SNS COLLEGE OF TENHNOLOGY

(AN AUTONOMOUS INSTITUTION)

Unit –II - Vector calculus

1. Find the unit normal vector to the surface $x^3 + y^2 - z$ at (1,1,2).
2. Find the unit normal vector of the surface $x^2 + y^2 - z = 1$ at (1, 1,1).
3. Find the unit vector normal to the surface $x^2 + y^2 = z$ at (1, -2, 5)
4. Find the unit normal vector to $xy=z^2$ at (1,1,-1).
5. Using Greens theorem in the plane,find the area of the circle $x^2 + y^2 = a^2$
6. Using Greens theorem evaluate $\int_C (xdy - ydx)$ where C is the circle $x^2+y^2=1$ in the xy plane
7. Prove that Curl(grad□) = 0
8. Find $\nabla(\nabla.((x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}))$ at the point (1,-1,2).
9. Find $Curl\vec{F}$ if $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$
10. Evaluate $\int_c (yz\vec{i} + xz\vec{j} + xy\vec{k}).\vec{dr}$ where C is the boundary of the surface S.
11. What is the greatest rate of $\phi = xyz^2$ at (1,0,3) .
12. The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located (1 ,1, 2)desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?
13. If $\vec{F}$=(x+3y)$\vec{i}$+(y-2z)$\vec{j}$+(x+2kz)$\vec{k}$ has divergence zero, find the unknown value of k.
14. State Green's theorem in a plane
15. State Stokes' theorem

PART –C

1 Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z^2 = x^2 + y^2 - 3$ at the point (2,-1,2).

2 Find the angle between the normals to the surfaces $x^2 = yz$ at the points(1,1,1) and (2,4,1).

3 A fluid motion is given by $\vec{V} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$. Is this motion is irrotational and is possible for an incompressible fluid?

4 If $\nabla\varphi = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$. find $\varphi(x, y, z)$ given that $\varphi(1,-2,2) = 4$

5 Find the constants a,b,c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} - (4x + cy + 2z)\vec{k}$ is irrotational.For those values of a,b,c.Find its scalar potential .

6 Find the constants a,b,c so that $\vec{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}-(4x+cy+2z)\vec{k}$ is irrotational.For those values of a,b,c.Find its scalar potential .

7 Show that $\vec{F}=2xyz^3\vec{i}+x^2z^3\vec{j}+3x^2yz^2\vec{k}$ is irrotational . Find the scalar potential ϕ and F= gradϕ

8 Show that $\vec{F}=(y^2+2xz^2)\vec{i}+(2xy-z)\vec{j}+(2x^2z-y+2z)\vec{k}$ is irrotational and hence find its scalar potential

9 Prove that $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ is a conservative field and find the scalar potential of $\vec{F}$.

10 Prove that $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ is irrotational and hence find its scalar potential.

11 Prove that $\vec{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+3xz^2\vec{k}$ is irrotational and find its scalar potential.

12 Verify Green's theorem for $\vec{F}=(x^2+y^2)\vec{i}-2xy\vec{j}$ taken around the rectangle bounded by the lines x = ±a, y=0 and y=b.

13 Using Green's theorem in a plane $\int_C\left[x^2(1+y)dx+(x^3+y^3)dy\right]$ where C is the square formed by $x=\pm1$ and $y=\pm1$.

14 Appply Green's theorem to evaluate $\int_c (xy-x^2)dx + x^2y\,dy$ along the closed curve C formed by y=0, x=1 and y=x.

15 Using Greens theorem, evaluate $\int_C\left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the boundary of the triangle formed by the lines x=0,y=0, x+y=1 in the xy plane.

16 Verify Stoke's theorem for $\vec{F}=(x^2-y^2)\vec{i}+2xy\vec{j}$,where S is the rectangle in the xy-plane formed by the lines x=0 ,x=a,y=0 and y=b.

17 Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{i}+2xy\vec{j}$ where S is the rectangle in the xy-plane formed by the lines x=0,x=a,y=0,y=b.

18 Verify Divergence theorem for $\vec{F}=4xz\vec{i}-y^2\vec{j}+yz\vec{k}$ taken over the cube bounded by the planes x=0,x=a,y=0,y=a,z=0,z=a.

19 Verify Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$, wher S is the surface of the cuboid formed by the planes $x=0, x=a, y=0, y=b, z=0, z=c$

20 Verify Gauss divergence theorem for $\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$. and S is the surface of the rectangular parallelepiped bounded by $x=0, x=a, y=0, y=b, z=0\, and\, z=c$.

21 Verify Gauss divergence Theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ taken over the cube bounded by the planes x= 0, x = 1, y = 0, y = 1, z = 0, z = 1

22 Evaluate $\iint\vec{F}.\hat{n}$ dS using Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ taken over the cube bounded by the planes x=0, y=0, z=0, x=1, y=1, z=1.

23 Find the values of constants a,b,c so that the maximum value of the directional derivative of $\varphi=axy^2+byz+cz^2x^3$ at (1,2,-1) has a magnitude 64 in the direction parallel to z-axis.

24 Find the directional derivative of $\phi=4xz^2+x^2yz$ at(1,-2,1) in the direction of $2\vec{i}+3\vec{j}+4\vec{k}$

25 Find 'a' and 'b' so that the surfaces $ax^3-by^2z=(a+3)x^2$ and $4x^2y-z^3=11$ cut orthogonally at 2, -1,-3)