PART-A

1. **Write the iterative formula of Newton-Raphson method.**

Solution:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

2. Newton Raphson method is also known as

Solution:

Method of Iteration (or)

Newton's Iteration Method.

3. Derive Newton's algorithm for finding the p^{th} root of a number N.

Solution:

If $x = N^{\frac{1}{p}}$

Then $X^p = N$ $\Rightarrow X^p - N = 0$ **is the equation.**

$\therefore f(x) = x^p - N$

$f'(x) = px^{p-1}$

By Newton's Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^p - N}{px_n^{p-1}}$$

$$= \frac{px_n^p - x_n^p + N}{px_n^{p-1}}$$

$$= \frac{(p-1)x_n^p + N}{px_n^{p-1}}$$

4. Show that the Newton's Raphson formula to find $\sqrt{a}$ can be expressed in the form

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{a}{x_n}\right], n = 0,1,2....$$

Solution:

If $x = \sqrt{a}$

Then $x^2 - a = 0$ is the equation.

$$\therefore f(x) = x^2 - a$$

$$f'(x) = 2x$$

By Newton's Raphson rule for n^{th} iterate,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - a}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + a}{2x_n}$$

$$= \frac{x_n^2 + a}{2x_n}$$

$$= \frac{1}{2}\left[x_n + \frac{a}{x_n}\right], n = 0,1,2...$$

5. What are the merits of Newton's method of iteration?

Solution:

Newton's method is successfully used to improve the result odtained by other methods. It is applicable to the solution of equations involving algebrical functions as well as transcendental functions.

6. State the order of convergence and convergence condition for Newton's Raphson method.

Solution:

Order of convergence is 2.[ie quadratic]

Condition for convergence is $|f(x)f''(x)| < |f'(x)|^2$

7. What is the condition for applying the fixed point iteration to find the real root?

Solution:

Let x=r be a root of x=g(x). Let I be an interval combining the point x=r.

The condition for fixed point iteration method is $|g'(x)|<1$ for all x in I, the sequence of approximation $x_0, x_1, \ldots, x_n$ will converge to the root r.

8. Check whether the fixed point method is applicable to the equation $x^3-2x+5=0$

Solution:

$$f(x)=x^3-2x+5=0$$

$$f(0)=-5=-ve$$

Given $$f(1)=-6=-ve$$

$$f(2)=-1=-ve$$

$$f(3)=16=+ve$$

The root lies between 2 & 3.

The given equation f(x) =0 is written as x=g(x)

$$x^3=2x+5$$

$$x=(2x+5)^{\frac{1}{3}}=g(x)$$

$$\Rightarrow g'(x)=\frac{1}{3}(2x+5)^{-\frac{2}{3}}.2$$

$$=\frac{2}{3}\frac{1}{(2x+5)^{\frac{2}{3}}}$$

$$|g'(x)|=\left|\frac{2}{3}\frac{1}{(2x+5)^{\frac{2}{3}}}\right|<1$$

We can apply fixed point iteration method..

9. What is the order of convergence for fixed point iteration?

Solution:

The convergence is linear and the order of convergence is 2.

10. Write the two method to solve simultaneous linear algebraic equations:

Solution:

By Direct Method:

1)Gauss Elimination Method and

2) Gauss Jordan Method

By Indirect (or) Iterative Method:

1)Gauss Jacobi Method and

2) Gauss Seidal Method

11. Compare Gauss elimination & Gauss Jordan method.

Gauss Elimination Method	Gauss Jordan Method
1)Coefficient matrix is transformed into upper triangular matrix.	Coefficient matrix is transformed into upper diagonal matrix
2)Direct method,need the back substitution method to obtain the soltution	Indirect method,no need of back substitution method

12. Write a sufficient condition for Gauss-seidal & Jacobi method to converge.

Solution:

Let the linear equation be,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b3y + c_3z = d_3$$

Then sufficient condition is

$$|a_1| > |b_1| + |c_1|$$
$$|b_2| > |a_2| + |c_2|$$
$$|c_3| > |a_3| + |b_3|$$

ie The coefficient of matrix should be diagonally dominant.

13. State the iterative formula for Gauss Jacobi method.

Solution:

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$
$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$
$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

with the initial condition x=y=z=0.

14. State the iterative formula for Gauss Seidal method.

Solution:

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$
$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$
$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

with the initial condition y=z=0.

15. When will iteration method succeed?

Solution:

Iteration method may succeed,the equation of system must contain one large coefficient and it should be along the leading diagonal of the matrix of the coefficient.

16. Whether the given system of equation is solvable using iterative method.

Solution:

$$x + 3y + 52z = 173.61$$
$$41x - 2y + 3z = 65.46$$
$$x - 27y + 2z = 71.31$$

As the sufficient condition is not satisfied by the system of equations[ie the coefficient matrix is not diagonally dominant],we write the equation as,

$$41x - 2y + 3z = 65.46$$
$$x - 27y + 2z = 71.31$$
$$x + 3y + 52z = 173.61$$

Now the diagonal elements are dominant in the coefficient matrix is solvable using iterative method.

17. Compare Gauss Jacobi and Gauss seidal methods.

Solution:

Gauss Jacobi method	Gauss Seidal method
1)Convergence rate is slow	The rate of convergence of Gauss Seidal method is roughly twice that of Gauss Jacobi.
2)Iterative method	Iterative method
3)Condition for convergence is the coefficient matrix is diagonally dominant	Condition for convergence is the coefficient matrix is diagonally dominant

18. Why Gauss-Seidal method is a better method than Jacobi's iterative method.

Solution:

Since the current value of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration, the convergence in Gauss Seidal method will be more rapid than in Gauss Jacobi method.

19. State the merits and demerits of Elimination and Iterative methods for solving a system of equations.

Solution:

Elimination method involves a certain amount of fixed computation and they are exact solutions.

Iterative method is those in which the solution is got by successive approximations and they are approximate solutions.

20. Find the inverse of the coefficient matrix by Gauss Jordan elimination method

$5x-2y=10; 3x+4y=12.$

Solution:

The coefficient matrix is

$$A=\begin{bmatrix}5 & -2\\ 3 & 4\end{bmatrix}$$

$$[A,I]=\begin{bmatrix}5 & -2 & 1 & 0\\ 3 & 4 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -2/5 & 1/5 & 0\\ 3 & 4 & 0 & 1\end{bmatrix} \qquad R_1\to R_1/5$$

$$=\begin{bmatrix}1 & -2/5 & 1/5 & 0\\ 0 & 26/5 & -3/5 & 1\end{bmatrix} \qquad R_2\to R_2-3R_1$$

$$=\begin{bmatrix}1 & -2/5 & 1/5 & 0\\ 0 & 1 & -3/26 & 5/26\end{bmatrix} \qquad R_2\to R_2*5/26$$

$$=\begin{bmatrix}1 & 0 & 2/13 & 1/13\\ 0 & 1 & -3/26 & 5/26\end{bmatrix} \qquad R_1\to R_1+(2/5)R_2$$

$$\therefore A^{-1}=\begin{bmatrix}2/13 & 1/13\\ -3/26 & 5/26\end{bmatrix}$$

$$=\frac{1}{26}\begin{bmatrix}4 & 2\\ -3 & 5\end{bmatrix}$$

21. Find the power method, the largest Eigen value of $\begin{bmatrix}4 & 1\\ 1 & 3\end{bmatrix}$ correct to 2 decimal places, choose $[1,1]^T$ as the initial eigen vector.

Solution:

Let

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = 5X_2$$

$$AX_2 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 4.8 \\ 3.4 \end{bmatrix} = 4.8\begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = 4.8X_3$$

$$AX_3 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.71 \end{bmatrix} = \begin{bmatrix} 4.71 \\ 3.13 \end{bmatrix} = 4.71\begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = 4.71X_4$$

$$AX_4 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 4.67 \\ 3.01 \end{bmatrix} = 4.67\begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = 4.67X_5$$

$$AX_5 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 4.65 \\ 2.95 \end{bmatrix} = 4.65\begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = 4.65X_6$$

$$AX_6 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.63 \end{bmatrix} = \begin{bmatrix} 4.63 \\ 2.89 \end{bmatrix} = 4.63\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.63X_7$$

$$AX_7 = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = \begin{bmatrix} 4.62 \\ 2.86 \end{bmatrix} = 4.62\begin{bmatrix} 1 \\ 0.62 \end{bmatrix} = 4.62X_8$$

Eigen value=4.62 &corresponding Eigen vector= $\begin{bmatrix} 1 \\ 0.62 \end{bmatrix}$.

22.Determine the largeset Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ correct to two decimal places using power method.

Solution:

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_2$$

$$AX_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2X_3$$

Largest Eigen value=2 and Corresponding Eigen vector=$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.