PERCENTAGE

The word “percent” is derived from the latin words “per centum”, which means “per hundred”.

A percentage is a fraction with denominator hundred, It is denoted by the symbol %.

Numerator of the fraction is called the **rate per cent.**

VALUE OF PERCENTAGE:

Value of percentage always depends on the quantity to which it refers:

Consider the statement, “65% of the students in this class are boys”. From the context, it is understood that boys from 65% of the total number of students in the class. To know the value of 65%, the value of the total number of student should be known. If the total number of student is 200, then,

The number of boys $=\frac{200\times65}{100}=130$;

It can also be written as (200) × (0.65) =130.

Note that the expressions 6%, 63%, 72%, 155% etc. Do not have any value intrinsic to themselves. Their values depend on the quantities to which they refer.

To express the fraction equivalent to %:

Express the fraction with the denominator 100, then the numerator is the answer.

Example 1:

Express the fraction $\frac{11}{12}$ into the per cent.

Solution:

$$\frac{11}{12}=\frac{\frac{11}{12}\times100}{100}=\frac{91\frac{2}{3}}{100}=91\frac{2}{3}\%$$

To express % equivalent to fraction:

$a\% = \frac{a}{100}$

Example 2:

Express $45\frac{5}{6}\%$ into fraction.

Solution:

$$45\frac{5}{6}\%=\frac{45\frac{5}{6}}{100}=\frac{275}{6\times100}=\frac{11}{24}$$

Fraction Equivalents of %	
$1\% = \frac{1}{100}$	$33\frac{1}{3}\% = \frac{1}{3}$
$2\% = \frac{1}{50}$	$40\% = \frac{2}{5}$
$4\% = \frac{1}{25}$	$50\% = \frac{1}{2}$
$5\% = \frac{1}{20}$	$66\frac{2}{3}\% = \frac{2}{3}$
$6\frac{1}{4}\% = \frac{1}{16}$	$60\% = \frac{3}{5}$
$10\% = \frac{1}{10}$	$75\% = \frac{3}{4}$
$11\frac{1}{3}\% = \frac{17}{150}$	$80\% = \frac{4}{5}$
$12\frac{1}{2}\% = \frac{1}{8}$	$96\% = \frac{24}{25}$
$16\% = \frac{4}{25}$	$100\% = 1$
$16\frac{2}{3}\% = \frac{1}{6}$	$115\% = \frac{23}{20}$
$20\% = \frac{1}{5}$	$133\frac{1}{3}\% = \frac{4}{3}$
$25\% = \frac{1}{4}$	

❖

$$\text{Increase }\% = \frac{\text{Increas value}}{\text{Original value}}\times100$$

Example 3:

Rent of the house is increased from ` 7000 to `7700. Express the increase in price as a percentage of the original rent.

Solution:

Increase value = Rs 7700 – Rs 7000 = Rs 700

Increase % = $\frac{\text{Increas value}}{\text{Original value}} \times 100 = \frac{700}{7000} \times 100 = 10$

∴ Percentage rise = 10%

❖

Example 4:

The cost of a bike last year was Rs19000. Its cost this year is Rs 17000. Find the per cent decrease in its cost.

Decrease % = $\frac{\text{Decreas value}}{\text{Original value}} \times 100$

% decrease = $\frac{19000-17000}{19000} \times 100$

$= \frac{2000}{19000} \times 100$= 10.5%.

∴ Percentage decrease = 10.5%.

❖ If A is x % if C and B is y % of C, then A is $\frac{x}{y} \times 100$% of B.

Example 5:

A positive number is divided by 5 instead of being multiplied by 5. By what per cent is the result of the required correct value?

Solution:

Let the number be 1, then the correct answer = 5

The incorrect answer that was obtained $=\frac{1}{5}$.

∴ The required % = $\frac{1}{5\times5} \times 100$ = 4%

❖ If two numbers are respectively x% and y% more than a third number, then the first number is $\left(\frac{100+x}{100+y} \times 100\right)$% of the second and the second is $\left(\frac{100+y}{100+x} \times 100\right)$% of the first.

❖ If two numbers are respectively x% and y% less than a third number, then the first number if $\left(\frac{100-x}{100-y} \times 100\right)$% of the second and the second is $\left(\frac{100-y}{100-x} \times 100\right)$% of the first.

❖ x% of a quantity is taken by the first, y% of the remaining is taken by the second and z% of the remaining is taken by third person. Now, if A is left in the fund, then the initial amount $= \frac{A\times100\times100\times100}{(100-x)(100-y)(100-z)}$ in the beginning.

❖ x% of a quantity is added. Again, y% of the increased quantity is added. Again z% of the increased quantity is added. Now it becomes A, then the initial amount

$$= \frac{A \times 100 \times 100 \times 100}{(100+x)(100+y)(100+z)}$$

Example 6:

3.5% income is taken as tax and 12.5% of the remaining is saved. This leaves Rs. 4,053 to spend. What is the income?

Solution:

By direct method,

Income = $\frac{4053\times100\times100}{(100-3.5)(100-12.5)}$ = Rs 4800.

❖ If the price of a commodity increases by r%, then reduction in consumption, so as not to increase the expenditure is $\left(\frac{r}{100+r} \times 100\right)$%.

❖ If the price of a commodity decreases by r%, then the increase in consumption, so as not to decrease the expenditure is $\left(\frac{r}{100-r} \times 100\right)$%.

Example 7:

If the price of coal be raised by 20%, then find by how much a householder must reduce his consumption of this commodity so as not to increase his expenditure?

Solution:

Reduction in consumption = $\left(\frac{20}{100+20} \times 100\right)\%$

$= \left(\frac{20}{100+20} \times 100\right)\% = 16.67\%$

POPULATION FORMULA

- If the original population of a town is P, and the annual increase is r%, then the population after n years is $P\left(1+\frac{r}{100}\right)^n$ and population before n years = $\frac{P}{\left(1+\frac{r}{100}\right)^n}$
- If the annual decrease be r%, then the population after n years is $P\left(1-\frac{r}{100}\right)^n$ and population before n years = $\frac{P}{\left(1+\frac{r}{100}\right)^n}$

Example 8:

The population of a certain town increased at a certain rate per cent annum. Now it is 456976. Four years ago, it was 390625. What will it be 2 years hence?

Solution:

Suppose the population increases at r % per annum. Then, $390625\left(1+\frac{r}{100}\right)^4 = 456976$

$\therefore \left(1+\frac{r}{100}\right)^2 = \sqrt{\frac{456976}{390625}} = \frac{676}{625}$

Population 2 years hence = $456976\left(1+\frac{r}{100}\right)^2$

$= 456976 \times \frac{676}{625} = 494265$ approximately.

Example 9:

The population of a city increase at the rate of 4% per annum. There is an additional annual increase of 1% in the population due to the influx of job seekers. Find percentage increase in the population after 2 years.

Solution:

The net annual increase = 5%

Let the initial population be 100.

- Then, population after 2 years = 100×1.05×1.05 = 110.25

Therefore, % increase in population = (110.25-100) = 10.25%

If a number A is increased successively by x% followed by y% and then z%, then the final value of A will be

$$A\left(1+\frac{x}{100}\right)\left(1+\frac{y}{100}\right)\left(1+\frac{z}{100}\right)$$

In case a given value decreases by an percentage then we will use negative sign before that.

- **First Increase and then decrease:**

If the value is first increased by x% and then decreased by y% then there is $\left(x-y-\frac{xy}{100}\right)\%$ increase or decrease, according to the +ve or –ve sign respectively.

If the value is first increased by x% and then decreased by x% then there is only decrease which is equal to $\left(\frac{x^2}{100}\right)$.

Example 10:

A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease per cent.

Solution:

% change = $\frac{10\times10}{100} = 1\%$

i.e. 1% decrease.

❖ Average percentage rate of change over a period.

$= \frac{(\text{New Value} - \text{Old Value})}{\text{Old Value}} \times \frac{100}{n}\%$ where n = period.

❖ The percentage error = $\frac{\text{The Error}}{\text{True Value}} \times 100\%$

SUCCESSIVE INCREASE OR DECREASE

❖ In the value is increased successively by x% and y% then the final increase is given by

$\left(x + y + \frac{xy}{100}\right)\%$

❖ In the value is decreased successively by x% and y% then the final decrease is given by

$\left(-x - y - \frac{xy}{100}\right)\%$

Example 11:

The price of a car is decreased by 10% and 20% in two successive years. What per cent of price of a car is decreased after two years?

Solution:

Put x = -10 and y = -20, then

$-10-20+\frac{(-10)(-20)}{100} = -28\%$

∴ The price of the car decreases by 28%.

STUDENT AND MARKS

❖ The percentage of passing marks in an examination is x%. If a candidate who scores y marks fails by z marks, then the maximum marks M = $\frac{100(y+z)}{x}$

❖ A candidate scoring x% in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more then the minimum required passing marks. Then the maximum marks M = $\frac{100(a+b)}{y-x}$

❖ In an examination x% and y% students respectively fail in two different subjects while z% students fail in both subjects then the % age of student who pass in both the subjects will be {100-(x + y - z)}%

Example 12:

Vishal requires 40% to pass. If he gets 185 marks, falls short by 15 marks, what was the maximum he could have got?

Solution:

If Vishal has 15 marks more, he could have scored 40% marks.

Now, 15 marks more then 185 is 185+15 = 200

Let the maximum marks be x, then 40% of x = 200

$\Rightarrow \frac{40}{100} \times x = 200 \Rightarrow x = \frac{200\times100}{40} = 500$

Thus, maximum marks = 500

Alternate method:

Maximum marks $= \frac{100(185+15)}{40} = \frac{100\times200}{40} = 500$

Example 13:

A candidate scores 15% and fails by 30 marks, while another candidate who scores 40% marks, gets 20 marks more then the minimum required marks to pass the pass the examination. Find the maximum marks of the examination.

Solution:

By short cut method:

Maximum marks = $\frac{100(30+20)}{40-15}$ =200

2-DIMENSIONAL FIGURE AND AREA

❖ If the sides of a triangle, square, rectangle, rhombus or radius of a circle are increased by a%, its area is increased by $\frac{a(a+200)}{100}$%

❖ If the sides of a triangle, square, rectangle, rhombus or radius of a circle are decreased by a %

Then its area is decreased by $\frac{a(200-a)}{100}$%.

Example 14:

If the radius of a circle is increased by 10%, what is the percentage increase in its area?

Solution:

Let R be the radius of circle.

Area of Circle, A =πR^2

Now, radius is increased by 10%

New radius, R' = R + 10% of R = 1.1 R

New Area, A' = $\pi(1.1R)^2$= 1.21 πR^2

% increase in area =

$\frac{1.21\pi R^2-\pi R^2}{\pi R^2}$×100=21%

Shortcut Method:

Radius is increases by 10%.

So, Area is increased by $\frac{10(10+200)}{100}$ = 21%

❖ If the both sides of rectangle are changed by x% and y% respectively, then % effect on area = x + y+$\frac{xy}{100}$ (+/- according to increase or decrease)

Example 15:

If the length and width of a rectangular garden were each increased by 20%, then what would be the per cent increase in the area of the garden?

Solution:

By direct formula

% increase in area = $\frac{20\ (20+200)}{100}$ = 44%

❖ If A's income is r% more than that of B, then B's income is less than that of A by $\left(\frac{r}{100+r}\times 100\right)\%$

❖ If A's income is r% less than that of B, then B's income is more than that of A by $\left(\frac{r}{100-r}\times 100\right)\%$

Example 16:

If A's salary is 50% more than B's, then by what percent B's salary is less than A's salary?

Solution:

Let B's salary be *Rs* x

Then, A's salary = x + 50% of x = 1.5x

B's salary is less than A's salary by

$\left(\frac{1.5x-x}{1.5x}\times 100\right)\%$ = $\frac{100}{3}$ = 33.33%

Shortcut method,

B's salary is less than A's salary by $\left(\frac{50}{100+50}\times 100\right)\%$

=$\frac{50}{150}\times 100\%$ = 33.33%

Example 17:

Ravi's weight is 25% that of Meena's and 40% that of Tara's. What percentage of Tara's weight is Meena's weight.

Solution:

Let Meena's weight be x kg and Tara's weight be y kg. Then Ravi's weight = 25% of Meena's weight

$= \frac{25}{100} \times x$ .....(i)

Also, Ravi's weight = 40% of Tara's weight

$= \frac{40}{100} \times y$ .....(ii)

From (i) and (ii), we get

$\frac{25}{100} \times x = \frac{40}{100} \times y$

⇒ 25x = 40y

⇒ 5x = 8y ⇒ x = $\frac{8}{5}$ y

Meena's weight as the percentage of Tara's weight

$= \frac{x}{y} \times 100 = \frac{\frac{8}{5}y}{y} \times 100$

$= \frac{8}{5} \times 100 = 160$

Hence, Meena's weight is 160% of Tara's weight.

Example 18:

The monthly salaries of A and B together amount to `50,000. A spends 80% of his salary and B spends 70% of his salary. If now their saving are the same, then find the salaries of A and B.

Solution:

Let A's salary by x, then B's salary (50,000-x)

A spends 80% of his salary and saves 20%

B spends 70% of his salary and saves 30%

Given that

20% of x = 30% of (50,000-x)

$\frac{20}{100} \times x = \frac{30}{100} \times (50{,}000 - x)$

$\frac{50x}{100} = \frac{30 \times 50{,}000}{100}$

⇒ x = $\frac{30 \times 50{,}000 \times 100}{100 \times 50} = 30{,}000$

A's salary Rs 30,000

B's salary = Rs 50,000 – Rs 30,000 = Rs20,000