Homogeneous Equations:

A linear partial derivatives involved are of same order, the equation is called <u>homogenous</u> equation with constant coefficients

(i) $(D^2 - DD' - 20D'^2)z = e^{x+2y}$

A.E. is $m^2 - m - 20 = 0$

$m = 5, m = -4$, $m_1 \neq m_2$, the roots are distinct.

C.F. is $z = f_1(y+5x) + f_2(y-4x)$

$$P.I = \frac{1}{D^2 - DD' - 20D'^2} \cdot e^{x+2y}$$

$$= \frac{1}{1 - 2 - 20 \cdot 2^2} e^{x+2y}$$

$$= \frac{1}{-81} e^{x+2y}$$

∴ Complete soln. is $z = C.F. + P.I$

$$= f_1(y+5x) + f_2(y-4x) - \frac{1}{81} e^{x+2y}.$$

② $(D^2+2DD'+D'^2)z = e^{x-y}$.

A.E $m^2+2m+1 = 0$

$\Rightarrow (m+1)(m+1) = 0$

$\Rightarrow m=-1, m=-1$

C.F. $z = f_1(y-x) + x f_2(y-x)$

$$P.I. = \frac{1}{D^2+2DD'+D'^2} e^{x-y}$$

$$= \frac{1}{1-2+1} e^{x-y}$$

$$= \frac{1}{0} e^{x-y}.$$

Since Dr. is '0' then (D.w.r to 'D') in the Dr. and (multiply by x in the Nr.)

$$= x \frac{1}{2D+2D'} e^{x-y}$$

$$= x\frac{1}{2-2} e^{x-y}.$$

Since Dr. is '0' then again (D.w.r. to 'D' in the Nr.) & (multiply by x in the Nr.)

$$= \frac{1}{2} x^2 e^{x-y}.$$

Complete soln. is $z = C.F + P.I$

$$= f_1(y-x) + x f_2(y-x) + \frac{x^2}{2} e^{x-y}.$$

① $(D^2 - 5DD' + 6D'^2)z = \sin(x+3y)$

A.E. $m^2 - 5m + 6 = 0$

$\Rightarrow m = 2, 3.$

C.F. is $z = f_1(y+3x) + f_2(y+2x)$

$$P.I = \frac{1}{D^2 - 5DD' + 6D'^2} \cdot \sin(x+3y)$$

$$= \frac{1}{-1-5(-3)+6(-9)} \sin(x+3y)$$

$D^2 = -(1)^2 = -1$

$D'^2 = -(3)^2 = -9$

$DD' = -(1\times3) = -3$

$$= \frac{1}{-40} \sin(x+3y)$$

C.S is $z = f_1(y+3x) + f_2(y+2x) - \frac{1}{40}\sin(x+3y)$

② $(D^2 + 2DD' + D'^2)z = \cos(x-y)$

A.E. is $m^2 + 2m + 1 = 0$

$\Rightarrow m = -1, m = -1$

C.F. is $z = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2+2DD'+D'^2}\cos(x-y)$$

$D^2 = -(1)^2 = -1$

$D'^2 = -(-1)^2 = -1$

$DD' = -(1\times -1) = 1$

$$= \frac{1}{-1+2-1=0}\cos(x-y)$$

D.w.r. to 'D' in 'Dr' & multi. by x in the 'Nr'.

$$= x\frac{1}{2D+2D'}x\cos(x-y)$$

$$= \frac{1}{2D+2D'}\times\frac{2D-2D'}{2D-2D'}x\cos(x-y)$$

$\left\{\begin{array}{l} D \to D.w.r.\text{ to } x \\ D' \to D.w.r.\text{ to } y \end{array}\right.$

$$= \frac{2x[D-D']\cos(x-y)}{4D^2-4D'^2}$$

$D^2 = -(1)^2 = -1$

$D'^2 = -(-1)^2 = -1$

$$= \frac{2x[D(\cos(x-y)) - D'\cos(x-y)]}{4(-1)-4(-1)}$$