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## DEPARTMENT OF ELECTRONICS \& COMMUNICATION ENGINEERING

# 19ECB202 - LINEAR AND DIGITAL CIRCUITS 

II YEAR/ III SEMESTER

UNIT 3 - GATES AND MINIMIZATION TECHNIQUES
TOPIC 6 - KARNAUGH MAP MINIMIZATION

## WHY KARNAUGH MAP MINIMIZATION?

> K-map simplification technique is simpler and less error-prone compared to the method of solving the logical expressions using Boolean laws.
$>$ Its main purpose is to simplify Boolean algebraic expressions.


## KARNAUGH MAP

$>$ KARNAUGH MAP is also named as K map
$>$ K map was introduced by Dr. Maurice karnaugh in the year 1953


## KARNAUGH MAP - Rules

$>$ Karnaugh map is a pictorial method of grouping together expressions with common factors and then eliminating unwanted variables.
$>$ Karnaugh map uses the following rules for the simplification of expressions by grouping together adjacent cells containing ones .

1. Groups may not include any cell containing a zero


KARNAUGH MAP - Rules
2. Groups may be horizontal or vertical, but not diagonal.


## KARNAUGH MAP - Rules

3. Groups must contain $1,2,4,8$, or in general $2^{n}$ cells. If $n=1$, a group will contain two $1^{\prime} s$ since $2^{1}=2$. If $n=2$, a group will contain four $1^{\prime} s$ since $\mathbf{2}^{\mathbf{2}}=4$.


## KARNAUGH MAP - Rules

4. Each group should be as large as possible.


KARNAUGH MAP - Rules
5. Each cell containing a one must be in at least one group.

6. Groups may overlap.

7. Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell

8. There should be as few groups as possible, as long as this does not contradict any of the previous rules


## K MAP - Rules

$>$ No zeros allowed.
$>$ No diagonals.
$>$ Only power of 2 number of cells in each group.
$>$ Groups should be as large as possible.
$>$ Every one must be in at least one group.
$>$ Overlapping allowed.
$>$ Wrap around allowed.
$>$ Fewest number of groups possible.

Karnaugh Map is most suitable for Minimizing Boolean expressions of 2 Variable
3 Variable
4 Variable
5Variable
$>$ The number of cells in 2 variable K-map is four, since the number of variables is two.
$\Rightarrow$ There is only one possibility of grouping 4 adjacent min terms.
$>$ The possible combinations of grouping 2 adjacent min terms are $\left\{\left(m_{0}, m_{1}\right),\left(m_{2}, m_{3}\right),\left(m_{0}, m_{2}\right)\right.$ and $\left.\left(m_{1}, m_{3}\right)\right\}$.

or

| $Y Z$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
| $m_{0}$ | $m_{1}$ | $m_{3}$ | $m_{2}$ |  |

## KARNAUGH MAP - $\mathbf{3}$ Variable

$>$ The number of cells in 3 variable K-map is eight, since the number of variables is three.
$>$ There is only one possibility of grouping 8 adjacent min terms.
$>$ The possible combinations of grouping 4 adjacent min terms are $\left\{\left(m_{0}, m_{1}, m_{3}\right.\right.$, $\left.m_{2}\right),\left(m_{4}, m_{5}, m_{7}, m_{6}\right),\left(m_{0}, m_{1}, m_{4}, m_{5}\right),\left(m_{1}, m_{3}, m_{5}, m_{7}\right),\left(m_{3}, m_{2}, m_{7}, m_{6}\right)$ and $\left(m_{2}\right.$, $\left.\left.m_{0}, m_{6}, m_{4}\right)\right\}$.
$>$ The possible combinations of grouping 2 adjacent min terms are $\left\{\left(m_{0}, m_{1}\right),\left(m_{1}\right.\right.$, $\left.m_{3}\right),\left(m_{3}, m_{2}\right),\left(m_{2}, m_{0}\right),\left(m_{4}, m_{5}\right),\left(m_{5}, m_{7}\right),\left(m_{7}, m_{6}\right),\left(m_{6}, m_{4}\right),\left(m_{0}, m_{4}\right),\left(m_{1}, m_{5}\right)$, $\left(m_{3}, m_{7}\right)$ and $\left(m_{2}, m_{6}\right)$.
>If $\mathrm{x}=0$, then 3 variable K-map becomes 2 variable K-map.

## KARNAUGH MAP - $\mathbf{3}$ Variable



## KARNAUGH MAP - 4 Variable

$>$ The number of cells in 4 variable K-map is sixteen, since the number of variables is four.

| $W X$ | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | $\mathrm{m}_{0}$ | $m_{1}$ | $\mathrm{m}_{3}$ | $\mathrm{m}_{2}$ |
| 01 | $\mathrm{m}_{4}$ | $\mathrm{m}_{5}$ | $\mathrm{m}_{7}$ | $\mathrm{m}_{6}$ |
| 11 | $\mathrm{m}_{12}$ | $\mathrm{m}_{13}$ | $\mathrm{m}_{15}$ | $\mathrm{m}_{14}$ |
| 10 | $\mathrm{m}_{8}$ | $\mathrm{m}_{9}$ | $\mathrm{m}_{11}$ | $\mathrm{m}_{10}$ |

## KARNAUGH MAP - 4 Variable

$>$ There is only one possibility of grouping 16 adjacent min terms.
$>$ Let $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}$ and $\mathrm{R}_{4}$ represents the min terms of first row, second row, third row and fourth row respectively. Similarly, $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$ and $\mathrm{C}_{4}$ represents the min terms of first column, second column, third column and fourth column respectively. The possible combinations of grouping 8 adjacent min terms are $\left\{\left(R_{1}, R_{2}\right),\left(R_{2}, R_{3}\right),\left(R_{3}\right.\right.$, $\left.\left.R_{4}\right),\left(R_{4}, R_{1}\right),\left(C_{1}, C_{2}\right),\left(C_{2}, C_{3}\right),\left(C_{3}, C_{4}\right),\left(C_{4}, C_{1}\right)\right\}$.
>If w=0, then 4 variable K-map becomes 3 variable K-map

## KARNAUGH MAP - 5 Variable

$>$ The number of cells in 5 variable K-map is thirty-two, since the number of variables is 5 .

| $V=0$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $w X V^{Y}$ | 00 | 01 | 11 | 10 |
| 00 | $\mathrm{m}_{0}$ | $\mathrm{m}_{1}$ | $\mathrm{m}_{3}$ | $\mathrm{m}_{2}$ |
| 01 | $\mathrm{m}_{4}$ | $\mathrm{m}_{5}$ | $\mathrm{m}_{7}$ | $\mathrm{m}_{6}$ |
| 11 | $\mathrm{m}_{12}$ | $\mathrm{m}_{13}$ | $\mathrm{m}_{15}$ | $\mathrm{m}_{14}$ |
| 10 | $\mathrm{m}_{8}$ | $\mathrm{m}_{9}$ | $\mathrm{m}_{11}$ | $\mathrm{m}_{10}$ |


| $V=1$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $w X^{Y Z}$ | 00 | 01 | 11 | 10 |
| 00 | $\mathrm{m}_{16}$ | $\mathrm{m}_{17}$ | $\mathrm{m}_{19}$ | $\mathrm{m}_{18}$ |
| 01 | $\mathrm{m}_{20}$ | $\mathrm{m}_{21}$ | $\mathrm{m}_{23}$ | $\mathrm{m}_{22}$ |
| 11 | $\mathrm{m}_{28}$ | $\mathrm{m}_{29}$ | $\mathrm{m}_{31}$ | $\mathrm{m}_{30}$ |
| 10 | $\mathrm{m}_{24}$ | $\mathrm{m}_{25}$ | $\mathrm{m}_{27}$ | $\mathrm{m}_{26}$ |

## KARNAUGH MAP - 5 Variable

$>$ There is only one possibility of grouping 32 adjacent min terms.
$>$ There are two possibilities of grouping 16 adjacent min terms. i.e., grouping of min terms from $\mathrm{m}_{0}$ to $\mathrm{m}_{15}$ and $\mathrm{m}_{16}$ to $\mathrm{m}_{31}$.
$>$ If $\mathrm{v}=0$, then 5 variable K-map becomes 4 variable K-map.

THANK YOU

