GREEN'S THEOREM

If R is a closed region of the x-y plane bounded by a simple closed curve C and if M and N are continuous functions of x and y having continuous partial derivatives in R then

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy,$$

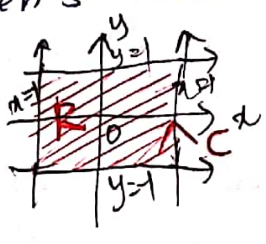
where C is a curve traversed in the anti-clockwise direction.

Problem :

Evaluate by Green's Theorem $\int_C (xy + x^2)\,dx + (x^2+y^2)\,dy$ where C is a square formed by the lines $x = 1$; $x = -1$; $y = 1$; $y = -1$.

Solution :

Using Green's Theorem, we have



$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \quad \text{①}$$

Comparing the given with the LHS of ①

$M = (xy + x^2)$; $N = (x^2 + y^2)$

we have to find,

$\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$

$$\frac{\partial M}{\partial y} = x \; ; \; \frac{\partial N}{\partial x} = 2x.$$

∴ The given integral is

$$\int_C (xy+x^2)\,dx + (x^2+y^2)\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} (2x - x)\,dx\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} x\,dx\,dy$$

$$= \int_{-1}^{1} \left[\frac{x^2}{2}\right]_{x=-1}^{x=1} dy$$

$$= \int_{-1}^{1} \left(\frac{1}{2} - \frac{1}{2}\right) dy$$

$$= \int_{-1}^{1} 0\,dy$$

$$= 0$$

∴ Given $\boxed{I = 0}$

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