FOURIER SERIES

PERIODIC FUNCTIONS:

A function $f : R \to R$ is said to be periodic if there exists a positive no. ω such that $f(x+\omega) = f(x)$, for all real numbers x & ω is called a period of f.

Eg: $f(x) = \sin x = \sin(x+2\pi) = \sin(x+4\pi) = \ldots$ is said to be periodic with period 2π

$|||^{ly}$ $f(x) = \cos x$ is a periodic function with period 2π.

$f(x) = \tan x$ is a periodic function with period π.

FOURIER SERIES:

A periodic function $f(x)$ which satisfies certain condition can be expressed as cosine and sine series of the form,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

is called Fourier series of $f(x)$ and a_0, a_n, b_n $(n=1,2,\ldots)$ are called Fourier co-efficients of $f(x)$.

DETERMINING THE FOURIER CO-EFFICIENTS :

The Fourier Series for the function $f(x)$ in the interval $c < x < c+2\pi$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx .$$

where $a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx \, dx$$

$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx \, dx$ are known as Euler's Formulae.

DIRICHLET'S CONDITION :

Any function $f(x)$ can be developed as a Fourier series if

(i) $f(x)$ is periodic, single valued and finite.

(ii) $f(x)$ is continuous or piecewise continuous with a finite no. of discontinuities in any one period.

(iii) $f(x)$ has a finite no. of maxima or minima in any one period.