SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB202 – LINEAR AND DIGITAL CIRCUITS

II YEAR/ III SEMESTER

UNIT 3 – GATES AND MINIMIZATION TECHNIQUES

TOPIC 2 - Basic Theorems and Properties of Boolean Algebra

Boolean Algebra

- Boolean algebra was invented by **George Boole** in 1854.
- Boolean algebra is a mathematical system for the manipulation of variables that can have one of two values.
  - In formal logic, these values are “true” and “false.”
  - In digital systems, these values are “on” and “off,” 1 and 0, or “high” and “low.”
- Boolean Algebra uses a set of Laws and Rules to define the operation of a digital logic circuit.
- It helps to reduce the number of logic gates needed to perform a particular logic operation
- Resulting in a list of functions or theorems known as the **Laws of Boolean Algebra**.

Boolean Operator

- A Boolean operator can be completely described using a truth table.
- The truth table for the Boolean operators AND and OR are shown at the right.
- The AND operator is also known as a Boolean product. The OR operator is the Boolean sum.

X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

Rule in Boolean Algebra

- Variable used can have only two values.

  Binary 1 for HIGH and Binary 0 for LOW

- Complement of a variable is represented by an overbar (-).

  Thus, complement of variable B is represented as $\overline{B}$

  if B = 0 then $\overline{B} = 1$ and B = 1 then $\overline{B} = 0$

- ORing of the variables is represented by a plus (+) sign between them.

  For example ORing of A, B, C is represented as A + B + C

- Logical ANDing of the two or more variable is represented by writing a dot between them such as A.B.C. Sometime the dot may be omitted like ABC.

Boolean Law

There are six types of Boolean Laws.

1. Commutative law

Any binary operation which satisfies the following expression is referred to as commutative operation

$$(i)\ A.B = B.A \qquad (ii)\ A + B = B + A$$

Commutative law states that changing the sequence of the variables does not have any effect on the output of a logic circuit.

2. Associative law

This law states that the order in which the logic operations are performed is irrelevant as their effect is the same.

$$(i)\ (A.B).C = A.(B.C) \qquad (ii)\ (A + B) + C = A + (B + C)$$

Boolean Law

3.Distributive law

Distributive law states the following condition $A.(B + C) = A.B + A.C$

4. AND law

These laws use the AND operation. So called as AND laws

(i) $A.0 = 0$ (ii) $A.1 = A$

(iii) $A.A = A$ (iv) $A.\overline{A} = 0$

5.OR law

These laws use the OR operation. So called as OR laws.

(i) $A + 0 = A$ (ii) $A + 1 = 1$

(iii) $A + A = A$ (iv) $A + \overline{A} = 1$

6.INVERSION law

This law uses the NOT operation. The inversion law states that double inversion of a variable results in the original variable itself. $\overline{\overline{A}} = A$

Properties of Boolean Algebra

- The basic **Laws of Boolean Algebra** that relate to the Commutative Law allowing a change in position for addition and multiplication,
- The Associative Law allowing the removal of brackets for addition and multiplication,
- The Distributive Law allowing the factoring of an expression, are the same as in ordinary algebra.
- Each of the Boolean Laws above are given with just a single or two variables, but the number of variables defined by a single law is not limited to this as there can be an infinite number of variables as inputs too the expression.
- These Boolean laws detailed above can be used to prove any given Boolean expression as well as for simplifying complicated digital circuits.

Boolean Function

- A **Boolean Function** is described by an algebraic expression called **Boolean expression**
- It consists of binary variables, the constants 0 and 1, and the logic operation symbols.
- Consider the following example.

$$\underbrace{F(A, B, C, D)}_{\text{Boolean Function}} = \underbrace{A + \overline{B}C + ADC}_{\text{Boolean Expression}}$$

left side of the equation represents the output Y

$$Y = A + \overline{B}C + ADC$$

Description of the Laws of Boolean Algebra

De Morgan´s Theorem – There are two “de Morgan´s” rules or theorems,

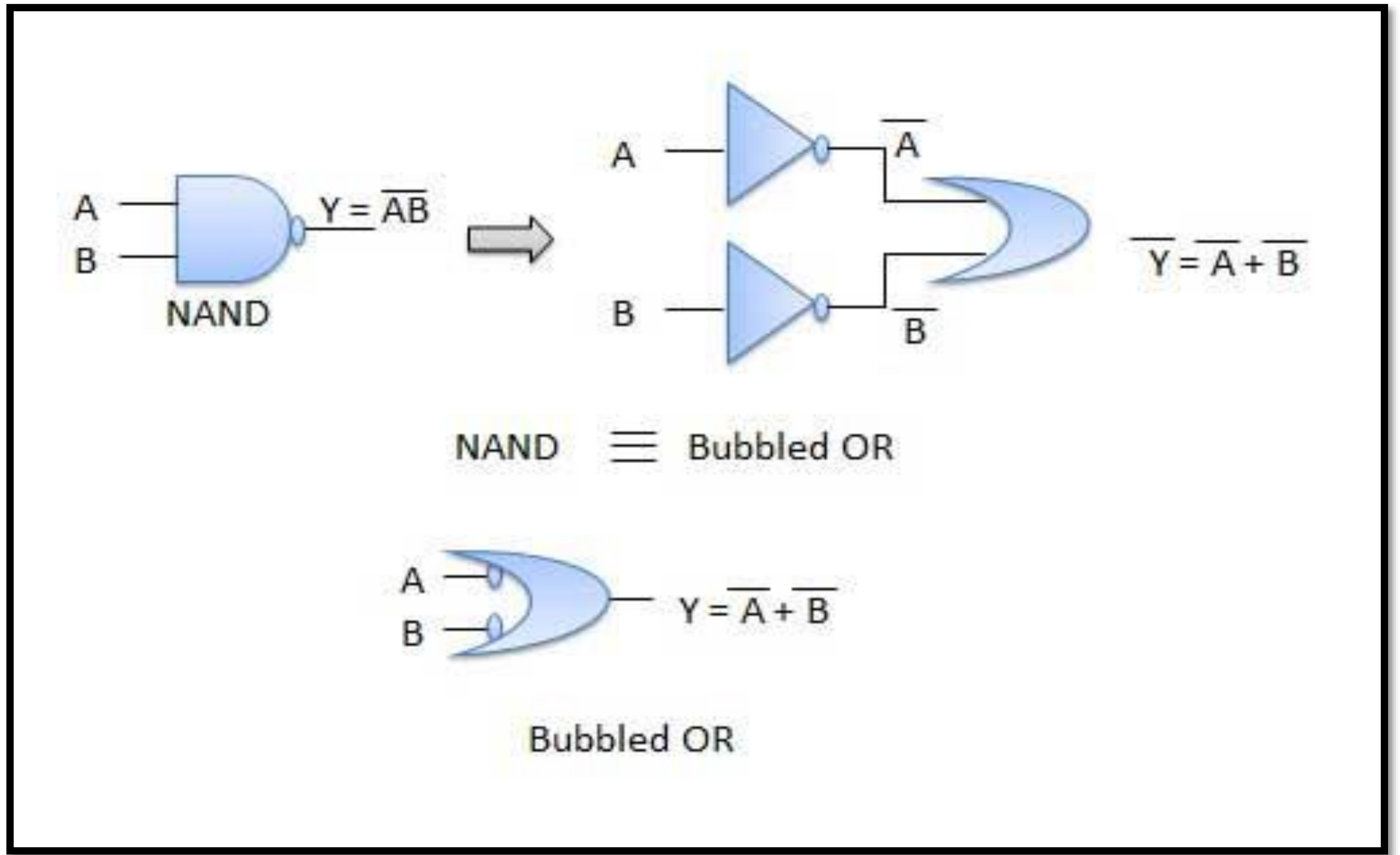
(1) Theorem 1

$$\overline{A.B} = \overline{A} + \overline{B}$$

NAND = Bubbled OR

- The left hand side (LHS) of this theorem represents a NAND gate with inputs A and B,
- The right hand side (RHS) of the theorem represents an OR gate with inverted inputs.

This OR gate is called as Bubbled OR.



A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

Truth table

Description of the Laws of Boolean Algebra

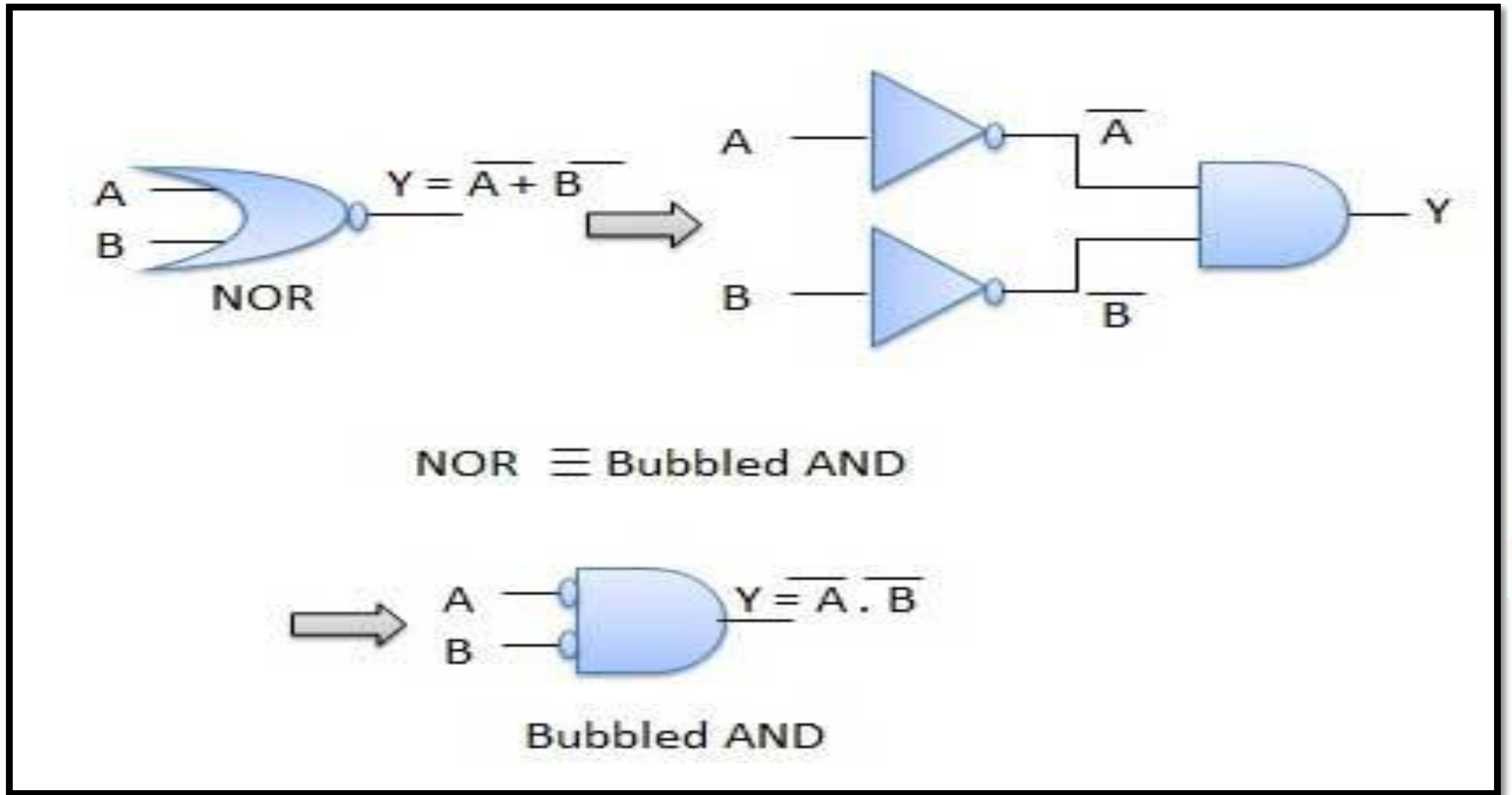
De Morgan´s Theorem

(2) Theorem 2 $\overline{A+B} = \overline{A}.\overline{B}$

NOR = Bubbled AND

- The LHS of this theorem represents a NOR gate with inputs A and B
- The RHS represents an AND gate with inverted inputs.

This AND gate is called as **Bubbled AND**



A	B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}.\overline{B}$
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Truth table

THANK YOU