

Applications of Stack

- * Evaluating arithmetic expressions
- * Balancing the symbols
- * Function calls
- * Towers of Hanoi
- * 8 - Queens Problem.
- * Applied Recursion function

Different Types of notation to represent arithmetic expression

There are three different ways of expressing algebraic expressions.

- * Infix notation
- * Postfix notation
- * Prefix notation

Infix notation:-

The arithmetic operator appears between 2 operands to which it is being applied.

Eg

A | B + C

Postfix

The arithmetic operator appears directly after two operands to which it applies.
also called reverse polish notation.

Eg. abt

Examples

Convert infix to postfix

①

(- (A | B) + C)

① Apply it on inner

② outer brackets

(AB | + C)

AB | C +

②

a + (b / c)

(i) (a + (b / c))

(ii) (a + b c /)

[a b c | +]

③ $a + b * c$

(i) Apply brackets based on Priority

$$(a + (b * c))$$

(ii) Inner bracket $(b * c)$

$$(a + (b * c))$$

$$(a + \underline{bc *})$$

$$\boxed{abc * +}$$

④ $(a + (b * (c - d)) / (p - r)))$

(i) $(a + (b * cd -) / (p - r))$

(ii) $(a + (b * cd -) / pr -)$

(iii) $(a + (b * cd - * pr - /))$

(iv) $(abcd - * pr - / +)$

abcd - * pr - / +

Prefix :-

The arithmetic operator is placed before the 2 operands to which it applies also called polish notation.

$$\text{Bq} \quad +ab \quad ((a+b))$$

$$① \quad (a+b) \quad \boxed{+ab}$$

$$② \quad (a+(b*c)) \quad \boxed{+ * b c}$$

$$(a+*(bc))((b-c)+d)+e) \quad \boxed{+ * b c} \quad (i)$$

$$③ \quad ((b+((c*3)/2))-4) \quad (ii)$$

$$((b+(*c3/2))-4) \quad (iii)$$

$$((b+1*c3/2)-4) \quad (iv)$$

$$(+b/*c3/2-4) \quad (v)$$

$$-+b/*c3/24$$

Exercise

Convert given infix notation to postfix & prefix

$$\text{a} + \text{b} * \text{c} + (\text{d} * \text{e})$$

$$\textcircled{1} \quad \text{A} + \text{B}$$

$$\textcircled{2} \quad \text{A} + \text{B} - \text{C} \quad \text{A} + \text{B} * (\text{C} - \text{D}) / (\text{E} - \text{F})$$

$$\textcircled{3} \quad (\text{A} + \text{B}) * (\text{C} - \text{D}) \quad (\text{A} + \text{B}) * \text{C} - (\text{D} - \text{E}) * (\text{F} + \text{G})$$

$$\textcircled{4} \quad \text{A} \$ \text{B} * \text{C} - \text{D} + \text{E} / \text{F} / (\text{G} + \text{H})$$

$$\textcircled{5} \quad ((\text{A} + \text{B}) * \text{C} - (\text{D} - \text{E})) \$ (\text{F} + \text{G})$$

$$\textcircled{6} \quad \text{A} - \text{B} / (\text{C} * \text{D} \$ \text{E})$$

$$\textcircled{7} \quad (\text{A} + \text{B}) * (\text{C} - \text{D}) \$ (\text{E} * \text{F})$$

$$\textcircled{8} \quad (\text{A} + \text{B}) * (\text{C} \$ (\text{D} - \text{E}) + \text{F}) - \text{G}$$

$$\textcircled{9} \quad \text{A} + ((\text{B} - \text{C}) * (\text{D} - \text{E}) + \text{F}) / \text{G} \$ (\text{H} - \text{J})$$

Evaluating Arithmetic Expressions

To evaluate the arithmetic expression, first convert the infix notation to postfix notation and then evaluate using stack.

evaluating postfix using stack

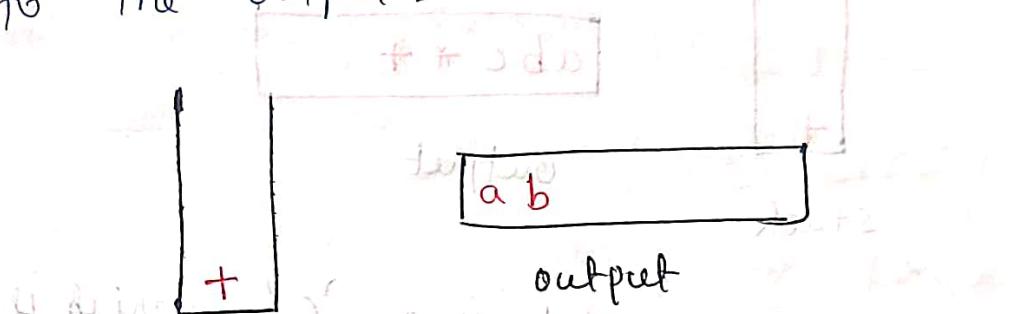
Infix to postfix notation

- * If the character is an operand placed on to the output.
- * If the character is an operator pushed on to the stack.
- * If the stack operator has higher or equal priority, then input operator, pop that operator from the stack & placed on to the output.
- * If the character is left parenthesis pushed on to the stack.
- * If the character is right parenthesis pop all the operators from the stack, fill characters' left parenthesis, discard both parenthesis in the output.

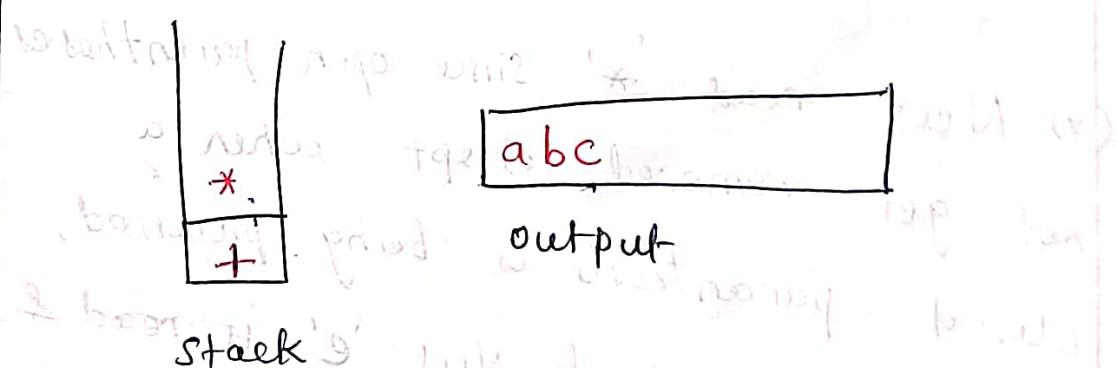
Input:- $a + b * c + (d * e + f) * g$

- (i) First, the symbol 'a' is read, so it is passed through to the output. Then '+' is read and pushed on to stack.

Next 'b' is read and passed through to the output.



(ii) Next '*' is read. The top entry on the operator stack has lower precedence than '*', so nothing is output and '*' is put on the stack. Next 'c' is read & output

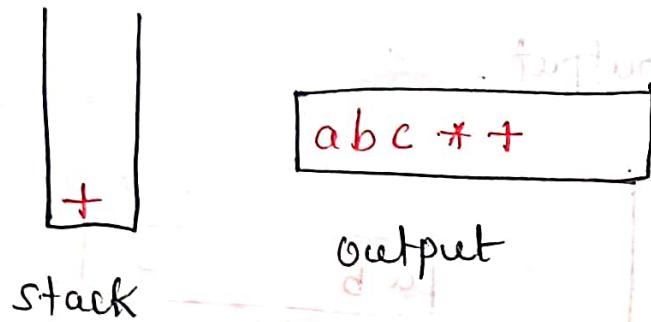


(iii) The next symbol is a '+'. Checking the stack, we find that we will pop

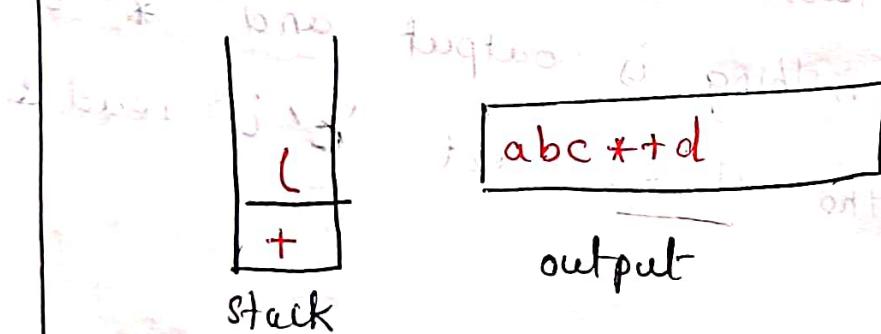
a '*' and place it on the output.

pop + also which is not of lower but equal priority, so the stack. and then push the '+'

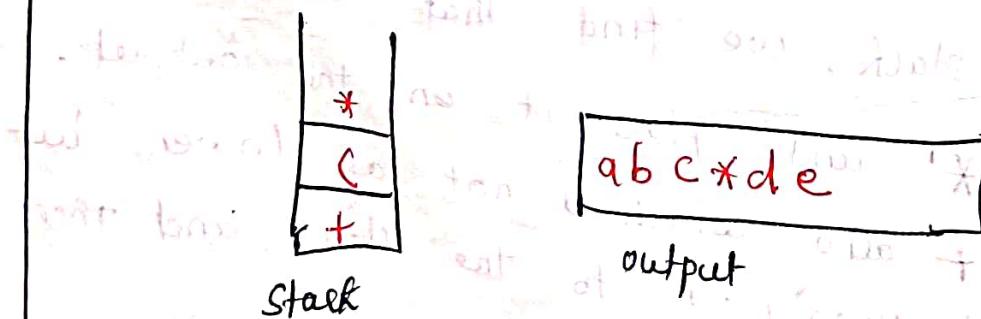
infix to postfix conversion of 'abc * + d'



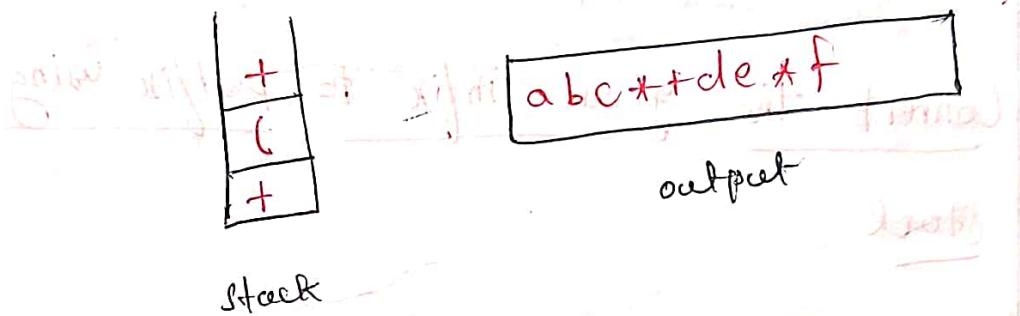
(iii) The next symbol is a ''C'', which is being of highest precedence, is placed on the stack. Then 'd' is read & output



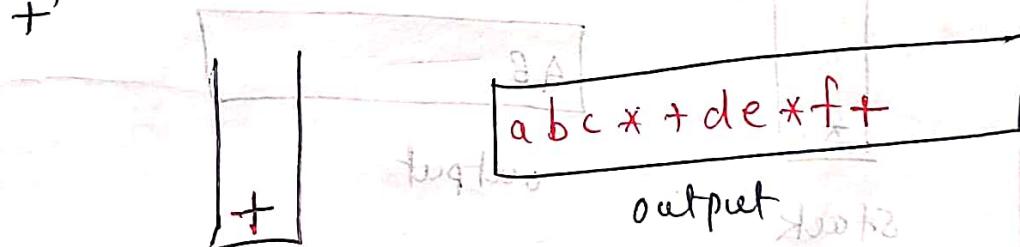
(iv) Next read '*' since open parentheses do not get removed except when a closed parenthesis is being processed, there is no output. Next 'e' is read & output.



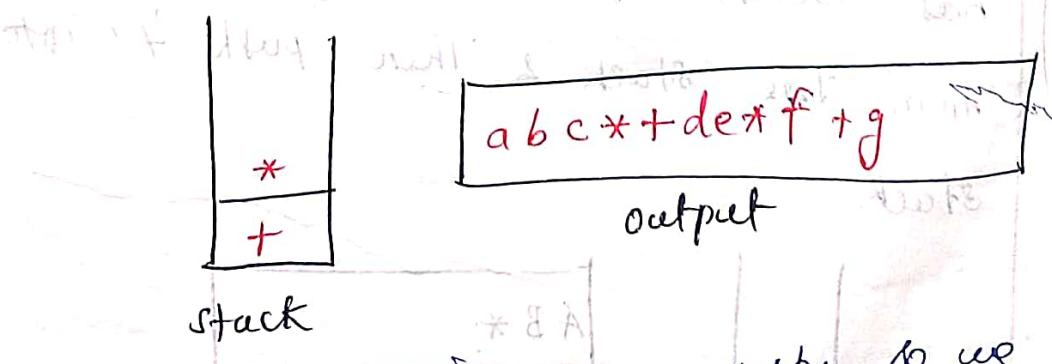
(vi) The next symbol read is a '+'
we pop and output '*' and then push '+'
Then we read & output f.



(vii) Now we need ')' to close the stack
is empties back to the '(' we output a
'+'



(viii) We read a '*' next, if it is pushed on
to the stack. Then g will read and output



(ix) The input is now empty, so we
pop and output symbols from the stack
until it is empty.



$\boxed{abc * + def * + g * +}$

output

stack

Convert the given infix to postfix using

stack

$A * B + (C - D / E)$

- (i) Insert 'A' put in output and read '*' push it on to stack. Then read 'B' & output



\boxed{AB}

output

stack



- (ii) Read '+' compare with operator already exists in stack. compared to '*' '+' has lowest precedence. Then pop '*' from the stack & then push '+' into stack

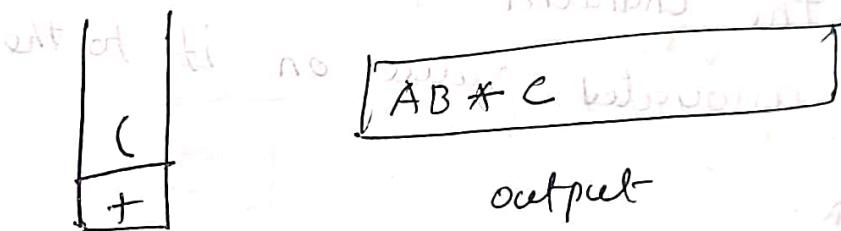


$\boxed{AB*}$

output

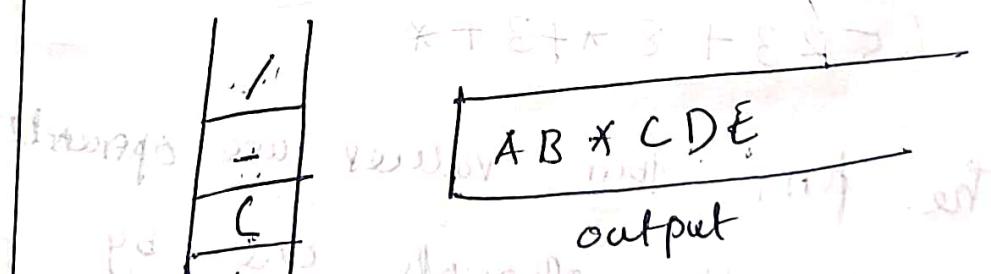
stack

(iii) Next read the symbol 'C' and push it on to stack. Then read 'C & output'.

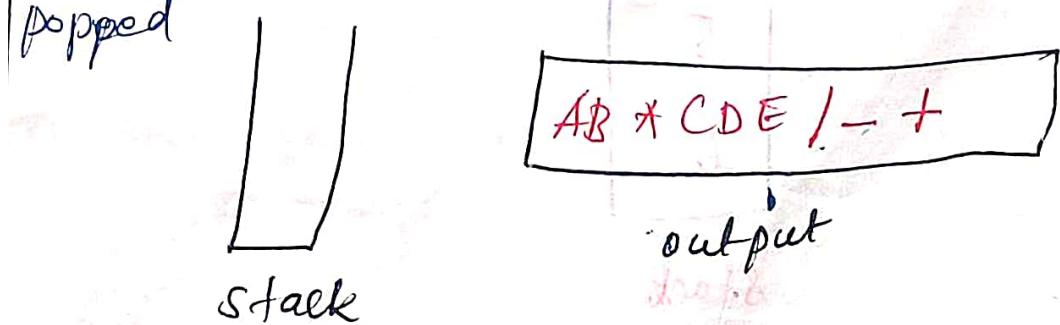


so go to stack and read '-' symbol push it to stack.

(iv) Then read '-' symbol. Next then read 'D' and output. Next read the symbol 'Y', it has higher precedence than '-' so push 'Y' in to stack. Then read E & output



(v) Next operators inside open and close brackets pop up. Finally '+' symbol popped



Evaluating Postfix Expression

* Read one character at a time.

if the character is an operand push it associated value on it to the stack.

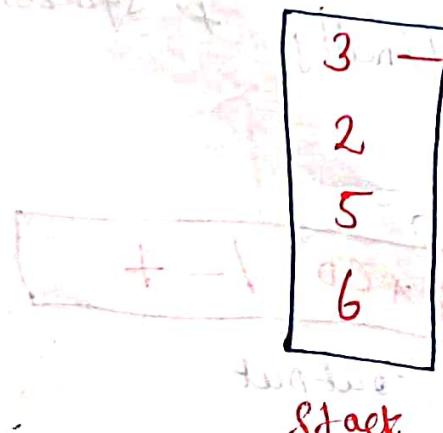
* If the character is an operator, pop two values from the stack apply the operator to them and then push the result on to the stack.

Example 1 \Rightarrow Postfix given as an input

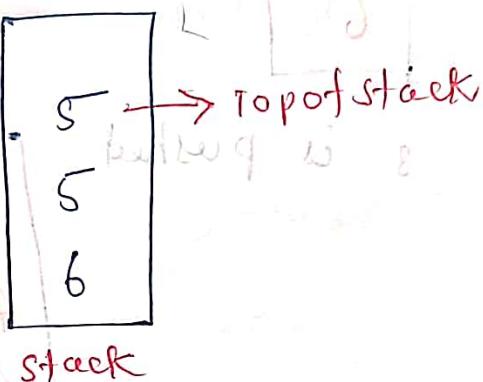
A B C D + E * + D + *

6 5 2 3 + 8 * + 3 + *

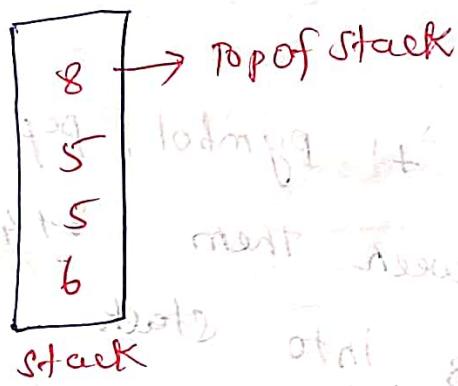
① The first four values are operands so push the elements one by one. The resulting stack is



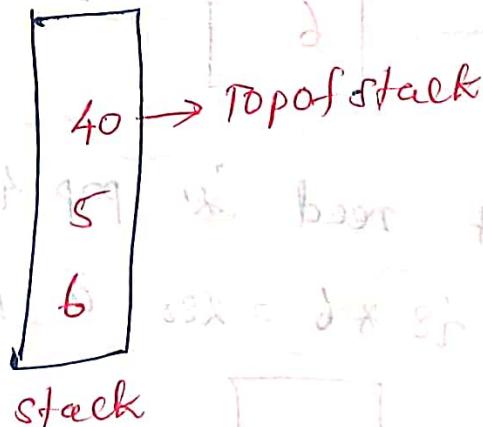
- ② Next a '+' is read, so 3 & 2 are popped from the stack and their sum, 5 is pushed.



- ③ Next 8 is pushed



- ④ Next read '*', so pop 8 and 5 from the stack and $5 * 8 = 40$ is again pushed

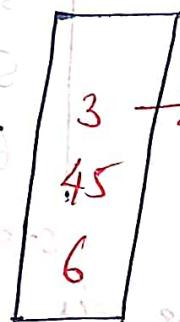


- ⑤ Next read '#' pushed on to the stack. apply $40 +$ between 40 & 5 to get 245 pushed to stack.



Top of stack with 45 & 6

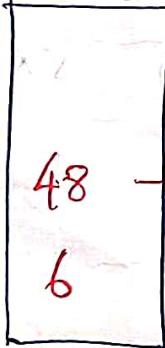
⑥ Now 8 is pushed



Top of stack

⑦ Read '+' symbol, pop 3 & 45 apply
+ between them $3 + 45 = 48$.

push 48 into stack



Top of stack

⑧ Next read '*' pop 48 & 6 from stack
then $48 * 6 = 288$ is pushed.



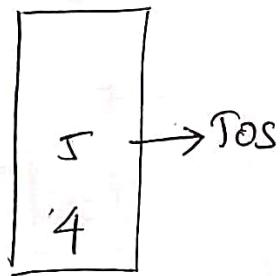
Top of stack

The time to evaluate a postfix expression is $O(N)$, because processing each element in the input consists of stack operations and thus takes constant time.

Example 2

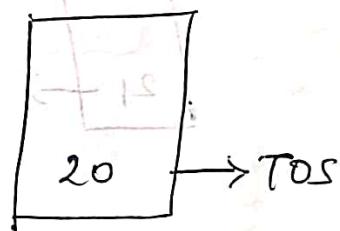
$AB * CDE / - +$
~~15 * 28 58 2 1 - today's book~~

①

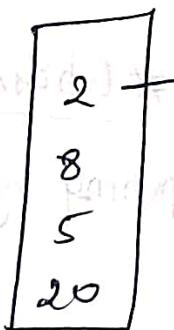


② Read * pop 4 & 5

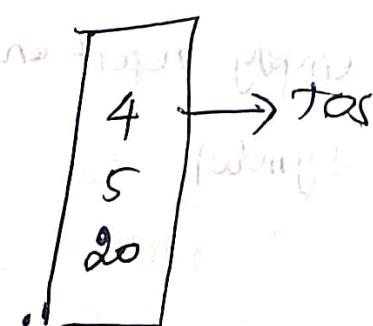
$4 * 5 = 20$ push



③ Read value 5, 8, 2 one by one



④ Read (one by one) POP 2 & 8 $8 / 2 = 4$. push 4



⑤ Read '-' symbol pop 5 & 4

push - value 1 no minus

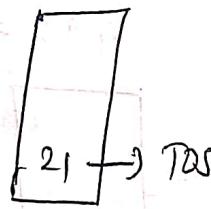
$$5-4 = 1$$

error with an unmatched closing brace



⑥ Read '+' symbol pop 20 & 1

20+1 = 21, push 21



Balancing the Symbols

* Read one character at a time until

it encounters the delimiters < # (hash)

* If the character is an opening symbol

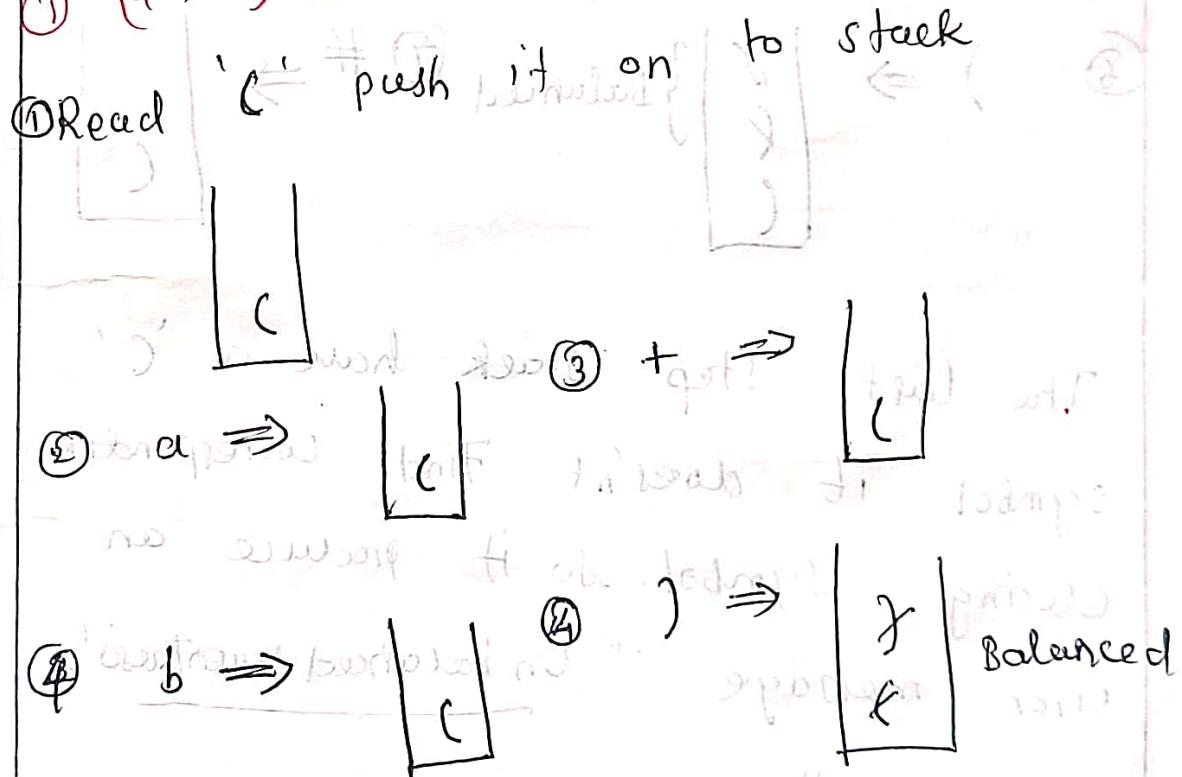
pushed on to the stack.

* If the character is a closing symbol

and if the stack is empty report an
error missing opening symbol

- * If it is a closing symbol and it has corresponding opening symbol in the stack, pop it from the stack otherwise report an error mismatch symbols.
- * At the end of file if the stack is not empty report an error as missing closing symbols.

① $(a+b) \#$



⑤ $\# \Rightarrow$ stack is empty.

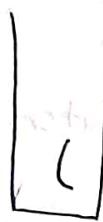
so both opening & closing validation exit the

stack after validating symbols of last expression.

In above answer pair left & right parentheses are balanced.

② $((a+b) \#)$

① C \Rightarrow



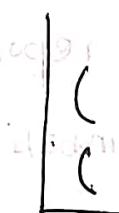
② + C \Rightarrow



③ a \Rightarrow



④ + \Rightarrow



⑤ b \Rightarrow



⑥) \Rightarrow



⑦ # \Rightarrow



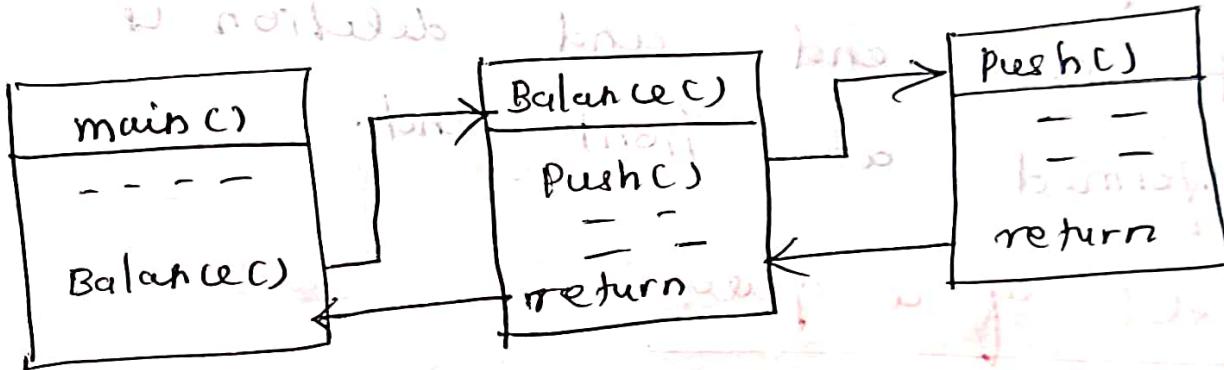
The last step stack have a 'C'
symbol it doesn't find corresponding
closing symbol. so it produce an
error message "Unbalanced parenthesis".

Function calls :-

* when call is made to a new function
all the variables local to calling routine
need to be saved. otherwise new function
will overwrite the calling routine.

Variables -

- * Similarly the current location address in the routine must be saved.
- * So the new function knows where to go after it is completed.



int fact (int n)

```

{
    int s;
    if (n == 1)
        return 1;
    else
        s = n * fact (n - 1);
}
```

Power

(except

except no factors

else if

Applications of Queue:-

- * Batch processing in OS
- * To implement priority Queues
- * Priority Queues can be used to sort the elements using heap sort
- * Simulation
- * Queue in Theory
- * Computer networks where the server takes the job of client as per the Queue strategy.