



DEPARTMENT OF MATHEMATICS

UNIT-III PARTIAL DIFFERENTIAL EQUATIONS

HOMOGENEOUS EQUATIONS:

A linear partial derivatives involved are of same order, the equation is called homogenous equation with constant coefficients.

$$(i) (D^2 - DD' - 20D'^2)z = e^{x+2y}$$

A.E. is $m^2 - m - 20 = 0$

$m = 5, m = -4, m_1 \neq m_2$, The roots are distinct.

C.F. is $z = f_1(y+5x) + f_2(y-4x)$

P.I. = $\frac{1}{D^2 - DD' - 20D'^2} \cdot e^{x+2y}$

$$= \frac{1}{1 - 2 - 20 \cdot 2^2} e^{x+2y}$$

$$= \frac{1}{-81} e^{x+2y}$$

\therefore Complete soln. is $z = C.F. + P.I.$

$$= f_1(y+5x) + f_2(y-4x) - \frac{1}{81} e^{x+2y}$$



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② $(D^2 + 2DD' + D'^2)z = e^{x-y}$ P.E

A.E. $m^2 + 2m + 1 = 0$

$\Rightarrow (m+1)(m+1) = 0$

$\Rightarrow m = -1, m = -1$

C.F. $z = f_1(y-x) + x f_2(y-x)$

P.I. = $\frac{1}{D^2 + 2DD' + D'^2} e^{x-y}$

= $\frac{1}{1 - 2 + 1} e^{x-y}$

= $\frac{1}{0} e^{x-y}$

Since D.R. is '0' then (D.w.r to 'D') and (multiply by x in the Nr. in the Dr.)

$(P.I.) = \frac{x \cdot 1}{2D + 2D'} e^{x-y}$

= $\frac{x \cdot 1}{2-2} e^{x-y}$

Since D.R. is '0' then again (D.w.r to 'D') in the Nr. & (multiply by x in the Nr.)

= $\frac{1}{2} x^2 e^{x-y}$

complete soln. is $z = C.F. + P.I.$

= $f_1(y-x) + x f_2(y-x) + \frac{x^2}{2} e^{x-y}$



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$$\textcircled{1} (D^2 - 5DD' + 6D'^2)z = \sin(x+3y)$$

$$\text{A.E. } m^2 - 5m + 6 = 0$$

$$\Rightarrow m = 2, 3$$

$$\text{C.F. is } z = f_1(y+3x) + f_2(y+2x)$$

$$\text{P.I.} = \frac{1}{D^2 - 5DD' + 6D'^2} \cdot \sin(x+3y)$$

$$= \frac{1}{-1 - 5(-3) + 6(-9)} \sin(x+3y)$$

$$D^2 = -(1)^2 = -1$$

$$+D'^2 = -(3)^2 = -9$$

$$DD' = -(1 \times 3) = -3$$

$$= \frac{1}{-40} \sin(x+3y)$$

$$\text{C.S. is } z = f_1(y+3x) + f_2(y+2x) - \frac{1}{40} \sin(x+3y)$$

$$\textcircled{2} (D^2 + 2DD' + D'^2)z = \cos(x-y)$$

$$\text{A.E. is } m^2 + 2m + 1 = 0$$

$$\Rightarrow m = -1, m = -1$$

$$\text{C.F. is } z = f_1(y-x) + x f_2(y-x)$$



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$$P.I = \frac{1}{D^2 + 2DD' + D'^2} \cos(x-y)$$

$$= \frac{1}{-1 + 2 - 1} \cos(x-y)$$

$$= \frac{1}{0} \cos(x-y)$$

$D^2 = -(1)^2 = -1$
 $+D'^2 = -(-1)^2 = -1$
 $DD' = -(1 \times -1) = 1$

D.w.r. to 'D' in 'Dr' & multi. by x in the 'Nr'.

$$= x \frac{1}{2D + 2D'} x \cos(x-y)$$

$$= \frac{1}{2D + 2D'} \times \frac{2D - 2D'}{2D - 2D'} x \cos(x-y)$$

$$= \frac{2x[D - D'] \cos(x-y)}{4D^2 - 4D'^2}$$

$$= \frac{2x [D(\cos(x-y)) - D'(\cos(x-y))]}{4(-1) - 4(-1)}$$

const. func.

$\frac{1}{2} x \cos(x-y)$

$\frac{1}{-1+1} \cos(x-y)$

$\begin{cases} D \rightarrow \text{D.w.r. to } x \\ D' \rightarrow \text{D.w.r. to } y \end{cases}$

$D^2 = -(1)^2 = -1$

$D'^2 = -(-1)^2 = -1$

$4(-1) - 4(-1)$