Half Range cosine Series and Half range Sine Series

The half range cosine series in the interval $(0, \ell)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\ell}$$

where $a_0 = \frac{2}{\ell}\int_0^{\ell} f(x)\,dx$

$$a_n = \frac{2}{\ell}\int_0^{\ell} f(x)\cos\left(\frac{n\pi x}{\ell}\right)dx$$

The half range Sine Series is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right)$$

where $b_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx$

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(1) Find half range cosine series for $f(x) = x(2-x)$ in $0 \le x \le 2$

Soln: $\ell = 2$

Half range cosine series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{\ell}\right)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2}\right) \rightarrow (1)$$

To find a_0:

$$a_0 = \frac{2}{2}\int_0^2 x(2-x)\,dx$$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \left(\frac{2x^2}{2} - \frac{x^3}{3}\right)_0^2$$

$$\boxed{a_0 = \frac{4}{3}}$$

To find a_n :

$$a_n = \frac{2}{2}\int_0^2 (2x - x^2)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \left[(2x - x^2)\sin\left(\frac{n\pi x}{2}\right)\frac{2}{n\pi} + (2-2x)\frac{4}{\pi^2 n^2}\cos\left(\frac{n\pi x}{2}\right) - 2\cdot\frac{8}{n^3\pi^3}\sin\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$\boxed{a_n = \frac{-8}{n^2\pi^2}\left[1 + (-1)^n\right]}$$

$$f(x) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-8}{n^2\pi^2}\left[1 + (-1)^n\right]\cos\left(\frac{n\pi x}{2}\right)$$

(2) Find half range Sine Series for

$f(x) = x(\pi - x)$ in $(0, \pi)$

Soln:

$$b_n = \frac{-4}{\pi n^3}\left[(-1)^n - 1\right]$$

(3) Find half range Cosine series for $f(x) = l - x$ in $(0, l)$.

Soln:

$$a_0 = l \;,\quad a_n = \frac{2l}{n^2\pi^2}\left[1-(-1)^n\right]$$

(4) Find half range Cosine series for $f(x) = x \sin x$ in $(0, \pi)$.

Soln:

$$a_0 = 2 \;,\quad a_n = \frac{-2(-1)^n}{n^2-1} \;,\quad a_1 = \frac{-1}{4}.$$

(5) Find half range Fourier Sine series for

$$f(x) = \begin{cases} x & , 0 \le x \le l/2 \\ l-x & , l/2 \le x \le l \end{cases}.$$

Soln:

$$b_n = \frac{4l}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

(6) Find half range Sine series for $f(x) = 1 - x$ in $(0, 1)$.

Soln:

$$b_n = \frac{-2}{n\pi}$$

(7) Find the half range Sine series for $f(x) = x$ in $0 < x < l$. and

Soln:

$$b_n = \frac{-2l}{n\pi}(-1)^n$$

ROOT MEAN SQUARE VALUE
[RMS VALUE]

The root mean square value or RMS value of $f(x)$ over the interval (a, b) is defined as,

$$RMS = \sqrt{\frac{\int_a^b (f(x))^2 dx}{b-a}}$$

(1) Find RMS value for $f(x) = x^2$ in $(0, 2\pi)$

Soln:

$$RMS = \sqrt{\frac{\int_0^{2\pi} x^4 \, dx}{2\pi}} = \frac{1}{\sqrt{2\pi}} \sqrt{\left(\frac{x^5}{5}\right)_0^{2\pi}}$$

$$= \frac{\sqrt{32\pi^5}}{\sqrt{2\pi}\sqrt{5}}$$

(2) Find RMS value for $f(x) = x$ in $(-1, 1)$

Soln:

$$RMS = \frac{1}{\sqrt{3}}$$