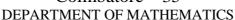


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UNIT- I FOURIER SERIES GENERAL FOURIER SERIES



Find the Fourier series for the Function
$$f(x) = \frac{(\pi - x)^2}{2} \text{ in } 0 \le x \le 2\pi.$$

$$\frac{S_0(n)}{a}$$
 = $\frac{(\pi - \infty)^2}{a}$

Fourier series for the function from in the interval [0,27] is

To Land ao:

$$a_0 = \frac{1}{\pi} \int_{-2\pi}^{2\pi} f(\alpha) d\alpha$$

$$= \frac{1}{\pi} \int_{-2\pi}^{2\pi} \frac{(\pi - \alpha)^2 d\alpha}{\alpha} d\alpha$$

$$= \frac{1}{2\pi} \int_{-2\pi}^{2\pi} \frac{(\pi - \alpha)^2 d\alpha}{\alpha} d\alpha$$

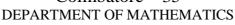
$$= \frac{1}{2\pi} \int_{-2\pi}^{2\pi} \frac{(\pi - \alpha)^2 d\alpha}{\alpha} d\alpha$$

$$= \frac{1}{2\pi} \int_{-2\pi}^{2\pi} \frac{(\pi - \alpha)^3}{\alpha} d\alpha$$



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To find an:

$$a_{n} = \frac{1}{\pi} \int_{0}^{4\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{0}^{2\pi} \frac{(\pi - x)^{2}}{2} (\cos nx \, dx)$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} (\pi - x)^{2} (\cos nx \, dx)$$

$$u = (\pi - x)^{2} \qquad \forall x = \cos nx$$

$$u' = 2(\pi - x)(-1) \qquad \forall x = +\frac{\sin x}{n}$$

$$= -2(\pi - x) \qquad \forall x = -\frac{\cos nx}{n^{2}}$$

$$u'' = -2(-1) \qquad \forall x = -\frac{\sin x}{n^{2}}$$

$$u''' = 0$$

$$a_{n} = \frac{1}{2\pi} \left[(\pi - x)^{2} \frac{\sin nx}{n} - \left[-2(\pi - x) \right] \left[-\frac{\cos nx}{n^{2}} \right] \right]$$

$$= \frac{1}{2\pi} \left[0 - 2(-\pi) \frac{\cos nx\pi}{n^{2}} - 0 - 0 \right]$$

$$= \frac{1}{2\pi} \left[\frac{2\pi}{n^{2}} + \frac{2\pi}{n^{2}} \right]$$

$$= \frac{1}{2\pi} \left[\frac{x\pi}{n^{2}} \right]$$



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To find bn:
$$b_{n} = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin nx dx$$

$$= \frac{1}{\pi} \int_{0}^{2\pi} (\pi - x)^{2} \sin nx dx$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} (\pi - x)^{2} \sin nx dx$$

$$u = (\pi - x)^{2} \qquad \forall x = \sin x$$

$$u' = 2(\pi - x)(-1) \qquad \forall x = -\frac{\cos x}{n}$$

$$= -2(\pi - x)$$

$$= -2(\pi - x)$$

$$= 2 \qquad \forall x = +\frac{\cos nx}{n^{2}}$$

$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} \frac{\cos nx}{n^{2}} - \left[-\frac{2(\pi - x)}{n^{2}} \right] - \frac{\sin x}{n^{2}} \right]$$

$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} \frac{\cos nx}{n^{2}} - 0 + \frac{2\cos nx}{n^{2}} \right]$$

$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} + \frac{2\cos nx}{n^{2}} - 0 + \frac{2\cos nx}{n^{2}} \right]$$

$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} + \frac{2\cos nx}{n^{2}} - \frac{2\cos nx}{n^{2}} \right]$$

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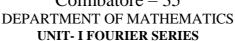
$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} + \frac{2\cos nx}{n^{2}} - \frac{2\cos nx}{n^{2}} \right]$$

$$= \frac{1}{2\pi} \left[-(\pi - x)^{2} + \frac{2\cos nx}{n^{2}} - \frac{2\cos nx}{n^{2}} \right]$$



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The fourier series is
$$f(\alpha) = \frac{\pi^2}{3} + \frac{2}{n^2} \cos n\alpha + 0$$

$$= \frac{\pi^2}{6} + \frac{2}{n^2} \cos n\alpha$$

$$= \frac{\pi^2}{6} + \frac{2}{n^2} \cos n\alpha$$



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UNIT- I FOURIER SERIES

GENERAL FOURIER SERIES

Interval! [0,28]



Find the Fourier Series for the function
$$f(x) = x^2$$
 in $(0,2-1)$.

Soluntion!
$$f(x) = x^2$$
 in $(0,20)$

The Fourier Series is given by
$$f(\alpha) = \frac{a_0}{2} + \underbrace{\sum_{n=1}^{\infty} a_n \cos(n\pi x)}_{l} + \underbrace{\sum_{n=1}^{\infty} b_n \sin(\frac{n\pi x}{l})}_{l}$$

Fund ao! To

$$a_0 = \frac{1}{2} \int_{0}^{2} f(x) dx$$

$$= \frac{1}{2} \int_{0}^{2} x^2 dx$$

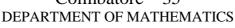
$$= \frac{1}{2} \int_{0}^{2} \frac{x^3}{3} dx$$

$$a_0 = \frac{8l^2}{3}$$



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$$an = \frac{1}{2} \int_{0}^{2\pi} \frac{1}{f(\alpha)} \frac{\cos(n\pi x)}{2} dx$$

$$= \frac{1}{2} \int_{0}^{2\pi} x^{2} \cos(\frac{n\pi x}{2}) dx.$$

$$V = \frac{1}{2} \int_{0}^{2\pi} x^{2} \cos(\frac{n\pi x}{2}) dx.$$

$$u = x^2$$

$$u' = 2x$$

$$u'' = 2$$

$$V = \cos\left(\frac{n\pi x}{\ell}\right)$$

$$Y_{l} = \frac{\sin\left(\frac{n\pi x}{\ell}\right)}{(n\pi/\ell)}$$

$$V_2 = - \cos\left(\frac{n\pi x}{x}\right)$$

$$\left(\frac{n\pi}{x}\right)^2$$

$$\frac{\sqrt{3} = -\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^{3}}$$

$$a_n = \frac{1}{2} \left[\frac{2^2}{2} \frac{\sin(\frac{n\pi x}{2})}{(n\pi/2)} - 2x \left[\frac{\cos(\frac{n\pi x}{2})}{(\frac{n\pi}{2})^2} \right] \right]$$

$$+2\left[-\frac{\sin\left(\frac{n\pi}{2}\right)}{(\frac{n\pi}{2})^3}\right]^{2\frac{2}{2}}$$

$$=\frac{1}{2}\left[0+2(2)\cos\left(\frac{n\pi 2}{2}\right)-0\right]$$

$$-0-0+0$$



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UNIT- I FOURIER SERIES





$$=\frac{1}{\ell}\left[4\ell \cos(n2\pi)\ell^{2}\right]$$

$$=\frac{1}{\ell}\left[4\ell^{2}\right]$$

$$=\frac{1}{\ell}\left[4\ell^{2}\right]$$

$$=\frac{1}{n^{2}\pi^{2}}$$

$$=\frac{1}{n^{2}\pi^{2}}$$

$$=\frac{1}{\ell}\left[4\ell^{2}\right]$$

$$=$$



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UNIT- I FOURIER SERIES



$$u = x^2$$

$$u' = 2x$$

$$u'' = 2$$

$$u''' = 0$$

$$V = \frac{\sin(\frac{n\pi x}{2})}{(\frac{n\pi x}{2})}$$

$$V_1 = -\frac{\cos(\frac{n\pi x}{2})}{(\frac{n\pi x}{2})}$$

$$V_2 = \frac{\sin(\frac{n\pi x}{2})}{(\frac{n\pi x}{2})^2}$$

$$V_3 = \frac{\cos(\frac{n\pi x}{2})}{(\frac{n\pi x}{2})^3}$$

$$b_{n} = \frac{1}{2} \left[-\frac{2c^{2} \cos\left(\frac{n\pi\alpha}{2}\right)}{\frac{n\pi}{2}} - 2\alpha \left[-\frac{\sin\left(\frac{n\pi\alpha}{2}\right)}{\frac{n\pi}{2}} \right] + 2\cos\left(\frac{n\pi\alpha}{2}\right) \right]$$

$$+ 2\cos\left(\frac{n\pi\alpha}{2}\right)$$

$$\frac{(n\pi)^{3}}{2}$$

$$=\frac{1}{2}\int_{-4}^{2}\frac{1}{4}\int_{-4}^{2}\frac{\cos\left(n\pi \frac{2}{2}\right)}{2}+2(22)\int_{-4}^{2}\frac{\sin\left(n\pi \frac{2}{2}\right)}{\left(n\pi \frac{2}{2}\right)}$$

$$+2\cos\left(n\pi \frac{2}{2}\right)$$

$$+2\cos\left(n\pi \frac{2}{2}\right)$$

$$+\cos\left(n\pi \frac{2}{2}\right)$$

$$\left(\frac{n\pi}{2}\right)^{3}$$

$$\begin{array}{c|c}
-2 & CB = O \\
\hline
-1 & CB = O
\end{array}$$
d with
$$\begin{array}{c|c}
(\frac{N}{2})^3 \\
\hline
2 & O
\end{array}$$



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$$=\frac{1}{2}\left[\frac{-4\ell^{2}}{(n\pi/2)}+0+\frac{2}{(n\pi/2)^{2}}-\frac{2}{(n\pi/2)^{2}}\right]$$

$$=\frac{1}{2}\left[-4\ell^{2}\times\ell\right]$$

$$=\frac{1}{2}\left[-4\ell^{2}\times\ell\right]$$

$$=\frac{1}{2}\left[-4\ell^{2}\times\ell\right]$$

$$=\frac{1}{2}\left[-4\ell^{2}\times\ell\right]$$

$$=\frac{1}{2}\left[-4\ell^{2}\times\ell\right]$$

$$=\frac{2\ell^{2}}{n\pi}$$

$$=\frac{2\ell^{2}}{n\pi}\left[\frac{4\ell^{2}}{n\pi}\right]\cos\left(\frac{n\pi\alpha}{\ell}\right)$$

$$=\frac{4\ell^{2}}{n\pi}\left[\frac{4\ell^{2}}{n\pi}\right]\sin\left(\frac{n\pi\alpha}{\ell}\right)$$

$$=\frac{4\ell^{2}}{n\pi}\left[\frac{4\ell^{2}}{n\pi}\right]\cos\left(\frac{n\pi\alpha}{\ell}\right)$$

$$=\frac{4\ell^{2}}{n\pi}\left[\frac{4\ell^{2}}{n\pi}\right]\sin\left(\frac{n\pi\alpha}{\ell}\right)$$

$$=\frac{4\ell^{2}}{n\pi}\left[\frac{4\ell^{2}}{n\pi}\right]\sin\left(\frac{n\pi\alpha}{\ell}\right)$$

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