1)} + y_{(n)} = 2$

and $\Delta y_{(n+1)} + \Delta^2 y_{(n-1)} = 1$ are difference equations.

Order of a difference equation: The order of a difference equation is the difference between the largest and the smallest arguments occuring in the difference equation divided by the unit of increment.

① Form the difference equation corresponding to the family of curves $y = ax + bx^2$.

Soln:

$$y_x = ax + bx^2 \longrightarrow (1)$$

$$y_{x+1} = a(x+1) + b(x+1)^2 \longrightarrow (2)$$

$$y_{x+2} = a(x+2) + b(x+2)^2 \longrightarrow (3)$$

Eliminating a and b from (1), (2) & (3), we get

$$\begin{vmatrix} y_x & x & x^2 \\ y_{x+1} & x+1 & (x+1)^2 \\ y_{x+2} & x+2 & (x+2)^2 \end{vmatrix} = 0$$

$$y_x\left[(x+1)(x+2)^2 - (x+2)(x+1)^2\right] - y_{x+1}\left[x(x+2)^2 - x^2(x+2)\right]$$

$$+ y_{x+2}\left[x(x+1)^2 - x^2(x+1)\right] = 0$$

$$y_x(x+1)(x+2) - y_{x+1}\, 2x(x+2) + y_{x+2}\, x(x+1) = 0$$

$$(x^2+3x+2)y_x - 2(x^2+2x)\, y_{x+1} + (x^2+x)\, y_{x+2} = 0$$

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(2) ~~Form~~ From $y_n = a\cdot 2^n + b(-2)^n$, derive a difference eqn by eliminating the constants.

Soln:

$$y_n = a\cdot 2^n + b(-2)^n \longrightarrow (1)$$

$$y_{n+1} = a\,2^{n+1} + b(-2)^{n+1} = 2a\,2^n - 2b(-2)^n \longrightarrow (2)$$

$$y_{n+2} = a\,2^{n+2} + b(-2)^{n+2}$$

$$= a\,2^n\cdot 4 + b(-2)^n\cdot 4 = 4a\,2^n + 4b(-2)^n \longrightarrow (3)$$

Eliminating $a(2^n)$ & $b(-2)^n$ from (1), (2) & (3),

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & -2 \\ y_{n+2} & 4 & 4 \end{vmatrix} = 0$$

$$y_n(8+8) - 1(4y_{n+1} + 2y_{n+2}) + 1(4y_{n+1} - 2y_{n+2}) = 0$$

$$16y_n - 4y_{n+1} - 2y_{n+2} + 4y_{n+1} - 2y_{n+2} = 0$$

$$16y_n - 4y_{n+2} = 0$$

$$y_{n+2} - 4y_n = 0.$$

(3) Derive the difference equation from $y_n = (A+Bn)2^n$

Soln:

$$y_n = (A+Bn)\,2^n = A\,2^n + B\,n\cdot 2^n \longrightarrow (1)$$

$$y_{n+1} = A\,2^{n+1} + B(n+1)\,2^{n+1}$$

$$= 2A\,2^n + 2B(n+1)\,2^n \longrightarrow (2)$$

$$y_{n+2} = A\,2^{n+2} + B(n+2)\,2^{n+2}$$

$$= 4A\,2^n + 4B(n+2)\,2^n \longrightarrow (3)$$

Eliminating $A\cdot 2^n$ and $B\cdot 2^n$ from (1), (2) & (3),

$$\begin{vmatrix} y_n & 1 & n \\ y_{n+1} & 2 & 2(n+1) \\ y_{n+2} & 4 & 4(n+2) \end{vmatrix} = 0$$

$$y_n\left[8(n+2)-8(n+1)\right]-1\left[4(n+2)y_{n+1}-2(n+1)y_{n+2}\right]$$
$$+n\left[4y_{n+1}-2y_{n+2}\right]=0$$

$$y_n\left[8n+16-8n-8\right]-\left[(4n+8)y_{n+1}-(2n+2)y_{n+2}\right]$$
$$+4ny_{n+1}-2ny_{n+2}=0$$

$$8y_n-(4n+8)y_{n+1}+(2n+2)y_{n+2}+4ny_{n+1}-2ny_{n+2}=0$$

$$2y_{n+2}-8y_{n+1}+8y_n=0$$

$$y_{n+2}-4y_{n+1}+4y_n=0$$