# NUMBER SYSTEM AND CODES

- The term digital refers to a process that is achieved by using discrete unit.
- In number system there are different symbols and each symbol has an absolute value and also has place value.

## RADIX OR BASE:-

The radix or base of a number system is defined as the number of different digits which can occur in each position in the number system.

## RADIX POINT :-

The generalized form of a decimal point is known as radix point. In any positional number system the radix point divides the integer and fractional part.

$N_r$ = [ Integer part **.** Fractional part ]

↑

Radix point

## NUMBER SYSTEM:-

In general a number in a system having base or radix ‘ r ’ can be written as

$$a_n \; a_{n-1} \; a_{n-2} \; \ldots\ldots\ldots\ldots\ldots \; a_0 \; . \; a_{-1} \; a_{-2} \ldots\ldots\ldots\ldots\ldots \; a_{-m}$$

This will be interpreted as

$$Y = a_n \times r^n + a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots\ldots\ldots + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2} + \ldots\ldots\ldots\ldots + a_{-m} \times r^{-m}$$

where Y = value of the entire number

$a_n$ = the value of the $n^{th}$ digit

r = radix

## TYPES OF NUMBER SYSTEM:-

There are four types of number systems. They are

1. Decimal number system
2. Binary number system
3. Octal number system
4. Hexadecimal number system

## DECIMAL NUMBER SYSTEM:-

- The decimal number system contain ten unique symbols 0,1,2,3,4,5,6,7,8 and 9.
- In decimal system 10 symbols are involved, so the base or radix is 10.
- It is a positional weighted system.
- The value attached to the symbol depends on its location with respect to the decimal point.

In general,

$$d_n \quad d_{n-1} \quad d_{n-2} \quad \ldots\ldots\ldots\ldots\ldots \quad d_0 \;.\; d_{-1} \quad d_{-2} \ldots\ldots\ldots\ldots\ldots d_{-m}$$

is given by

$$(d_n \times 10^n) + (d_{n-1} \times 10^{n-1}) + (d_{n-2} \times 10^{n-2}) + \ldots + (d_0 \times 10^0) + (d_{-1} \times 10^{-1}) + (d_{-2} \times 10^{-2}) + \ldots + (d_{-m} \times 10^{-m})$$

**For example:-**

9256.26 = 9 x 1000 + 2 x 100 + 5 x 10 + 6 x 1 + 2 x (1/10) + 6 x ( 1/100)

$= 9 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 6 \times 10^{-2}$

## BINARY NUMBER SYSTEM:-

- The binary number system is a positional weighted system.
- The base or radix of this number system is 2.
- It has two independent symbols.
- The symbols used are 0 and 1.
- A binary digit is called a bit.
- The binary point separates the integer and fraction parts.

In general,

$$d_n \quad d_{n-1} \quad d_{n-2} \quad \ldots\ldots\ldots\ldots\ldots \quad d_0 \;.\; d_{-1} \quad d_{-2} \ldots\ldots\ldots\ldots\ldots d_{-k}$$

is given by

$$(d_n \times 2^n) + (d_{n-1} \times 2^{n-1}) + (d_{n-2} \times 2^{n-2}) + \ldots. + (d_0 \times 2^0) + (d_{-1} \times 2^{-1}) + (d_{-2} \times 2^{-2}) + \ldots. + (d_{-k} \times 2^{-k})$$

## OCTAL NUMBER SYSTEM:-

- It is also a positional weighted system.
- Its base or radix is 8.
- It has 8 independent symbols 0,1,2,3,4,5,6 and 7.
- Its base $8 = 2^3$ , every 3- bit group of binary can be represented by an octal digit.

## HEXADECIMAL NUMBER SYSTEM:-

- The hexadecimal number system is a positional weighted system.
- The base or radix of this number system is 16.
- The symbols used are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E and F
- The base 16 = 24 , every 4 – bit group of binary can be represented by an hexadecimal digit.

## CONVERSION FROM ONE NUMBER SYSTEM TO ANOTHER :-

1. **BINARY NUMBER SYSTEM:-**

### (a) Binary to decimal conversion:-

In this method, each binary digit of the number is multiplied by its positional weight and the product terms are added to obtain decimal number.

**For example:**

**(i) Convert $(10101)_2$ to decimal.**

**Solution :**

| | |
|---|---|
| (Positional weight) | $2^4$ $2^3$ $2^2$ $2^1$ $2^0$ |
| Binary number | 10101 |
| | $= (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$ |
| | $= 16 + 0 + 4 + 0 + 1$ |
| | $= (21)_{10}$ |

**(ii) Convert $(111.101)_2$ to decimal.**

**Solution:**

$$(111.101)_2 = (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$
$$= 4 + 2 + 1 + 0.5 + 0 + 0.125$$
$$= (7.625)_{10}$$

## (b) Binary to Octal conversion:-

For conversion binary to octal the binary numbers are divided into groups of 3 bits each, starting at the binary point and proceeding towards left and right.

| Octal | Binary | Octal | Binary |
|---|---|---|---|
| 0 | 000 | 4 | 100 |
| 1 | 001 | 5 | 101 |
| 2 | 010 | 6 | 110 |
| 3 | 011 | 7 | 111 |

**For example:**

**(i) Convert $(101111010110.110110011)_2$ into octal.**

**Solution :**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Group of 3 bits are | 101 | 111 | 010 | 110 | . | 110 | 110 | 011 |
| Convert each group into octal = | 5 | 7 | 2 | 6 | . | 6 | 6 | 3 |

The result is **$(5726.663)_8$**

**(ii) Convert $(10101111001.0111)_2$ into octal.**

**Solution :**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Binary number | 10 | 101 | 111 | 001 | . | 011 | 1 |
| Group of 3 bits are | = 010 | 101 | 111 | 001 | . | 011 | 100 |
| Convert each group into octal = | 2 | 5 | 7 | 1 | . | 3 | 4 |

The result is **$(2571.34)_8$**

## (c) Binary to Hexadecimal conversion:-

For conversion binary to hexadecimal number the binary numbers starting from the binary point, groups are made of 4 bits each, on either side of the binary point.

| Hexadecimal | Binary | Hexadecimal | Binary |
|---|---|---|---|
| 0 | 0000 | 8 | 1000 |
| 1 | 0001 | 9 | 1001 |
| 2 | 0010 | A | 1010 |
| 3 | 0011 | B | 1011 |
| 4 | 0100 | C | 1100 |
| 5 | 0101 | D | 1101 |
| 6 | 0110 | E | 1110 |
| 7 | 0111 | F | 1111 |

**For example:**
**(i) Convert $(1011011011)_2$ into hexadecimal.**

**Solution:**

| | | | |
|---|---|---|---|
| Given Binary number | 10 | 1101 | 1011 |
| Group of 4 bits are | 0010 | 1101 | 1011 |
| Convert each group into hex = | 2 | D | B |

The result is $(\mathbf{2DB})_{16}$

**(ii) Convert $(01011111011.011111)_2$ into hexadecimal.**

**Solution:**

| | | | | | | |
|---|---|---|---|---|---|---|
| Given Binary number | 010 | 1111 | 1011 | . | 0111 | 11 |
| Group of 3 bits are = | 0010 | 1111 | 1011 | . | 0111 | 1100 |
| Convert each group into octal = | 2 | F | B | . | 7 | C |

The result is $(\mathbf{2FB.7C})_{16}$

## 2. DECIMAL NUMBER SYSTEM:-

### (a) Decimal to binary conversion:-

In the conversion the integer number are converted to the desired base using successive division by the base or radix.

**For example:**
**(i) Convert $(52)_{10}$ into binary.**

**Solution:**

Divide the given decimal number successively by 2 read the integer part remainder upwards to get equivalent binary number. Multiply the fraction part by 2. Keep the integer in the product as it is and multiply the new fraction in the product by 2. The process is continued and the integer are read in the products from top to bottom.

| | | |
|---|---|---|
| 2 | 52 | |
| 2 | 26 | — 0 |
| 2 | 13 | — 0 |
| 2 | 6 | — 1 |
| 2 | 3 | — 0 |
| 2 | 1 | — 1 |
| | 0 | — 1 |

Result of $(52)_{10}$ is $\mathbf{(110100)_2}$

**(ii) Convert $(105.15)_{10}$ into binary.**

**Solution:**

| Integer part | | Fraction part |
|---|---|---|
| 2 \| 105 | | 0.15 x 2 = 0.30 |
| 2 \| 52 | — 1 | 0.30 x 2 = 0.60 |
| 2 \| 26 | — 0 | 0.60 x 2 = 1.20 |
| 2 \| 13 | — 0 | 0.20 x 2 = 0.40 |
| 2 \| 6 | — 1 | 0.40 x 2 = 0.80 |
| 2 \| 3 | — 0 | 0.80 x 2 = 1.60 |
| 2 \| 1 | — 1 | |
| 0 | — 1 | |

Result of $(105.15)_{10}$ is $\mathbf{(1101001.001001)_2}$

## (b) Decimal to octal conversion:-

To convert the given decimal integer number to octal, successively divide the given number by 8 till the quotient is 0. To convert the given decimal fractions to octal successively multiply the decimal fraction and the subsequent decimal fractions by 8 till the product is 0 or till the required accuracy is obtained.

**For example:**

**(i) Convert $(378.93)_{10}$ into octal.**

**Solution:**

| | | |
|---|---|---|
| 8 \| 378 | | 0.93 x 8 = 7.44 |
| 8 \| 47 | — 2 | 0.44 x 8 = 3.52 |
| 8 \| 5 | — 7 | 0.52 x 8 = 4.16 |
| 0 | — 5 | 0.16 x 8 = 1.28 |

Result of $(378.93)_{10}$ is $\mathbf{(572.7341)_8}$

## (c) Decimal to hexadecimal conversion:-

The decimal to hexadecimal conversion is same as octal.

**For example:**

**(i) Convert $(2598.675)_{10}$ into hexadecimal.**

**Solution:**

| | Remainder Decimal | Remainder Hex | | Hex |
|---|---|---|---|---|
| 16 \| 2598 | | | 0.675 x 16 = 10.8 | A |
| 16 \| 162 | — 6 | 6 | 0.800 x 16 = 12.8 | C |
| 16 \| 10 | — 2 | 2 | 0.800 x 16 = 12.8 | C |
| 0 | — 10 | A | 0.800 x 16 = 12.8 | C |

Result of $(2598.675)_{10}$ is $\mathbf{(A26.ACCC)_{16}}$

## 3. OCTAL NUMBER SYSTEM:-

## (a) Octal to binary conversion:-

To convert a given a octal number to binary, replace each octal digit by its 3- bit binary equivalent.

**For example:**

**Convert $(367.52)_8$ into binary.**

**Solution:**

| | | | | | | |
|---|---|---|---|---|---|---|
| Given Octal number is | 3 | 6 | 7 | . | 5 | 2 |
| Convert each group octal to binary | = 011 | 110 | 111 | . | 101 | 010 |

Result of $(367.52)_8$ is **$(011110111.101010)_2$**

## (b) Octal to decimal conversion:-

For conversion octal to decimal number, multiply each digit in the octal number by the weight of its position and add all the product terms

**For example: -**

**Convert $(4057.06)_8$ to decimal**

**Solution:**

$(4057.06)_8 = 4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$

$= 2048 + 0 + 40 + 7 + 0 + 0.0937$

$= (2095.0937)_{10}$

Result is **$(2095.0937)_{10}$**

## (c) Octal to hexadecimal conversion:-

For conversion of octal to Hexadecimal, first convert the given octal number to binary and then binary number to hexadecimal.

**For example :-**

**Convert $(756.603)_8$ to hexadecimal.**

**Solution :-**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Given octal no. | | 7 | 5 | 6 | . | 6 | 0 | 3 |
| Convert each octal digit to binary | = | 111 | 101 | 110 | . | 110 | 000 | 011 |
| Group of 4bits are | = | 0001 | 1110 | 1110 | . | 1100 | 0001 | 1000 |
| Convert 4 bits group to hex. | = | 1 | E | E | . | C | 1 | 8 |

Result is **$(1EE.C18)_{16}$**

## (4) HEXADECIMAL NUMBER SYSTEM :-

## (a) Hexadecimal to binary conversion:-

For conversion of hexadecimal to binary, replace hexadecimal digit by its 4 bit binary group.

**For example:**

**Convert $(3A9E.B0D)_{16}$ into binary.**

**Solution:**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Given Hexadecimal number is | 3 | A | 9 | E | . | B | 0 | D |
| Convert each hexadecimal digit to 4 bit binary | = 0011 | 1010 | 1001 | 1110 | . | 1011 | 0000 | 1101 |

Result of $(3A9E.B0D)_8$ is **$(0011101010011110.101100001101)_2$**

### (b) Hexadecimal to decimal conversion:-

For conversion of hexadecimal to decimal, multiply each digit in the hexadecimal number by its position weight and add all those product terms.

**For example: -**
**Convert $(A0F9.0EB)_{16}$ to decimal**

**Solution:**

$(A0F9.0EB)_{16}$ = $(10 \times 16^3)+(0 \times 16^2)+(15 \times 16^1)+(9 \times 16^0)+(0 \times 16^{-1})+(14 \times 16^{-2})+(11 \times 16^{-3})$
= 40960 + 0 + 240 + 9 + 0 +0.0546 + 0.0026
= $(41209.0572)_{10}$

Result is **$(41209.0572)_{10}$**

### (c) Hexadecimal to Octal conversion:-

For conversion of hexadecimal to octal, first convert the given hexadecimal number to binary and then binary number to octal.

**For example :-**
**Convert $(B9F.AE)_{16}$ to octal.**

**Solution :-**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Given hexadecimal no.is | | B | | 9 | | F | . | A | E | |
| Convert each hex. digit to binary | = | 1011 | | 1001 | | 1111 | . | 1010 | 1110 | |
| Group of 3 bits are | = | 101 | 110 | | 011 | 111 | . | 101 | 011 | 100 |
| Convert 3 bits group to octal. | = | 5 | 6 | | 3 | 7 | . | 5 | 3 | 4 |

Result is **$(5637.534)_8$**

## BINARY ARITHEMATIC OPERATION :-

1. **BINARY ADDITION:-**

The binary addition rules are as follows
0 + 0 = 0 ; 0 + 1 = 1 ; 1 + 0 = 1 ; 1 + 1 = 10 , i.e 0 with a carry of 1

**For example :-**

**Add $(100101)_2$ and $(1101111)_2$.**
**Solution :-**

```
      1 0 0 1 0 1
  +  1 1 0 1 1 1 1
    1 0 0 1 0 1 0 0
```

Result is **$(10010100)_2$**

2. **BINARY SUBTRACTION:-**

The binary subtraction rules are as follows
0 - 0 = 0 ; 1 - 1 = 0 ; 1 - 0 = 1 ; 0 - 1 = 1 , with a borrow of 1

**For example :-**
**Substract**
**(111.111)$_2$ from**
**(1010.01)$_2$.**
**Solution :-**

```
          1 0 1 0 . 0 1 0
  -        1 1 1 . 1 1 1
          0 0 1 0 . 0 1 1
```

Result is **(0010.011)$_2$**

**3. BINARY MULTIPLICATION:-**

The binary multiplication rules are as follows0 x 0 = 0 ; 1 x 1 = 1 ; 1 x 0 = 0 ; 0 x 1 = 0

**For example :-**

**Multiply (1101)$_2$ by (110)$_2$.**
**Solution :-**

```
        1 1 0 1
   x      1 1 0
        0 0 0 0
      1 1 0 1
 +  1 1 0 1
    1 0 0 1 1 1 0
```

Result is **(1001110)$_2$**

**4. BINARY DIVISION:-**

The binary division is very simple and similar to decimal number system. The division by '0' is meaningless.So we have only 2 rules

0 ÷ 1 = 0

1 ÷ 1 = 1

**For example :-**
**Divide**

**(10110**

**)$_2$ by**

**(110)$_2$.**

**Soluti**

**on :-**

```
110 ) 101101 ( 111.1
    -  110
        1010
         110
         1001
          110
          110
          110
          000
```

Result is $\mathbf{(111.1)_2}$