SNS COLLEGE OF TENHNOLOGY

(AN AUTONOMOUS INSTITUTION)

UNIT-III – COMPLEX VARIABLES

1. If f(z) is an analytic function of z prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\mathbf{f(z)}|^2 = 4|\mathbf{f'(z)}|^2$
2. Determine the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y - \cos 2x}$
3. Find the analytic function w=u+iv given $V = e^{-2xy}\sin(x^2 - y^2)$
4. Show that the function $U = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find its harmonic conjugate.
5. Find the analytic function whose imaginary part is $V = e^{2x}(y\cos 2y + x\sin 2y)$
6. Prove that the function $V = e^{-x}(x\cos y + y\sin y)$is harmonic and determine the corresponding analytic function f(z)
7. If f(z)=u+iv is analytic , find f(z) given that $u + v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$
8. If f(z)=u+iv is analytic , find f(z) given that $u + v = e^x(\cos y + \sin y)$
9. Construct the analytic function u + iv given that $2u + v = e^x(\cos y - \sin y)$
10. Find the image of the circle $|z| = 2$ under the transformation w = 3z
11. Find the image of x=1 under the transformation of $w = \dfrac{1}{z}$
12. Under the transformation $w = \dfrac{1}{z}$,find the image of $|z - 2i| = 2$
13. Find the bilinear transformation that maps **0,1,**∞ of the z-plane into -5,-1,3 of the w-Plane. What are the invariant points in this transformation?
14. Find the bilinear transformation which maps ∞,i,0 onto 0,i, ∞
15. Find the bilinear transformation which maps the points z=-1,0,1 into the points

    W=0,i,3i.
16. If f(z)=u+iv is analytic , find f(z) given that $u - v = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

17. What will be the image of a circle passing through the origin in the XY plane under the transformation $w = \frac{1}{z}$?