N/mm².

2. A tensile test was conducted on a mild steel bar. The following data was obtained from the test.

(i) Diameter of the steel bar = 3cm.

(ii) Gauge length of the bar = 20cm.

(iii) Load at elastic limit = 250kN.

(iv) Extension at a load of 150kN = 0.21mm.

(v) Maximum load = 380kN.

(vi) Total extension = 60mm.

(vii) Diameter of the rod at failure = 2.25cm.

Determine (a) the Young's Modulus

(b) the stress at elastic limit.

(c) the percentage elongation.

(d) the percentage decrease in area.

Sol: Area of the rod, $A = \frac{\pi D^2}{4}$

$= \frac{\pi \times 3^2}{4}$

$A = 7.0685 \text{ cm}^2$

(or)

$7.0685 \times 10^{-4} \text{ m}^2$

$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{\text{Load at ext.}}{\text{Area.}}$

$\sigma = \frac{P}{A} = \frac{150 \times 10^3}{7.0685 \times 10^{-4}} \text{ N/m}^2.$

$= 21220.9 \times 10^4\ N/m^2$

$$\text{Strain} = \frac{\text{Increase in length (Extension)}}{\text{Original length (or Gauge length)}}$$

$$e = \frac{\delta l}{l} \text{ (or) } \frac{dl}{l} = \frac{0.21}{20\times10} = 0.00105$$

(a) Young's Modulus $= E = \dfrac{\text{Stress}}{\text{Strain}}$

$$\sigma = E \times e$$

$$E = \frac{\sigma}{e} = \frac{21220.9 \times 10^4}{0.00105}$$

$$= 20209523 \times 10^4\ N/m^2$$

$$= 202.09523 \times 10^9\ N/m^2$$

$$= 202.095\ GN/m^2$$

(b) The stress at elastic limit:

$$\text{Stress} = \frac{\text{Load at elastic limit}}{\text{Area}}$$

$$\sigma = \frac{P}{A} = \frac{250 \times 10^3}{7.0685 \times 10^{-4}}$$

$$= 35368 \times 10^4\ N/m^2$$

$$= 353.68 \times 10^6\ N/m^2$$

$$= 353.68\ MN/m^2$$

(c) The percentage elongation is :

$$= \frac{\text{Total increase in length}}{\text{Original length (or Gauge length)}} \times 100$$

$$= \frac{60\ mm}{20 \times 10\ mm} \times 100 = 30\%.$$

(d) The percentage decrease in area is :

$$= \frac{\text{Original Area} - \text{Area at failure}}{\text{Original Area}} \times 100$$

$$\text{Original Area} = \frac{\pi D^2}{4}$$

$$= \frac{\pi \times 3^2}{4}$$

$$\text{Area at failure} = \frac{\pi \times 2.25^2}{4}$$

$$\text{Original Area} = \frac{\pi \times 3^2}{4}$$

$$\text{Percentage decrease in area} = \frac{\frac{\pi \times 3^2}{4} - \frac{\pi \times 2.25^2}{4}}{\frac{\pi \times 3^2}{4}} \times 100$$

$$= \left[\frac{3^2 - 2.25^2}{3^2}\right] \times 100$$

$$= \left[\frac{9 - 5.063}{9}\right] \times 100.$$

$= 43.75\%$

3. Find the minimum diameter of a steel wire, which is used to raise a load of 4000 N if the stress in the rod is not to exceed 95 MN/m².

Given:

Load, $P = 4000\,N$

Stress, $\sigma = 95\ MN/m^2$

$\cancel{= 95 \times 10^3\ KN/m^2}$

$= 95 \times 10^6\ N/m^2 = 95\ N/mm^2$

Area, $A = \frac{\pi}{4} D^2$

To find Dia of steel wire. (D)

Stress $\sigma = \frac{P}{A}$

$95 = \frac{4000}{\frac{\pi}{4} \times D^2}$

$D^2 = \frac{4000 \times 4}{\pi \times 95}$

$D^2 = 53.61$

$D = 7.32\ mm$

4. Find the Young's Modulus of a brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50kN when the extension of the rod is equal to 0.3mm.

Given

Dia of rod = 25 mm.

Tensile load (P) = 50kN.

Length of rod (l) = 250 mm.

Change in length dl = 0.3 mm.

Area of the rod (A) = $\frac{\pi D^2}{4}$

$= \frac{\pi \times 25^2}{4}$

$= 490.87 \, mm^2$

Tensile Load (P) = 50 kN

= 5000 N.

Stress, $\sigma = \frac{P}{A}$

$= \frac{5000}{490.87}$

$= 101.86 \, N/mm^2$.

Strain of the rod (e) $= \frac{dL}{L}$

$= \frac{0.3}{250}$

$= 0.0012$

Young's Modulus $E = \frac{\sigma}{e}$

$= \frac{101.86}{0.0012}$

$= 8488.33 \ N/mm^2$

$= 84.8 \times 10^9 \ N/m^2$

5. The safe stress, for a hollow steel column which carries an axial load of 2.1×10^3 kN is $125 \ MN/m^2$. If the ext. dia of th

Permissible stress,

Permissible stress $= \dfrac{480}{4}$

$= \underline{120\ N/mm^2}$.

Stress $\sigma = \dfrac{P}{A}$

$120 = \dfrac{1.9 \times 10^6}{\frac{\pi}{4}(200^2 - d^2)}$

$200^2 - d^2 = \dfrac{1.9 \times 10^6 \times 4}{\pi \times 120}$.

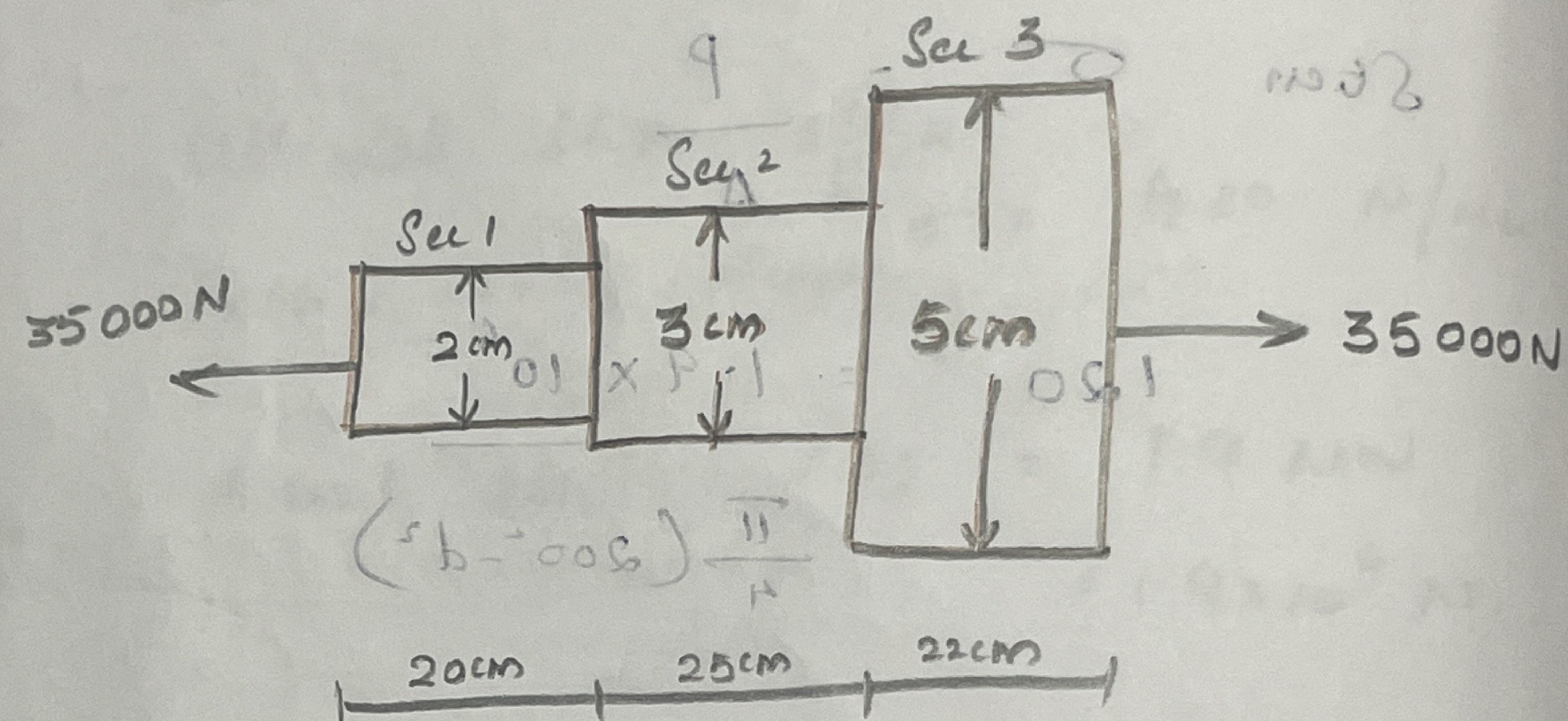
$d^2 = 200 - 2.016 \times 10^4$.

$= \underline{140.856\ mm}$

Analysis of Bars of Varying Sections

1) An axial pull of 35000N is acting on a bar consisting of three

lengths as shown in fig. If the Young's Modulus = 2.1×10^5 N/mm². Determine i) Stresses in each section ii) Total Extension of the bar.



Axial Pull (P) = 35,000 N.

Young's Modulus = 2.1×10^5 N/mm²

Section 1

length L_1 = 20 cm
= 200 mm

Diameter D_1 = 2cm
= 20mm

Area of the section $A_1 = \frac{\pi}{4} \times D_1^2$
$= \frac{\pi}{4} \times 20^2$
$= 100\pi$ mm²

Section 2

Length L_2 = 25 cm
= 250 mm.

Diameter D_2 = 3cm
= 30mm

Area of Section 2 $A_2 = \frac{\pi}{4} \times D_2^2$
$= \frac{\pi}{4} \times 30^2$
$= 225\pi$ mm²

Section 3

Length $L_3 = 220$ mm.

Diameter $D_3 = 5$ cm

$= 50$ mm.

Area of Section 3 $A_3 = \frac{\pi}{4} \times D_3^2$

$A = \frac{\pi}{4} \times 50^2$

$= 625\pi \ mm^2$

Youngs Modulus $E = 2.1 \times 10^5 \ N/mm^2$.

(i) Stresses in each section :

Stress in section 1 $\sigma_1 = \frac{P}{A_1}$

$= \frac{35{,}000}{100\pi} = 111.408 \ N/mm^2$

Section 2 $\sigma_2 = \frac{P}{A_2} = \frac{35{,}000}{225\pi}$

$= 49.51 \ N/mm^2$

Section 3 $\sigma_3 = \frac{P}{A_3} = \frac{35{,}000}{625\pi}$

$= 17.82 \ N/mm^2$.

ii) Total Extension of the bar :

$$\text{Total Extension} = \frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$= \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

Note :

Stress $\sigma = \frac{P}{A}$ — ①

$e = \frac{dl}{l}$ (or) $\frac{\delta l}{l}$ — ②

Acc. to Hooke's law

$\sigma = eE$

① ⇒ $eE = \frac{P}{A}$

$e = \frac{P}{AE}$

② ⇒ $\frac{\delta l}{l} = \frac{P}{AE}$

$$\boxed{\delta l \text{ or } dl = \frac{Pl}{AE}}$$

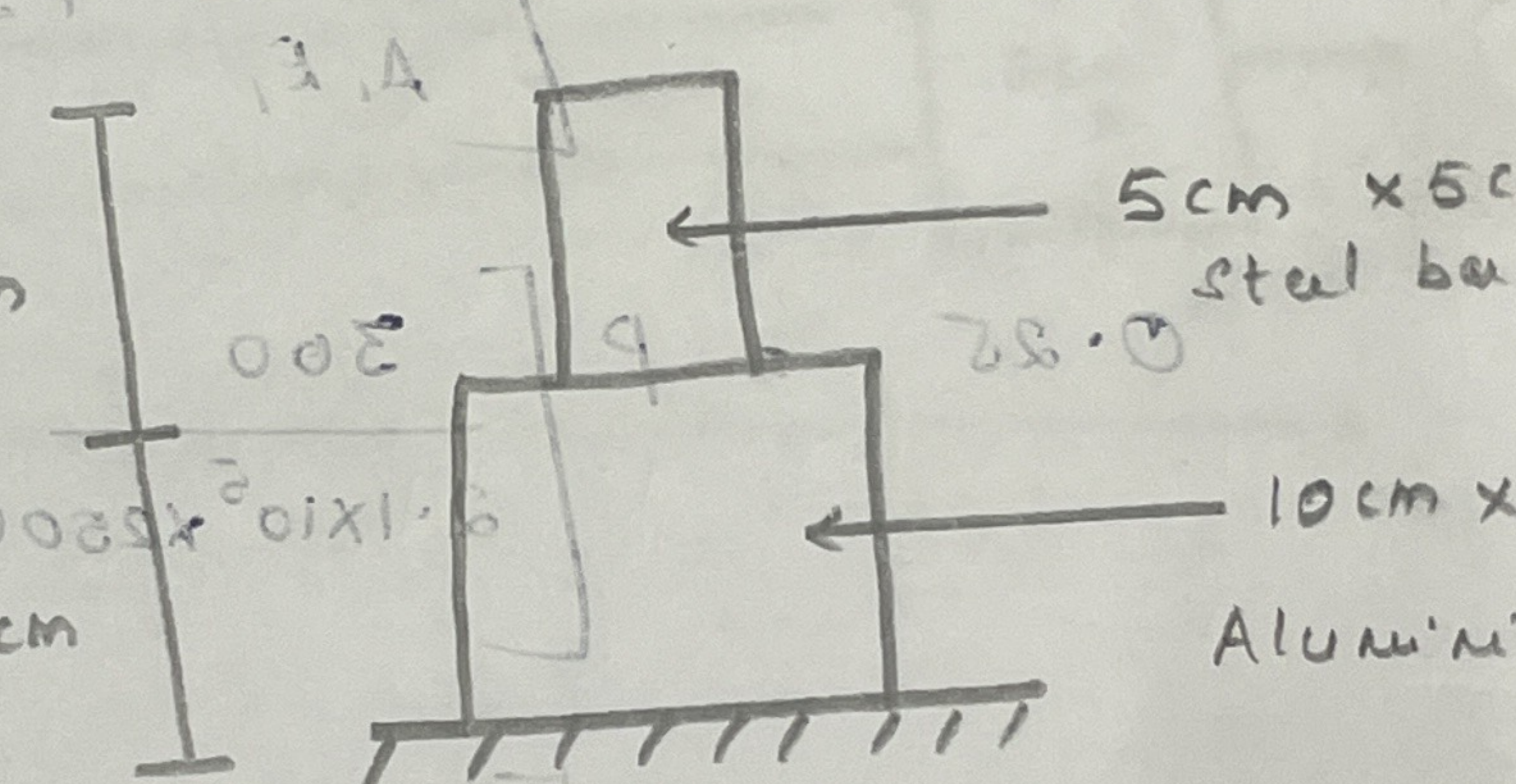
$$\text{Total Extension} = \frac{35000}{2.1\times10^5}\left[\frac{200}{100\pi} + \frac{250}{225\pi} + \frac{220}{625\pi}\right]$$

$$= \frac{35{,}000}{2.1\times10^5}(6.36 + 3.53 + 1.120)$$

$= \underline{0.183 \text{ mm}}$

$\frac{PL_3}{A_3E}$

2) A member formed by connecting a steel bar to an aluminium bar is shown in fig. Assuming the bars are prevented from building sideways, Calculate the magnitude of force P that will cause the total length of the member to decrease 0.25 mm. The values of elastic modulus for steel and aluminium are 2.1×10^5 N/mm² & 7×10^4 N/mm² respectively



Solution

Given: Length of steel bar L_1 = 30cm = 300mm.

Length of Aluminium bar L_2 = 38cm = 380mm

Youngs Modulus for Steel = 2.1×10^5 N/mm².

Aluminium = 7×10^4 N/mm².

$\frac{220}{625\pi}$

$+ 1.120$