SNS COLLEGE OF TECHNOLOGY

An Autonomous Institution

Coimbatore-35

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A++' Grade

Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECB301-ANALOG AND DIGITAL COMMUNICATION

III YEAR/ V SEMESTER

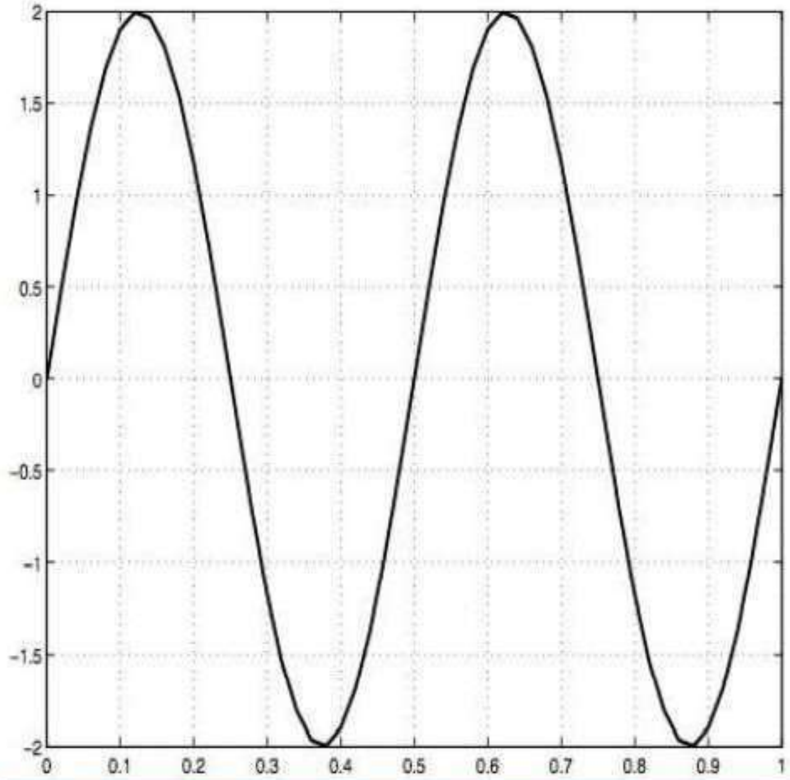
UNIT 3 – DIGITAL COMMUNICATION

TOPIC – SAMPLING

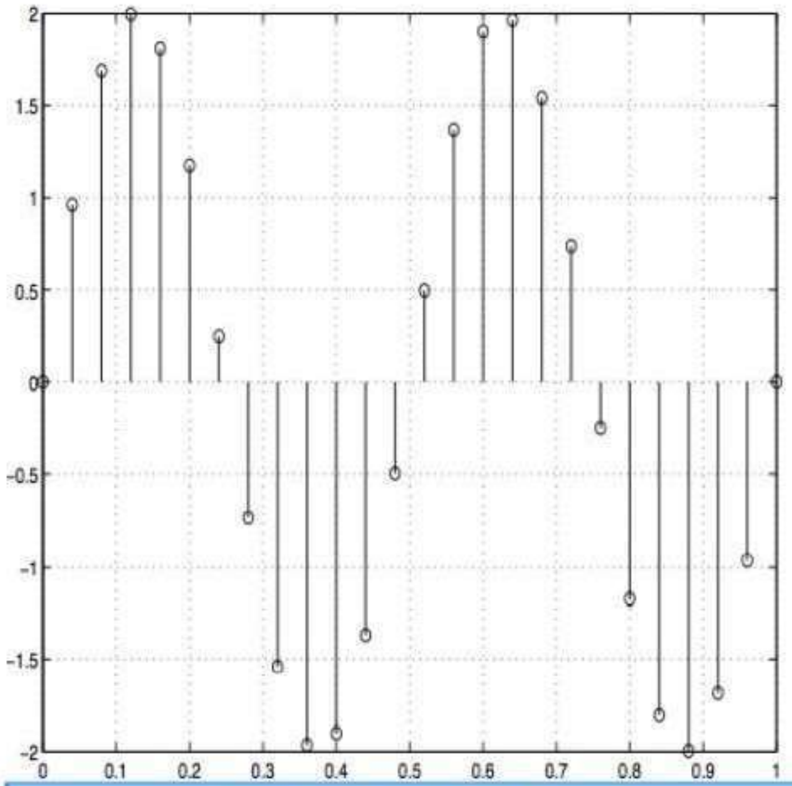
SAMPLING

- Process of converting the continuous time signal to a discrete time signal.
- Sampling is done by taking "Samples" at specific times spaced regularly.
  - $V(t)$ is an analog signal
  - $V(nT_s)$ is the sampled signal
    - T_s = positive real number that represent the spacing of the sampling time
    - n = sample number integer

SAMPLING



Original Analog Signal
“Before Sampling”



Sampled Analog Signal
“After Sampling”

SAMPLING

- The closer the Ts value, the closer the sampled signal resemble the original signal.
- Note that we have lost some values of the original signal, the parts between each successive samples.

- **Can we recover these values? And How?**
- **Can we go back from the discrete signal to the original continuous signal?**

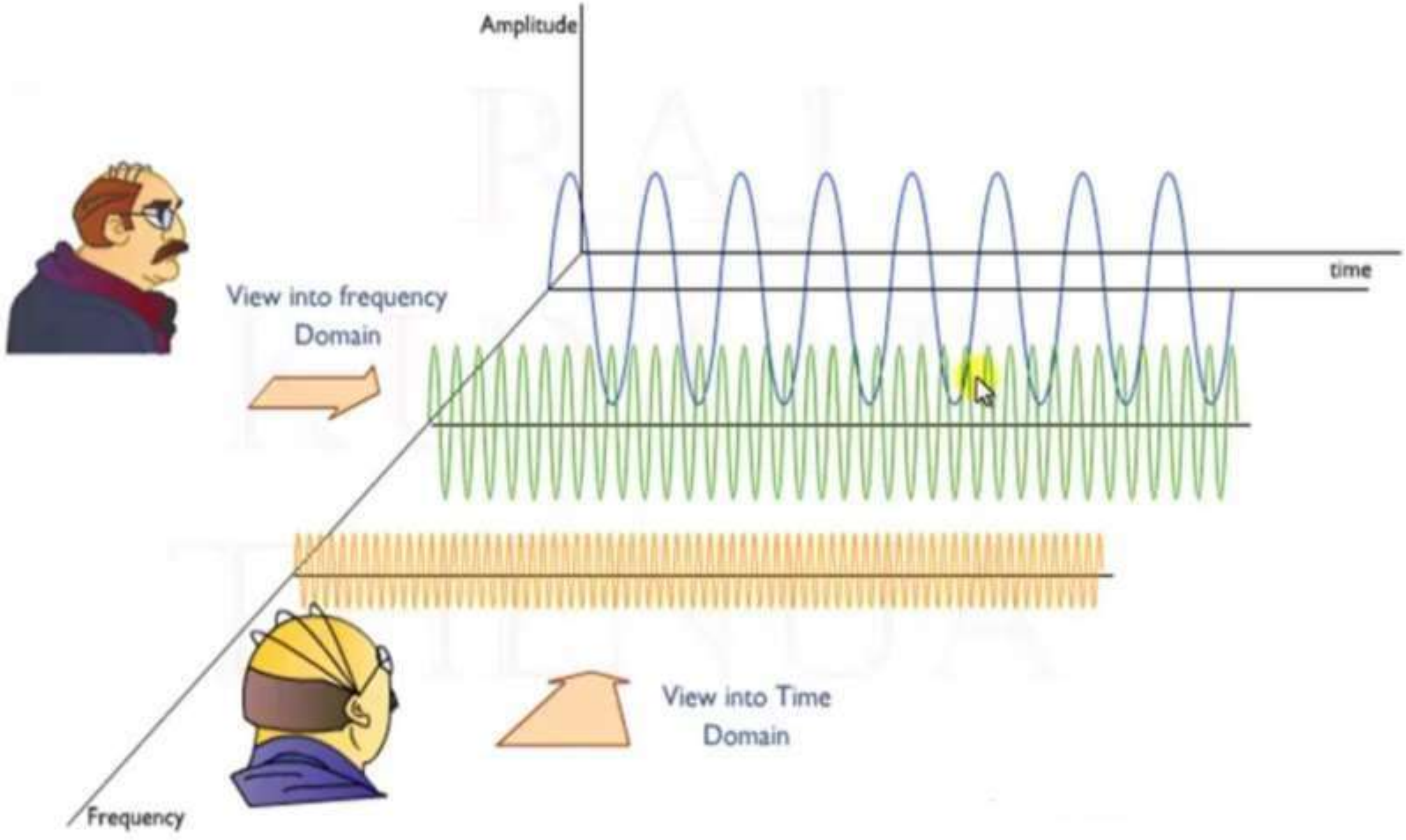
SAMPLING

- A bandlimited signal having no spectral components above f_{max} (Hz), can be determined uniquely by values sampled at uniform intervals of Ts seconds, where

- An analog signal can be reconstructed from a sampled signal without any loss of information if and only if it is:
  - Band limited signal
  - The sampling frequency is at least twice the signal bandwidth

$$Ts \leq \frac{1}{2f_{\max}}$$

TIME DOMAIN Vs FREQUENCY DOMAIN



Sampling theorem - Low pass sampling:-

Consider analog signal $g(t)$ Continuous in time and amplitude.

Infinite duration but finite energy.

Let the sample values of $g(t)$ at times $t = 0, \pm T_s, \pm 2T_s$ denoted by series as

$$g(nT_s), n = 0, \pm 1, \pm 2 \ldots \quad \text{--- (1)}$$

where T_s - Sampling Period

$f_s = 1/T_s$ Sampling rate

(1) Discrete time signal $g_\delta(t)$, results from the sampling process

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \quad \text{--- (2)}$$

$\delta(t - nT_s) \rightarrow$ Dirac delta function located at $t = nT_s$

Each delta function is weighted by the corresponding sample value.

I-7

from the definition of a delta function

$$g(nT_s)\,\delta(t-nT_s) = g(t)\,\delta(t-nT_s) \quad \text{--- (3)}$$

Using ~~this~~, rewrite eqn (2)

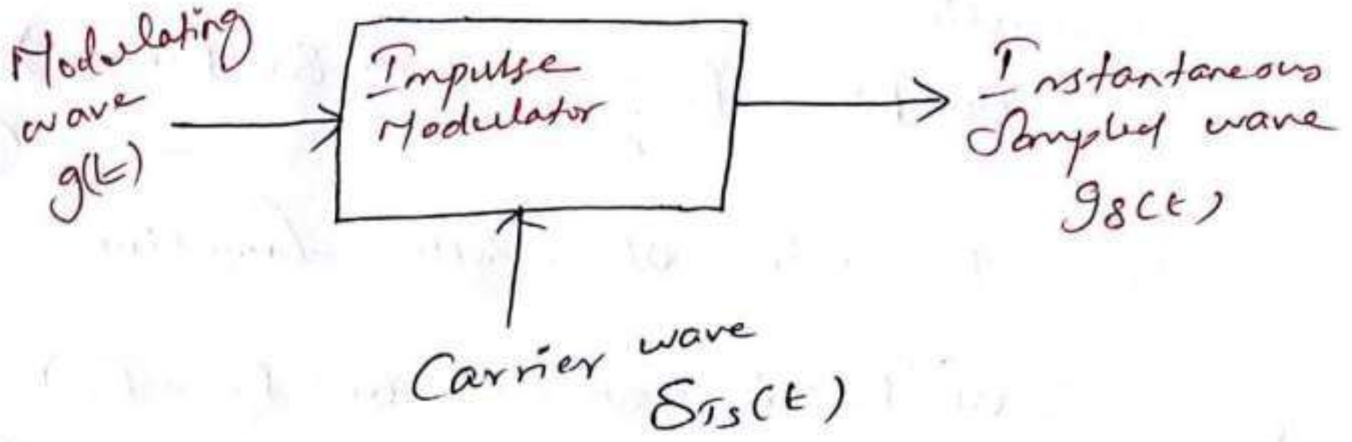
$$g_\delta(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$g_\delta(t) = g(t)\,\delta_{T_s}(t) \quad \text{--- (4)}$$

$\delta_{T_s}(t)$ = Dirac Combination (or) Ideal Sampling function.

from eqn 4

$g_\delta(t)$ = output of impulse Modulator operates with $g(t)$ mod. wave where $\delta_{T_s}(t)$ Carrier wave.



from fourier theorem

Multiplication of two time function is equivalent to the Convolution of their fourier transform

$G(f) =$ FT of $g(t)$

$G_\delta(t) =$ FT of $g_\delta(t)$

fourier transform of $\delta_{Ts}(t)$

$$F[\delta_{Ts}(t)] = f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s) \quad \text{—— (5)}$$

Transforming (4) into frequency domain

$$G_\delta(f) = G(f) * \left[f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s) \right] \quad \text{—— (6)}$$

$*$ - Convolution function

Interchanging order of Summation & Convolution

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f) * \delta(f - m f_s) \quad \text{——— (7)}$$

from properties of Delta function

$$G(f) * \delta(f - m f_s) = G(f - m f_s) \quad \text{——— (8)}$$

from eqn (8) Simplify eqn (7)

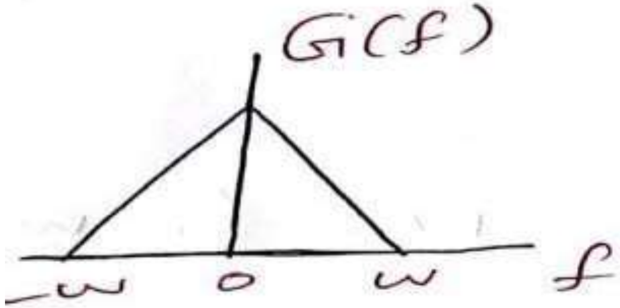
$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - m f_s) \quad \text{——— (9)}$$

In eqn ⑨

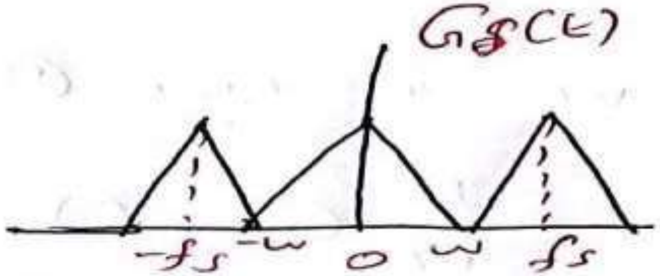
$G_\delta(f)$ - Represents a Spectrum that is periodic in the frequency f with period f_s

- Periodic extension of the Original Spectrum $G(f)$

Process of Uniformly sampling a signal in the time domain results in a periodic Spectrum in the frequency domain with a period equal to the sampling rate.



Spectrum of g(t)



Spectrum of sampled signal gδ(t) for a sampling rate $f_s = 2W$

Anothe representation obtained by taking fourier transform on both sides of eqn ②

and

$$F[\delta(t-nT_s)] = \exp[-j2\pi f n T_s]$$

So ② becomes by applying $F[\delta(t-nT_s)]$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp\left[-j2\pi f n T_s\right] \quad \text{——— (10)}$$

when signal has no frequency components higher than W hertz

then $G(f) = 0$ for $|f| \geq W$

Choose sampling period $T_s = 1/2W$

Sub in eqn (10)

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j2\pi n f}{2W}\right) \quad \text{——— (11)}$$

Sub $f_s = 2W$ in eqn (9)

$$G(f) = \frac{1}{2W} G_\delta(f) \qquad -W < f < W$$

So (10) becomes

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi n f}{W}\right) \qquad (12)$$

THANK YOU