# UNIT-I

## Fourier Series:

### Periodic Functions:

A function $f(x)$ is said to have a period $T$ if for all $x$, $f(x+T) = f(x)$, where $T$ is a positive constant. The least value of $T>0$ is called the period of $f(x)$

Ex: $\sin x$, $\cos x$, $\tan x$, $\sin nx$, $\cos nx$

period $2\pi$, $2\pi$, $\pi$, $\frac{2\pi}{n}$, $\frac{2\pi}{n}$

### Dirichlet Conditions:

A function $f(x)$ defined in $c \leq x \leq c+2l$ can be expanded as an infinite trigonometric series of the form $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

provided,

(i) $f(x)$ is defined and single valued except possibly at a finite no. of points in $(c, c+2l)$

(ii) $f(x)$ is periodic in $(c, c+2l)$

(iii) $f(x)$ and $f'(x)$ are piecewise continuous in $(c, c+2l)$

(iv) $f(x)$ has no or finite number of maxima or minima in $(c, c+2l)$.

### Fourier Series: Defn:

The infinite trigonometric Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

is called the Fourier series of $f(x)$ which satisfies Dirichlet's conditions in $c \leq x \leq c+2l$.

$a_0$, $a_n$ and $b_n$ are called Fourier coefficients and the values are given by Euler's formula.

where $a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$

$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx$

$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx$

} Euler's formulae.

**Problems:**

1) Write the (i) Euler's formula of $f(x)$ in $(c, c+2\pi)$

(ii) Fourier coefficients of $(0, 2\pi)$ $[a_0, a_n, b_n]$

(iii) Fourier's constants $a_0, a_n$ and $b_n$ in $(-\pi, \pi)$

(iv) Fourier coefficients of $(0, 2l)$ and $(-l, l)$

**Solution:**

(i) $a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x)\,dx$ ; $a_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx$

$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx$

Here $c = c$ and $c+2l = c+2\pi$

$\Rightarrow l = \pi$

$\therefore a_0 = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\,dx$ ; $a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos\frac{n\pi x}{\pi}\,dx$ ; $b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\sin\frac{n\pi x}{\pi}$

$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx\,dx$ $\quad b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\sin nx\,dx$

(ii) Here $c = 0$ ; $c+2l = 2\pi$

$\Rightarrow l = \pi$

$a_0 = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\cos\frac{n\pi x}{\pi}\,dx = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\sin\frac{n\pi x}{\pi}\,dx = \frac{1}{\pi}\int_{0}^{2\pi} f(x)\sin nx\,dx$

(iii) Here $c = -\pi$ ; $c+2l = \pi$

$-\pi + 2l = \pi$

$2l = \pi + \pi$

$l = \pi$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\frac{n\pi x}{\pi}\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

(iv) Here $c = -l$ $\qquad c + 2l = l$

$2l = 2l \Rightarrow l = l$

$(0, 2\pi) \Rightarrow l = \pi$

$(-l, l) \Rightarrow l = l$

$$a_0 = \frac{1}{l} \int_{-l}^{l} f(x)\, dx \; ; \quad a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l}\, dx \; ;$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l}\, dx .$$

$(-\pi, \pi) \Rightarrow l = \pi$

$(0, 2l) \Rightarrow l = l$