SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECE301 – IMAGE PROCESSING AND COMPUTER VISION

III B.E. ECE / V SEMESTER

UNIT 1 – DIGITAL IMAGE FUNDAMENTALS AND TRANSFORMS

TOPIC – DISCRETE COSINE TRANSFORM

DISCRETE COSINE TRANSFORMS

What is Transform Coding?

- Transform coding constitutes an integral component of contemporary image/video processing applications.
- Transform coding relies on the premise that pixels in an image exhibit a certain level of correlation with their neighboring pixels
- Similarly in a video transmission system, adjacent pixels in consecutive frames2 show very high correlation. Consequently, these correlations can be exploited to predict the value of a pixel from its respective neighbors.
- Transformation is a lossless operation, therefore, the inverse transformation renders a perfect reconstruction of the original image.

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Why Transform Coding?

- Better image processing
  - Take into account long-range correlations in space
  - Conceptual insights in spatial-frequency information. what it means to be “smooth, moderate change, fast change, …”
- Fast computation
- Alternative representation and sensing
  - Obtain transformed data as measurement in radiology images (medical and astrophysics), inverse transform to recover image
- Efficient storage and transmission
  - Energy compaction
  - Pick a few “representatives” (basis)
  - Just store/send the “contribution” from each basis

DISCRETE COSINE TRANSFORMS

The Discrete Cosine Transform

- Discrete Cosine Transform (DCT) has emerged as the image transformation in most visual systems. DCT has been widely deployed by modern video coding standards, for example, MPEG, JVT etc.
- It is the same family as the Fourier Transform
  - Converts data to frequency domain
- Represents data via summation of variable frequency cosine waves.
- Captures only real components of the function.
  - Discrete Sine Transform (DST) captures odd (imaginary) components → not as useful.
  - Discrete Fourier Transform (DFT) captures both odd and even components → computationally intense.

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Mathematical Basis

- 1D DCT:

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{\pi(2x+1)u}{2N}\right]$$

Where:

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & for \quad u = 0 \\ \sqrt{\frac{2}{N}} & for \quad u \neq 0. \end{cases}$$

- 1D DCT is ***O(n^2)***

- 2D DCT:

$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{\pi(2x+1)u}{2N}\right] \cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

  - Where α(u) and α(v) are defined as shown in the 1D case.
  - 2D DCT is ***O(n^3)***

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Properties of DCT:

- De correlation
- Energy Compaction
- Separability
- Symmetry
- Orthogonality

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De correlation:

- The principle advantage of image transformation is the removal of redundancy between neighboring pixels. This leads to uncorrelated transform coefficients which can be encoded independently.
- The amplitude of the autocorrelation after the DCT operation is very small at all lags. Hence, it can be inferred that DCT exhibits excellent de correlation properties.

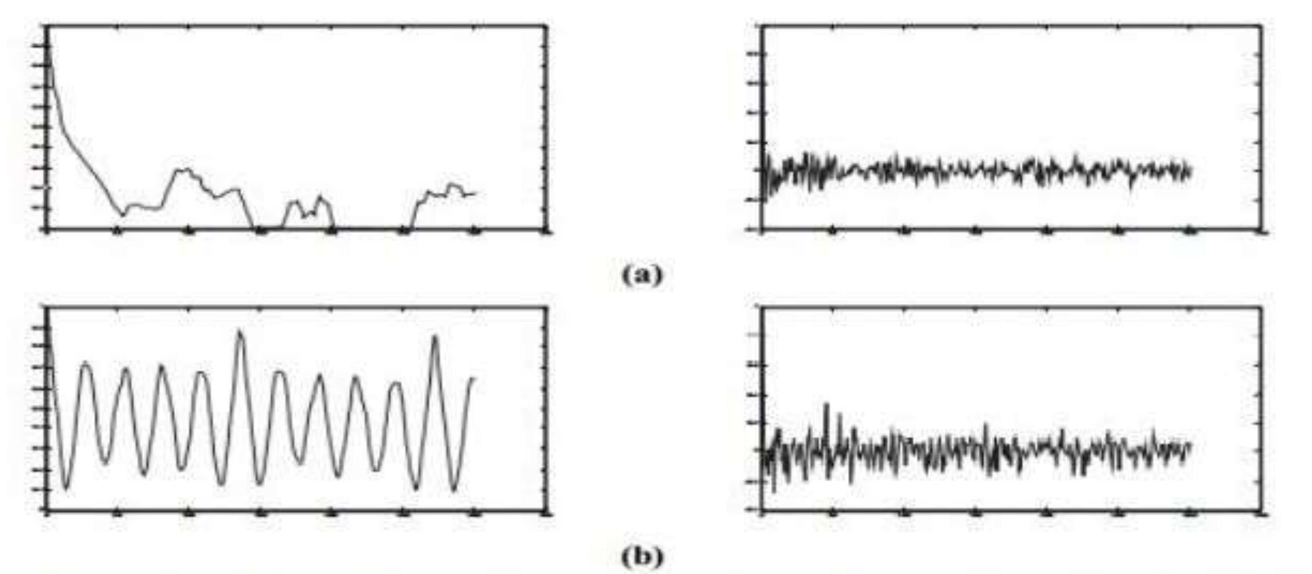

Figure 5. (a) Normalized autocorrelation of uncorrelated image before and after DCT; (b) Normalized autocorrelation of correlated image before and after DCT.

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Energy Compaction:

- Efficiency of a transformation scheme can be directly gauged by its ability to pack input data into as few coefficients as possible. This allows the quantizer to discard coefficients with relatively small amplitudes without introducing visual distortion in the reconstructed image. DCT exhibits excellent energy compaction for highly correlated images.

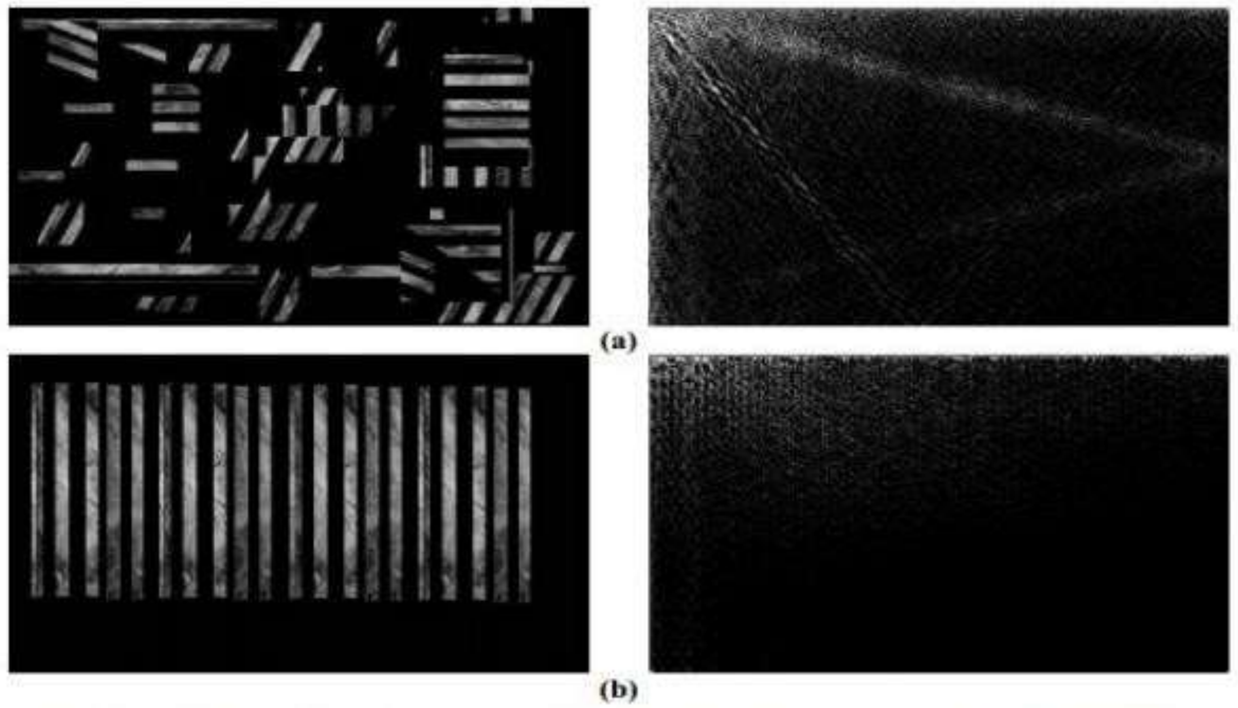

Figure 6. (a) Uncorrelated image and its DCT; (b) Correlated image and its DCT.

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Separability:

- The principle advantage that $C(u, v)$ can be computed in two steps by successive 1-D operations on rows and columns of an image.

$$C(u,v)=\alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{\pi(2x+1)u}{2N}\right]\cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

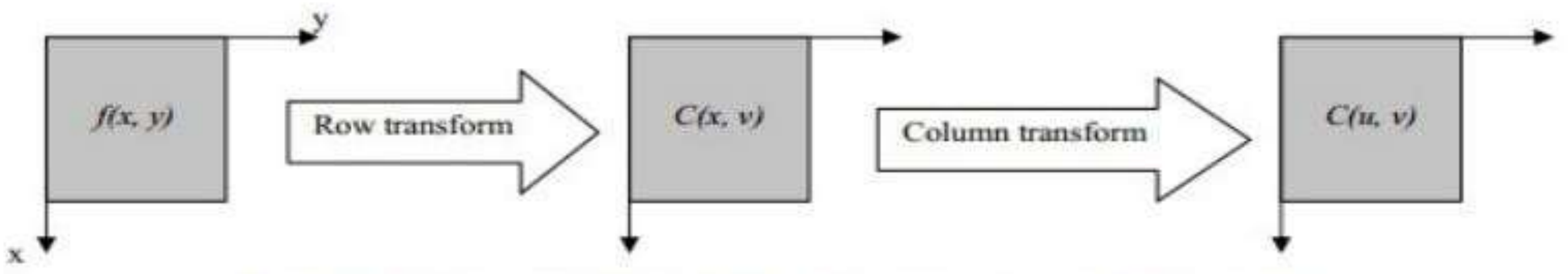

Figure 8. Computation of 2-D DCT using separability property.

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Symmetry:

- A separable and symmetric transform can be expressed in the form

$$T = AfA ,$$

where A is an $N \times N$ symmetric transformation matrix with entries

$a(i, j)$ given by,

$$a(i, j) = \alpha(j) \sum_{j=0}^{N-1} \cos\left[\frac{\pi(2j+1)i}{2N}\right],$$

and f is the $N \times N$ image matrix.

- This is an extremely useful property since it implies that the transformation matrix can be pre computed offline and then applied to the image thereby providing orders of magnitude improvement in computation efficiency.

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Orthogonality:

- The inverse transformation as

$$f = A^{-1}\, T\, A^{-1} .$$

DCT basis functions are orthogonal. The inverse transformation matrix of A is equal to its transpose i.e.

$$A^{-1} = A^{T}.$$

Therefore, and in addition to it de correlation characteristics, this property renders some reduction in the pre-computation complexity

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Significance/Where is this DCT used?

- Image Processing
  - Compression - Ex.) JPEG
  - Scientific Analysis - Ex.) Radio Telescope Data
- Audio Processing
  - Compression - Ex.) MPEG – Layer 3, aka. MP3
- Scientific Computing / High Performance Computing (HPC)
  - Partial Differential Equation Solvers

Thank you!