## What is a Decision Tree?

- A Visual Representation of Choices, Consequences, Probabilities, and Opportunities.
- A Way of Breaking Down Complicated Situations Down to Easier-to-Understand Scenarios.


Decision Tree

## Easy Example

- A Decision Tree with two choices.



## Notation Used in Decision Trees

$\begin{array}{ll}\text { - A box } & \text { sed to show a choice that the } \\ \text { manager } \quad \text { make. }\end{array}$

- A circle is used to show that a probability outcome ...r occur.
- Lines



## Easy Example - Revisited

What are some of the costs we should take into account when deciding whether or not to go to business school?

- Tuition and Fees
- Rent / Food / etc.
- Opportunity cost of salary
- Anticipated future earnings



## Simple Decision Tree Model



## The Yeaple Study (1994)

According to Ronald Yeaple, it is only profitable to go to one of the top 15 Business Schools otherwise you have a NEGATIVE NPV!
(Economist, Aug. 6, 1994)

Benefits of Learning

School
Harvard
Chicago
Stanford
MIT (Sloan)
Yale
\$83,775
Northwestern \$53,526
Berkeley \$54,101
Wharton \$59,486
UCLA \$55,088
Virginia \$30,046
Cornell $\$ 30,974$
Michigan \$21,502
Dartmouth \$22,509
Carnegie Mellon \$18,679
Texas \$17,459
Rochester - \$307
Indiana - $\$ 3,315$
North Carolina - $\$ 4,565$
Duke - \$17,631
NYU - $\$ 3,749$

## Things he may have missed

- Future uncertainty (interest rates, future salary, etc)

- Cost of Living differences
- Type of Job [utility function = f(\$, enjoyment)]
- Girlfriend / Boyfriend / Family concerns
- Others?

Utility Function $=f(\$$, enjoyment, family, location, type of job / prestige, gender, age, race) Human Factors Considerations

## Mary's Factory

Mary is a manager of a gadget factory. Her factory has been quite successful the past three years. She is wondering whether or not it is a good idea to expand her factory this year. The cost to expand her factory is $\$ 1.5 \mathrm{M}$. If she does nothing and the economy stays good and people continue to buy lots of gadgets she expects $\$ 3 \mathrm{M}$ in revenue; while only $\$ 1 \mathrm{M}$ if the economy is bad.
If she expands the factory, she expects to receive \$6M if economy is good and \$2M if economy is bad.
She also assumes that there is a $40 \%$ chance of a good economy and a 60\% chance of a bad economy.
(a) Draw a Decision Tree showing these choices.


## Decision Tree Example


$\mathrm{NPV}_{\text {Expand }}=(.4(6)+.6(2))-1.5=\$ 2.1 \mathrm{M}$
$\mathrm{NPV}_{\text {No Expand }}=.4(3)+.6(1)=\$ 1.8 \mathrm{M}$
$\$ 2.1>1.8$, therefore you should expand the factory

## Example 2 - Joe's Garage

Joe's garage is considering hiring another mechanic. The mechanic would cost them an additional $\$ 50,000$ / year in salary and benefits. If there are a lot of accidents in Providence this year, they anticipate making an additional $\$ 75,000$ in net revenue. If there are not a lot of accidents, they could lose \$20,000 off of last year's total net revenues. Because of all the ice on the roads, Joe thinks that there will be a $70 \%$ chance of "a lot of accidents" and a 30\% chance of "fewer accidents". Assume if he doesn't expand he will have the same revenue as last year.

Draw a decision tree for Joe and tell him what he should do.

## Example 2 - Answer



- Estimated value of "Hire Mechanic" =

NPV $=.7(70,000)+.3(-\$ 20,000)-\$ 50,000=-\$ 7,000$

- Therefore you should not hire the mechanic



## Mary's Factory With Options

A few days later she was told that if she expands, she can opt to either (a) expand the factory further if the economy is good which costs 1.5 M , but will yield an additional $\$ 2 \mathrm{M}$ in profit when economy is good but only $\$ 1 \mathrm{M}$ when economy is bad, (b) abandon the project and sell the equipment she originally bought for $\$ 1.3 \mathrm{M}$, or (c) do nothing.
(b) Draw a decision tree to show these three options for each possible outcome, and compute the NPV for the expansion.


## Decision Trees, with Options

Expand further - yielding \$8M (but costing \$1.5)

Stay at new expanded levels - yielding \$6M

Reduce to old levels - yielding \$3M (but saving \$1.3-sell equipment)
Expand further - yielding \$3M (but costing \$1.5)

Stay at new expanded levels - yielding \$2M

Reduce to old levels yielding $\$ 1 \mathrm{M}$ (but saving $\$ 1.3$ in equipment cost)

## Present Value of the Options

- Good Economy
- Expand further $=8 \mathrm{M}-1.5 \mathrm{M}=6.5 \mathrm{M}$
- Do nothing = 6M
- Abandon Project $=3 \mathrm{M}+1.3 \mathrm{M}=4.3 \mathrm{M}$
- Bad Economy
- Expand further $=3 \mathrm{M}-1.5 \mathrm{M}=1.5 \mathrm{M}$
- Do nothing = 2M
- Abandon Project = $1 \mathrm{M}+1.3 \mathrm{M}=2.3 \mathrm{M}$


## NPV of the Project

So the NPV of Expanding the factory is:
$\mathrm{NPV}_{\text {Expand }}=[.4(6.5)+.6(2.3)]-1.5 \mathrm{M}=\$ 2.48 \mathrm{M}$

Therefore the value of the option is
2.48 (new NPV) - 2.1 (old NPV) = \$380,000

You would pay up to this amount to exercise that option.


## Mary's Factory Discounting

Before Mary takes this to her boss, she wants to account for the time value of money. The gadget company uses a $10 \%$ discount rate. The cost of expanding the factory is borne in year zero but the revenue streams are in year one.
(c) Compute the NPV in part (a) again, this time account the time value of money in your analysis. Should she expand the factory?

## Time Value of Money



## Time Value of Money

- Recall that the formula for discounting money as a function of time is: $\mathrm{PV}=\mathrm{S}(1+\mathrm{i})^{-n}$ [where $\mathrm{i}=$ interest / discount rate; $\mathrm{n}=$ number of years / $S=$ nominal value]
- So, in each scenario, we get the Present Value (PV) of the estimated net revenues:
a) $\mathrm{PV}=6(1.1)^{-1}=\$ 5,454,454$
b) $\mathrm{PV}=2(1.1)^{-1}=\$ 1,818,181$
c) $P V=3(1.1)^{-1}=\$ 2,727,272$
d) $P V=1(1.1)^{-1}=\$ 0.909,091$


## Time Value of Money

- Therefore, the PV of the revenue streams (once you account for the time value of money) are:
$P V_{\text {Expand }}=.4(5.5 \mathrm{M})+.6(1.82 \mathrm{M})=\$ 3.29 \mathrm{M}$
PV Don't Ex. $=0.4(2.73)+0.6(.910)=1.638$
- So, should you expand the factory?

Yes, because the cost of the expansion is $\$ 1.5 \mathrm{M}$, and that means the NPV $=3.29-1.5=\$ 1.79>\$ 1.64$

- Note that since the cost of expansion is borne in year 0, you don't discount it.


## Stephanie's Hardware Store

Stephanie has a hardware store and she is deciding whether or not to buy
 Adler's Hardware store on Wickendon
Street. She can buy it for $\$ 400,000$; however it would take one year to renovate, implement her computer inventory system, etc.

The next year she expects to earn $\$ 600,000$ if the economy is good and only $\$ 200,000$ if the economy is bad. She estimates a 65\% probability of a good economy and a 35\% probability of a bad economy. If she doesn't buy Adler's she knows she will get \$0 additional profits.

Taking the time value of money into account, find the NPV of the project with a discount rate of $10 \%$


## Should she buy?

- NPV of purchase =

$$
\begin{aligned}
& -.65(600,000 / 1.1)+.35(200,000 / 1.1)-400,000 \\
& \quad=\$ 18,181.82
\end{aligned}
$$

- Therefore, she should do the project!
- What happens if the discount rate $=15 \%$ ?
- The NPV = 0 , so it probably is not worth it.
- What happens if the discount rate $=20 \%$ ?
- The NPV = - \$16,666.67; so you should not buy!

